

A Hybrid Approach to Addressing the Problem of Noncoherency in Multi-Area Reliability Models

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Abstract

Most analytical approaches to multi-area reliability evaluation depend on the coherency of the state space. If a network model is assumed which accommodates Kirchhoff's voltage law, then the state space becomes noncoherent when tie-line capacity variations occur. In such cases the only recourse has been to use Monte Carlo simulation, which often tends to be extremely time consuming. The method used in this paper uses a hybrid approach which assumes tie-line capacities not to change during an analytical phase, and then considers all other tie-line contingencies during a simulation phase. A DC flow model is assumed. The method is tested on 5-area and 10-area cases, and is shown to be several times faster than a purely simulation based approach.

Keywords: reliability, interconnected systems, coherency, Monte Carlo simulation

1 Introduction

Present day power systems almost invariably operate under interconnected conditions. The evolution of sophisticated control methodologies has been accompanied by an increase in the complexity of interconnections and exchange policies. These and other developments foreseeable in the immediate future, such as competitive markets, non-utility generation, and open transmission access, must be accompanied by the development of more powerful multi-area tools for the planning and analysis of large and complex interconnected systems.

Several methods have been proposed [1–9] for multi-area reliability analysis of looped configurations. Some of the proposed methods are purely analytical [1, 5], some are entirely based on Monte Carlo simulation [3, 4], while others are hybrid in nature [2, 6, 7, 8], with an analytical phase followed by a simulation phase. These methods have used two types of transmission models — the *capacity flow model*, and the *DC flow model*. The capacity flow

model [1, 2, 6, 7] uses only Kirchhoff's current law in the power balance and line flow constraints, ignoring the effect of tie-line admittances, while the DC flow model [5, 8] accommodates both Kirchhoff's current and voltage laws.

The transmission model used to describe the system behavior determines whether or not the resulting system model will be reliability-coherent. A system is said to be coherent if the failure or degradation of a component cannot in any way result in an improvement in system performance, and, likewise, the restoration of a failed component cannot in any way deteriorate the system performance. It is known [5, 8, 9] that when a DC load flow model is used in multi-area reliability calculations, the system is coherent for generation capacity changes but not under transmission capacity variations. This means that if the system is in an acceptable state and some tie capacities move to states with higher capacities, one or more areas may have loss of load. Similarly if the system is in system loss of load state and some tie-lines change to lower capacities, some areas which had loss of load may meet their load demands. This is because changes in capacities cause changes in line impedances, causing the network to redistribute loads. This noncoherency causes problems in the grouping of states [1] in the sets. The capacity flow model, however, precludes this kind of noncoherency, since the flow profile does not depend on the line susceptances, and an increase in the capacity of any line can only result in an increase in the throughput capability of the entire network. So while analytical or hybrid methods using the capacity flow model can accommodate changes in tie-line states, including tie-line failures, those using the DC flow model can deal with only a limited number of tie-line states, as explained in [8]. Thus if a multi-area model were desired which can accommodate both DC flow constraints as well as multiple tie-line states, sampling is the only recourse. However, for highly reliable systems, sampling can be extremely expensive, in terms of computational

burden. The method proposed in this paper alleviates this computational burden by using a hybrid approach, which consists of first separating out a suitably large set of acceptable states, and then subjecting the remainder of the state-space to sampling. This results in an accelerated convergence of the sampling procedure, since the removal of a large acceptable set leaves behind a set with a large proportion of loss of load probability.

The proposed method enables efficient computation of multi-area indices for larger and more complex interconnected systems, and at the same time uses a more accurate transmission model.

2 The Proposed Approach and its Justification

This section outlines the concepts which build up to the proposed approach, and then provides a theoretical justification of the hybrid methodology.

2.1 Characterization of State Space

For any given load scenario, the available area generations and the tie-line capacities will determine whether or not the area loads will be satisfied. We can, therefore, define a state space as the set of all possible combinations of generation levels and tie-line capacities.

The treatment of temporal load variations and planned outages of generators can be accommodated using clustering concepts. Clustering involves reducing the multi-area hourly load model to an equivalent cluster model comprising an arbitrary number of multi-area load vectors, each with an associated probability. Then a reference load state is defined, using which the generation model is modified for every load cluster, and these modified models are interleaved to construct an integrated generation model. Similar modification and subsequent integration is used to accommodate planned outages. The resulting generation model needs to use only one load state, which is the reference state. The method is described in detail in [7, 8].

