

Composite System Reliability Analysis Using Particle Swarm Optimization

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Abstract—This paper presents a new approach to composite system reliability analysis. This approach is based on multi-objective particle swarm optimization (PSO), which is used as an intelligent method for scanning the state space. This paper adapts traditional binary PSO to bi-objective binary PSO using load curtailment and state probability as the two objectives to better control the particle dynamics. Another novel feature of this method is that while it utilizes information from failure states to estimate loss of load indices, it also uses information from acceptable states that are encountered to accelerate the convergence of the estimate. The method is demonstrated on the modified IEEE Reliability Test System. It is also shown to compare very favorably with Monte Carlo Simulation.

Keywords— *composite system, reliability evaluation, multi-objective particle swarm optimization.*

I. INTRODUCTION

Composite system reliability evaluation consists of determining the reliability of a power system taking into account both its generation and its transmission components. It can also be viewed as a combined adequacy and security analysis, and is appropriate in systems where the internal transmission constraints play a significant role in determining overall system reliability [1]. In general, composite system reliability methods permit the determination of system-wide as well as locational reliability indices, and can be used for planning and operation studies.

Traditionally, analytical [2–4] and Monte Carlo [5], [6] methods have been used for composite system reliability evaluation. While analytical techniques are potentially faster, they tend to get difficult as system size and complexity increases. Monte Carlo methods, on the other hand, afford tremendous modeling flexibility, and can be used for systems with large size and complexity. However, Monte carlo methods tend to be extremely time consuming, particularly for reliable systems. For this reason, in recent years several methods have been proposed, based on so called computational intelligence or heuristic search methods. Recent work has used genetic algorithms (GA) [7], [8], artificial neural networks (ANN) [9], and particle swarm optimization (PSO) [10]. These approaches have generally performed well

in terms of computational accuracy, and have often been reported to be faster than traditional, i.e., analytical and Monte Carlo based methods.

This paper presents an innovative method of computing reliability indices of composite systems, and is based on a multi-objective particle swarm formulation. It uses two objectives—load curtailment and failure probability—as two objectives in the PSO fitness to exercise better control over the particle dynamics. A further innovation in this work consists of using information from acceptable system states to accelerate the convergence of indices determined from system failure states. Hence in this approach, both loss of load probability (LOLP) and availability (denoted “NLOLP” hereafter) of the system are calculated during the search process. Based on the trends of the successive estimates of the two indices, accelerated estimates of the LOLP are obtained using three different methods.

The IEEE Reliability Test System (RTS) [11] is used for testing the method. However, since the original IEEE-RTS was shown to be of limited use for composite system studies on account of its transmission system being too strong [12], its data was suitably modified. The proposed method is shown to compare favorably with prior methods.

II. LOSS OF LOAD PROBABILITY ESTIMATION

This work uses the LOLP as the reliability index of composite system, since it reflects the risk of the generation capacity failing to satisfy the load demand in the presence of transmission constraints and contingencies. Several factors have impact on LOLP, for example, component failure rate, network topology, generator dispatch, load demand, etc. The system LOLP can be obtained by following formula:

$$LOLP = \sum_{i \in S} p_i \quad (1)$$

$$p_i = \prod_{j \in R(S_i)} (1 - PG_j) \prod_{j \in W(S_i)} PG_j \prod_{j \in R(S_i)} (1 - PL_j) \prod_{j \in W(S_i)} PL_j \quad (2)$$

where

S is the set of states for which there is load curtailment;

p_i is probability of state i ;

$R(S_i)$ and $W(S_i)$ are the component sets in up or down states, respectively;

PG_j or PL_j is the failure probability of generating unit j or transmission line j .

In this work, we use another index that we denote as $NLOLP$, which estimates the system availability. According to our definition, $NLOLP$ is always equal to $1 - LOLP$.

The system state is decided by generating unit and line states. Most components in a composite system, such as generating units, lines, etc, can be modeled as two states units, i.e., any component can be “up” or “down”. So the total number of states in a system will be 2^m where m is the number of components in the system. For a system of realistic size, the number of states will be extremely large; however, most of those states play a small role in reliability evaluation. Some heuristic approaches get enough precision through visiting sufficient states of high probability and this approach has been shown to be effective.

III. THE DC FLOW MODEL AND LOAD CURTAILMENT

State analysis is the main part of composite system reliability evaluation, and it can be divided into three steps: (1) first calculate power flow, and decide if there are any failure situations, such as system disconnection and operation constraint violation; (2) next try to fix the failure through adjusting generator output; (3) finally, minimize load curtailment using the following redispatch model [13].

$$\text{Min } \sum_{i=1}^N C_i \quad (3)$$

Subject to:

$$B\theta + G + C = D \quad (4)$$

$$G^{\min} \leq G \leq G^{\max} \quad (5)$$

$$0 \leq C \leq D \quad (6)$$

$$|f| \leq f^{\max} \quad (7)$$

where

N is the number of system nodes;

B is the nodal admittance matrix;

θ is the vector of bus voltage angles;

G is the vector of dispatched generation output at all buses and lies in the interval $[G^{\min}, G^{\max}]$;

D and C are vectors of bus load demand and bus load curtailment, respectively;

f is the vector of branch power flows and $f^{\max}$ is the vector of branch capacities.

If we define G and θ as the variables of this optimization model and E the row vector $[1, 1, \dots, 1]_{1 \times N}$, and then (3) can be rewritten as (8), shown below, by the substitution of (4). By using simple algebraic operations, constraints (4)–(7) can be converted into equation (9), where I is a unit diagonal matrix, A is the branch-node incidence matrix, and b is the branch admittance matrix. So the load curtailment problem can be transformed into a linear programming problem.

$$\text{Min } [-E \quad -EB] \begin{bmatrix} G \\ \theta \end{bmatrix} + ED \quad (8)$$

Subject to:

$$\begin{bmatrix} -I & 0 \\ I & 0 \\ I & B \\ -I & -B \\ 0 & bA \\ 0 & -bA \end{bmatrix} \begin{bmatrix} G \\ \theta \end{bmatrix} \leq \begin{bmatrix} -G^{\min} \\ G^{\max} \\ D \\ 0 \\ f^{\max} \\ f^{\max} \end{bmatrix} \quad (9)$$

IV. DESCRIPTION OF THE APPROACH

A. Bi-Objective Particle Swarm Optimization

The particle swarm optimization (PSO) was originated from simulation of social animal's behavior, such as bird and fish, and was put forward by Kennedy and Eberhart [14]. A discrete binary particle swarm optimization (BPSO) was proposed by them for the purpose of solving combinatorial optimization problems [15].

PSO has been shown to have good performance in search ability, thus we use BPSO as a state searching tool and try to find states which have an influence on system reliability index as much as possible in a large state space. The best case is where particles only visit states which cause load curtailment and have high probability at the same time. The problem is that high consequence states (where many components are failed) usually have low probability. There are two directions of particles during the searching process: one is related to load curtailment and the other is related to state probability. In order to solve this problem, Particles that we design in this paper will take both directions into consideration.

The particle velocity v_i is the direction of movement of particle i from position x_i in one time step, and is given by the following [15]:

$$\begin{aligned} v_i = & v_i + \text{rand}() \times c_1 \times (Ppbest_i - x_i) \\ & + \text{rand}() \times c_2 \times (Pgbest - x_i) \\ & + \text{rand}() \times c_3 \times (Cpbest_i - x_i) \\ & + \text{rand}() \times c_4 \times (Cgbest - x_i) \end{aligned} \quad (10)$$

where

$Ppbest_i$ is the best position, in terms of highest state probability, that particle i has ever encountered;

$Pgbest$ is the position with the highest state probability encountered by any particle;

$Cpbest_i$ is the best position, in terms of highest load curtailment, that particle i has ever encountered;

$Cgbest$ is the position with the highest load curtailment encountered by any particle;

c_1, c_2, c_3 and c_4 are acceleration factors;

$\text{rand}()$ is a uniformly distributed value in the interval $[0, 1]$.