Since clustering will finally result in a model which uses a single load level, the proposed method will be described in this section in terms of a model using a single load level. The idea can be easily extended to include multiple load levels and planned maintenance.

In general, therefore, a system with N_a areas and N_t tie-lines will have a discrete state space of dimension $(N_a + N_t)$, each axis consisting of the area generation or tie-line capacity levels, zero levels included.

2.2 The Analytical Phase

As stated earlier, the proposed method comprises an analytical phase and a simulation phase. The analytical phase consists of identifying and removing suitably large portions of the state space, where no failure states are present. This results in an accelerated convergence of the simulation phase, since the remainder of the state space contains a larger proportion of loss of load states. The computation of reliability indices is performed in the simulation phase.

The analytical method implemented here uses the idea of partitioning vectors, and applies it in a form which is analogous to the idea of state space decomposition [2, 6, 7, 8]. The original state space is first treated as an unclassified set (U -set); based on the maximum capacity levels available in this U -set, the system load curtailment is minimized; then the combination of the lowest capacity states which yield zero total curtailment constitutes a partition vector which will be called the u -vector. The u -vector has the property that all capacity levels between and including the u -vector and the upper boundary of the U -set will be acceptable states and will constitute an acceptable set (A -set). Using the u -vector, the original U -set is now decomposed into an A -set, and $N_a + N_t - n_1$ disjoint U -sets, where n_1 is the number of single level components in the original U -set (i.e., the maximum generation or transmission level in the U -set coincides with the corresponding minimum level). If more A -sets are desired to be removed, more of the undecomposed U -sets may be decomposed.

If for a certain U -set the minimum curtailment is not zero, no A -set can be formed, and the entire U -set is set aside for Monte Carlo simulation. Otherwise when an arbitrarily large portion of the state space has been removed, decomposition is terminated, and the undecomposed U -sets are subjected to simulation. A large proportion of the sets generated have very low probabilities; thus their number can be kept manageable by deleting sets with very low probabilities (e.g., less than 10^{-16}).

2.3 Addressing the Noncoherency Problem

A necessary and sufficient condition for the validity of the method of decomposition outlined above is that the A -sets generated must satisfy the coherency property. This is not a problem if a capacity flow model is used, but if a DC or AC flow model is used, then the system remains coherent for generation level changes, but not for transmission level changes, because transmission capacity changes are accompanied by line susceptance changes, which alter the flow profile, and may cause transmission ca-

capacity violations, thereby producing a failure state in an otherwise acceptable set. It is, however, not always sufficient to use the capacity flow model in multi-area reliability problems. In using DC or AC flow models, therefore, the noncoherency problem must be addressed. The proposed method addresses this problem by performing decomposition over the generation levels, holding the transmission levels at the maximum capacity states. In other words, every time the u -vector is determined, the components of the u -vector corresponding to transmission lines are set at the maximum capacity levels.

This is explained as follows. Consider a two-area system, with area generations G_1 and G_2 , connected by a single tie-line of capacity T , as shown in FIGURE 1. Let T assume three discrete values, 0, t_1 , and t_2 . While G_1 and G_2 also assume discrete values, they are treated as continuous, for convenience. Then the state space of the system may be represented as shown. Now the subspace corresponding to $T = t_2$, i.e., the maximum capacity level of T , can be subjected to decomposition, since on this plane T is constant. Note that the term ‘subspace’ is loosely applied here, and refers to any set of points in the state space. After a desired number of A -sets have been removed from this plane by decomposition, the remainder of this plane, and the other two planes, are subjected to Monte Carlo simulation. It is true that any of the other two planes could have been selected for decomposition; however, since tie-lines are generally highly reliable, A -sets on the plane $T = t_2$ will have much higher probability than on any of the other planes, and decomposition will be considerably more effective on this plane.

The above argument is equally valid for larger systems: the transmission system is generally far more reliable than the generation system, and therefore the above approach is able to remove fairly large portions of the probability space.

2.4 The Simulation Phase

At termination of decomposition, all the disjoint undecomposed U -sets form the residual subspaces, and these are subjected to proportional sampling for determination of the reliability indices. First, the probabilities of all the residual subspaces are computed. Then each sampled state is selected as follows.

1. a subspace is randomly selected in such a manner that the probability of that subspace being selected is proportional to the probability of the subspace

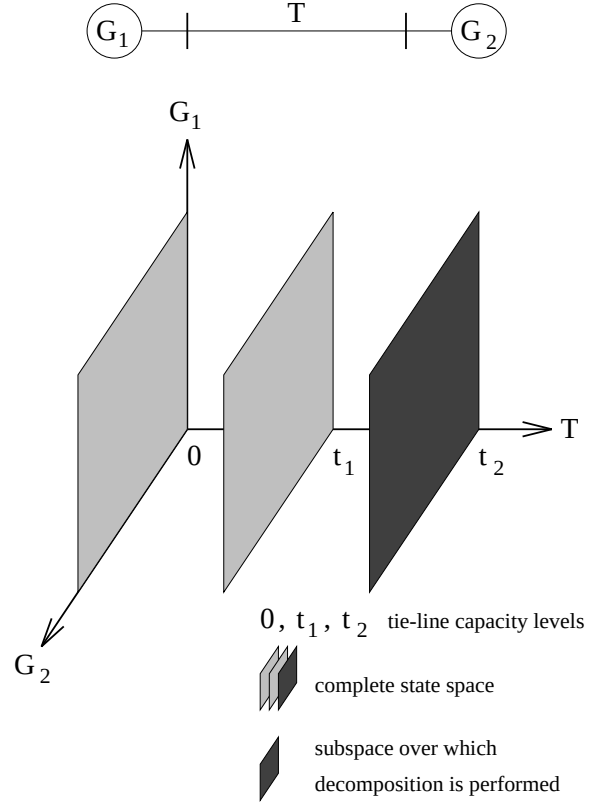


FIGURE 1: TWO-AREA SYSTEM AND STATE SPACE THEREOF

2. if a cluster model is used, a load cluster is randomly selected; otherwise the same load scenario is used
3. within the selected subspace, proportional sampling is used to select a generation level at every area, and a transmission level for every tie-line

The generation-transmission-load scenario thus selected constitutes the sampled state, which is tested for acceptability. If the state turns out to be a failure state, then the system and bus indices are updated. This is continued till the coefficients of variation of selected indices drop below prespecified tolerances.

The indices calculated in the work reported in this paper are the *Loss of Load Expectation (LOLE)* and *Expected Unserved Energy (EUE)* for the system as well as for every bus.

Notice that the question of noncoherency does not arise in the context of simulation.

2.5 Acceleration of Convergence

The justification for using the proposed method rests on the contention that it results in an acceleration

of convergence in the simulation phase, when data is collected for calculation of the reliability indices. This contention is validated as follows.

Assume that a certain level of decomposition has resulted in the removal of acceptable sets of total probability $(1 - \alpha)$, so that the probability of the residual state space is α ;

Consider calculating the *system Loss of Load Probability (LOLP)*, p ;

Let p be the estimate of p ;

Since failure states are Bernoulli distributed with probability p (a sampled state can be a failure state or a success state and the outcome of every sample is independent of others), the distribution has mean p and variance $p(1 - p)$;

Then the coefficient of variation of p is

$$\eta = \frac{1}{p} \sqrt{\frac{p(1-p)}{N}} \quad (1)$$

where N is the number of sampled states.

If sampling is performed over the residual state space of probability α , then the estimate of the *conditional LOLP* is

$$p' = \frac{p}{\alpha}$$

Then the coefficient of variation of p' is

$$\eta' = \frac{1}{p'} \sqrt{\frac{p'(1-p')}{N'}} = \frac{\alpha}{p} \sqrt{\frac{\frac{p}{\alpha}(1-\frac{p}{\alpha})}{N'}} \quad (2)$$

where N' is the number of states sampled from the residual state space.

Now if both the estimates p and p' are required to converge to the same tolerance, then

$$\frac{N'}{N} = \alpha \frac{1 - \frac{p}{\alpha}}{1 - p} \quad (3)$$

Note that (3) has been obtained by equating η and η' , i.e., the coefficients of variation of p and p' , which are actually estimates of different quantities. However, η and η' are the coefficients of variation within their respective state spaces over which sampling was performed, and it is therefore reasonable to equate them.