B. The Solution Algorithm

Our approach is mainly composed of three parts: state sampling, reliability index counting, and state analysis as

explained in section III. The whole reliability evaluation process can be divided into the following steps:

- 1) Initialize the particle positions x and velocities v . Binary numbers can be used to represent particles, 1 or 0 represent up or down status respectively. The number of bits equals to total number of generating units and lines. So each particle is corresponding to a certain system state.
- 2) For each particle in position x_i , use (2) to compute the exact state probability.
- 3) Determine whether particle probability p_i is smaller than ε , where ε is a predetermined threshold (very small) value. If so, go to the next step; otherwise, go to the next particle and continue to search. The threshold ε is a predetermined positive value and its value is determined according to two aspects: on one hand, it is related to computing accuracy directly; on the other hand, as some states with low probability plays a great role in the decision of particle direction, so ε is unlikely be a large number.
- 4) Compute the equivalent decimal value of each particle to enable storage in the state database. In order to avoid the situation that equivalent decimal value is too large, all bits in x_i can be divided into several groups, and then the equivalent decimal value can be computed for each group.
- 5) If the state (or particle position) already exists in database, set the probability of this particle to a small enough value (in our approach, we use ε) and load curtailment to zero, and then go to step 9). If not, add decimal value, x_i , p_i , and load curtailment to database and go to the next step.
- 6) As x_i contains status information of generating units and lines, update matrices $B(x_i)$, $f^{max}(x_i)$ and $G^{max}(x_i)$ accordingly. Then analyze state according to the approach proposed in section III. If load shedding is inevitable, using optimal load curtailment model to cut load as shown in (8) and (9).
- 7) If there is load shedding, then add p_i to the *LOLP* estimate, otherwise add p_i to the *NLOLP* estimate.
- 8) If all particles have been processed, go to next step; Otherwise, go to step 3).
- 9) Initialize or update $Ppbest_i$, $Pgbest$, $Cpbest_i$ and $Cgbest$, and then update particle velocity v_i using (10). Bit d of Particle position x_i is changed by following rule.
$$\begin{cases} x_{id} = 1, & rand() < \frac{1}{1 + e^{-v_{id}}} \\ x_{id} = 0, & otherwise \end{cases} \quad (11)$$
- 10) Test for convergence; if the estimate has converged, stop the iteration when the index is stable; otherwise, go to step 2).

C. Convergence Criterion

In the approach described above, two opposing criteria are used to control the particle dynamics and ensure that the particles do not rapidly collapse to one corner of the search space. It is new appropriate to define a criterion for stopping the search process. Two estimates, *LOLP* and *NLOLP* are derived. Both *LOLP* and *NLOLP* are monotonically increasing

functions of n , the number of visited states. In each iteration k , we perform the following:

- 1) It should be apparent that the values *LOLP* and $1 - \text{NLOLP}$ both estimate the same index, and as the search progresses their trajectories approach each other. Based on the sampled states in the interval $[1, n_k]$, fit the data to power functions, thereby smoothing the trajectories of these curves.
- 2) In the interval $[n_{k-k'}, n_k]$, compute the tangents to both curves, and then find the intersection point of the two tangents. k' is a default positive integer which is always less than k and dependent on sampling accuracy. We will take those intersection points as the final estimated value of the composite system reliability index (*LOLP*).
- 3) If the relative change of estimated value stays below the threshold value δ after visiting K new sampling states, stop the search process.

V. CASE STUDY

The modified IEEE reliability test system [12] is chosen to test the proposed approach. In this modified system, the peak annual load is 5130 MW and the total generating capacity is 6810 MW. For the purpose of comparison with prior methods reported in [10] and [13], the bus loads were held at their peak values.

The method described in section IV was implemented using Matlab®. The acceleration factors in (10) are taken as 0.1, 0.1, 0.01 and 0.01 respectively [10]. Probability threshold ε for selecting a state is 10^{-10} ; Iteration stopping rule is determined by two parameters: $\delta = 10^{-5}$, $K = 1000$.

Table I compares the performance of this method with those of [10] and [13]. The *LOLP* index estimated by this new approach is 0.07131, compared with 0.06784 obtained in [10] and 0.07122 obtained in [13]. However, in order to get this result, the approach described in this paper needed to search only 9985 states. Hence the computational effort required by the new method is only 23.77% and 19.31% that of the other two methods. If the result reported [13] is taken as a reference, the percentage error of the method is under 0.1264%.

TABLE I. COMPARISON OF DIFFERENT METHODS

Method	<i>LOLP</i>	n
PSO without acceleration [10]	0.06784	42001
Monte Carlo Simulation [13]	0.07122	51697
Bi-objective PSO with acceleration	0.07131	9985

This approach uses both the simulation data of *LOLP* and *NLOLP* to estimate loss of load probability. The trends of these two estimates are shown in Figure 1. It can be seen that *LOLP* increases as the search progresses, while $1 - \text{NLOLP}$ decreases. After carrying out 20 iterations ($k = 20$),

we plot both fitting curves of $LOLP$ and $1 - NLOLP$ using all visited states at interval $[n_{k-k'}, n_k]$ ($k'=10$). The intersection points of tangents are indicated with black spots which are employed to estimate loss of load probability.

From Figure 1, we can find three features of profile curve of the accelerated estimate from the intersection of the tangents: 1) It fluctuates at the beginning, but later converges better convergence than any other two fitting curves ($LOLP$ and $1 - NLOLP$). In the first 9985 states visited, neither the $LOLP$ curve nor the $1 - NLOLP$ curve is close to the system $LOLP$, but the accelerated estimate reaches the system $LOLP$ with negligible error. 2) At convergence, the sum of probability of states which have not been visited is only 0.08258. This proves that bi-objective BPSO proposed has a strong ability in seeking out states that have effectively contribute to the system $LOLP$. 3) Due to the monotonicity of the two fitting curves, sampling value of intersection point of tangents is always between corresponding $LOLP$ and $1 - NLOLP$. So this intersection point constitutes an effective accelerated estimate.

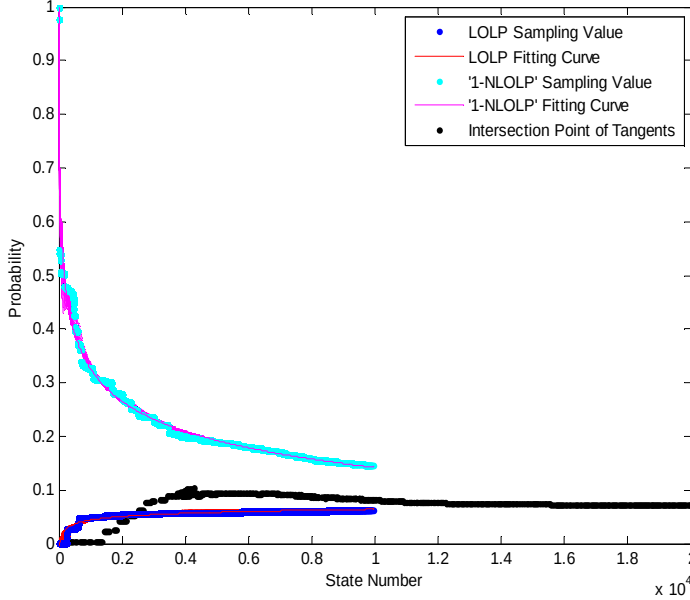


Figure 1. Trajectories of successive estimates

The trend of convergence is shown in Table II. The intersection point of the tangents to the fitting curves is used to evaluate $LOLP$ and the result is shown in column four of Table II. The second column of Table II shows the estimate from the accumulated probability of failed states and the third column shows the estimate from the accumulated values of the acceptable states (by subtracting this accumulated value from 1). It is evident that these two estimates approach the system $LOLP$ very slowly in comparison to the intersection of the tangents. The method used in [10] just uses the data in column two of Table II to estimate the reliability index ($LOLP$) and therefore it needs to search many more states to reach the same accuracy.