Note also how (3) indicates an approximate equality between the fraction N'/N and the residual probability α , since p is small. In other words, the acceleration in convergence is almost proportional to the probability of the residual state space.

It should also be realized that since all the failure states remain available for sampling in the residual

state space, and the convergence criterion remains unchanged, there is no loss of accuracy accompanying the reduction in computational effort.

3 Model Description

This section briefly describes the models used in the work reported in this paper. The generation and load models are described here in terms of single load levels; however, for the treatment of chronological load variations and planned maintenance, cluster models need to be constructed and integrated into a simultaneous decomposition framework [6, 7, 8].

3.1 Generation Model

Based on the capacity states and forced outage rates of units available in a given area, a discrete probability distribution function is constructed, for every area, using the *Unit Addition Algorithm* [10].

3.2 Transmission Line Model

The transmission line model can be constructed for every transmission line, in the form of a probability mass function, or a discrete probability distribution function, over all the capacity levels, including zero.

3.3 Load Model

The load model is constructed as a vector of size N_a , comprising the load levels at all the areas. If multiple load levels are to be accommodated, they are combined into a cluster load model [6, 7, 8], and the vector of maximum load levels is used as the reference load state in the decomposition phase.

3.4 DC Flow Model

The DC Flow model [8, 9] is used in the decomposition phase for determination of the partition vectors, and in the simulation phase to test a sampled state for acceptability.

In the decomposition phase, the u -vector is determined from the solution of the following linear programming problem:

$$\text{Loss of Load} = \text{Min} \sum_{i=1}^{N_a} C_i$$

subject to:

$$\begin{aligned} \hat{B}\theta + G + C &= D \\ G &\leq G^{\max} \\ C &\leq D \\ b\hat{A}\theta &\leq F_f^{\max} \\ -b\hat{A}\theta &\leq F_r^{\max} \\ G, C &\geq 0 \\ \theta &\text{ unrestricted} \end{aligned} \quad (4)$$

where

- N_a = number of areas
- N_t = number of tie-lines
- C = N_a -vector of area load curtailments
- C_i = i -th element of C , i.e., unsatisfied demand in area i
- D = N_a -vector of net negative injections for No Load Loss Sharing (NLLS) and actual area loads for Load Loss Sharing (LLS)
- G^{max} = N_a -vector of maximum available net positive injections for NLLS and available area generation for LLS; available generation levels coincide with upper bounds of current U -set
- F_f^{max} = N_t -vector of forward flow capacities of tie-lines
- F_r^{max} = N_t -vector of reverse flow capacities of tie-lines
- G = N_a -vector of net positive injections for NLLS and actual area generation for LLS
- θ = N_a -vector of node voltage angles
- b = $N_t \times N_t$ primitive (diagonal) matrix of tie-line susceptances
- $\hat{A}$ = $N_t \times N_a$ element-node incidence matrix
- $\hat{B}$ = $N_a \times N_a$ augmented node susceptance matrix
 $= \hat{A}^T b \hat{A}$

In the NLLS case, the solution for the G -vector needs to be modified to provide the actual area generation; in the LLS case this modification is unnecessary. The solution for the G -vector obtained from this model is basically the u -vector, unless one or more elements of G lie below the corresponding lower bounds of the current U -set, in which case they are set equal to the lower bounds.

In the simulation phase, states are randomly sampled from the undecomposed state space, as described in section 2.4, and tested for acceptability using model (4). The only difference is that the G^{max} vector is now determined from the sampled generation states. If the solution to (4) yields a zero loss of load value, the indices remain unaltered; otherwise the probability of this state is computed and the contribution of this state is added to the appropriate indices. This is continued till the coefficients of variation of the computed indices drop below prespecified tolerances.

4 Test Cases and Results

The proposed method was tested on the four configurations shown in FIGURE 2.