TABLE II. CONVERGENCE TREND

n	$LOLP$	$1 - NLOLP$	Accelerated estimate of system $LOLP$
1000	0.04842	0.32769	0.09320
3000	0.05628	0.22612	0.08509
5000	0.05823	0.19030	0.07655
7000	0.06051	0.16972	0.07345
8000	0.06084	0.15722	0.07251
9000	0.06183	0.15102	0.07173
9985	0.06327	0.14585	0.07131
10000	0.06327	0.14583	0.07131
10500	0.06397	0.14342	0.07131
11000	0.06681	0.14106	0.07131

Now we compare the proposed method, i.e., the method of intersection of tangents, to two other methods of interpolating between the estimates of $LOLP$ and $1 - NLOLP$ to obtain accelerated estimates. In one, we simply find the arithmetic mean, i.e., we use $\frac{1 - NLOLP + LOLP}{2}$ to estimate the system

$LOLP$. In the other, we assume that the system $LOLP$ bears the same ratio to the probability of the state space (which is 1.0) as does the accumulated probability of failed states to the total probability of states visited, i.e, $LOLP + NLOLP$; so the estimate of the system $LOLP$ is given by $\frac{LOLP}{LOLP + NLOLP}$.

The three accelerated estimates are compared in Table III and their convergence trends are shown in Figure 2. In the beginning, the third method converges most rapidly. However, it fluctuates within a certain degree, i.e., about $\pm 2\%$. The convergence of the second method is poor. The first method, i.e., the intersection of tangents, fluctuates a little in the beginning but shows the most stable convergence in the long run.

In other words, the method of intersection of tangents is appropriate if the calculation accuracy is of the most concern. Otherwise the third method, that of proportionate interpolation using $\frac{LOLP}{LOLP + NLOLP}$, shows the most rapid convergence.

TABLE III. COMPARISON OF METHODS OF ACCELERATED ESTIMATION

Estimator	$LOLP$	n
Intersection of tangents	0.07131	9985
Mean Value	0.07825	49902
$LOLP/(LOLP + NLOLP)$	0.07179	2008

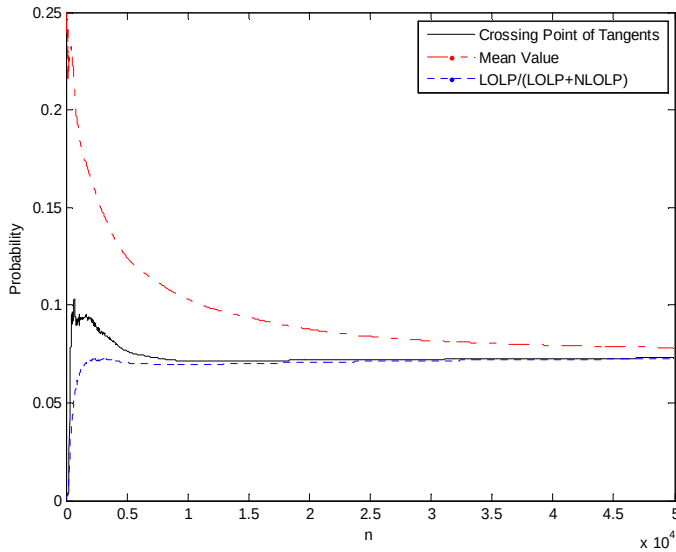


Figure 2. Convergence of accelerated estimates

VI. CONCLUSION

This paper presents an intelligent approach for the assessment of loss of load probability of a composite system. Bi-objective PSO is used as a search tool in state sampling. The use of opposing objectives of high failure probability and high loss of load for better control of swarm dynamics is shown to possess superior search ability. The proposed method makes use of all visited states, and not just the system failure states, to achieve an accelerated estimate of the system LOLP. The proposed approach thus possesses two innovative features: the implementation of bi-objective PSO and a method of accelerated convergence.

The proposed method is tested on the modified IEEE RTS system, and the results are compared with those obtained by other methods in the literature. It is shown that the proposed approach has excellent convergence characteristics, and compares favorably with other methods. The method should prove to be suitable for composite reliability analysis of large and complex power systems. Further research toward the determination of other reliability indices such as loss of load frequency is in progress and will be reported in due course.

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