Every area in the above configurations is identical to the IEEE-RTS [11], with a maximum area genera-

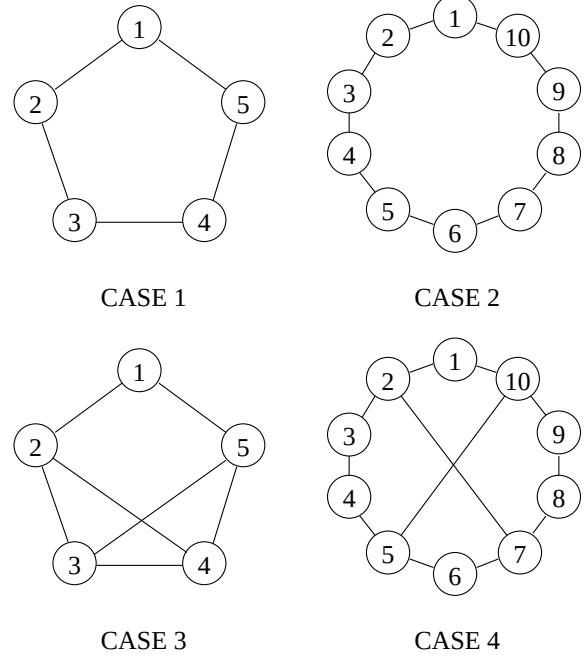


FIGURE 2: CONFIGURATION OF TEST CASES

tion of 3405 MW; the hourly load model is used with a peak of 3000 MW, i.e., with 13.5% reserve. All inter-area ties are identical; each tie-line is assumed to be a double circuit line, with each circuit of capacity 150 MW in both directions, susceptance $-60 pu$, and failure probability 0.04. In other words, each tie-line is assumed to have the states shown in TABLE I.

For all studies, 10-cluster load models were used; the generation and cluster load models were rounded off to integral multiples of 50 MW to enable proper integration into a simultaneous decomposition framework. No planned outage was considered. The NLLS policy was assumed.

TABLE I: TIE-LINE STATES

CAPACITY	SUSCEPTANCE	PROBABILITY
300 MW	$-120 pu$	0.9216
150 MW	$-60 pu$	0.0768
0 MW	$0 pu$	0.0016

The extent of decomposition was controlled by means of a threshold probability, p_0 . An U -set was taken up for decomposition only if its probability was larger than p_0 ; otherwise it was set aside for simulation. A small threshold probability therefore resulted in a large degree of decomposition; likewise, $p_0 = 1.0$ implied that the entire state space was subjected to simulation. In all the studies reported, the convergence criterion used in the simulation phase was a coefficient of variation of 2.5% of the AREA 1 LOLE.

TABLE II: RESULTS FOR CASE 1 AND CASE 2

Reliability Indices	CASE 1		CASE 2	
	SYSTEM	AREA	SYSTEM	AREA
LOLE (h/year)	1.625	0.357	2.974	0.308
EUE (MWh/year)	244.2	48.8	436.3	43.6

TABLE III: REDUCTION OF COMPUTATIONAL EFFORT

Threshold Probability p_0	CASE 1		CASE 2	
	Residual Probability	CPU time ¹	Residual Probability	CPU time ¹
	α	(hh:mm:ss)	α	(hh:mm:ss)
1.0	1.0	32 : 25 : 10	1.0	****
10^{-1}	0.3429	10 : 49 : 00	0.5675	****
10^{-2}	0.0691	2 : 18 : 00	0.2071	95 : 02 : 50
10^{-3}	0.0131	0 : 27 : 20	0.0626	29 : 01 : 10
10^{-4}	0.0062	0 : 13 : 50	0.0261	11 : 56 : 30
10^{-5}	0.0024	0 : 05 : 30	0.0113	5 : 42 : 10

TABLE IV: RESULTS FOR CASE 3 AND CASE 4

CASE 3	SYSTEM	AREA 1	AREAS 2 & 5	AREAS 3 & 4
LOLE (h/year)	0.544	0.324	0.090	0.085
EUE (MWh/year)	91.4	42.2	12.6	12.0
CASE 4	SYSTEM	AREAS 1 & 6	AREAS 2, 5, 7 & 10	AREAS 3, 4, 8 & 9
LOLE (h/year)	1.605	0.242	0.033	0.270
EUE (MWh/year)	211.4	29.1	4.2	34.1

The results for CASE 1 and CASE 2 are shown in TABLE II. Both the system indices and area indices are shown. Since these configurations consist of identical areas and identical interconnections, all areas have identical indices, so the results for only one area are given. As the threshold probability p_0 is varied, the results do not change in any significant manner, but the computation time does.

TABLE III demonstrates the effectiveness of the proposed method. As the threshold probability is lowered, the residual probability decreases and the CPU time is reduced. The first case for 5 areas corresponds to complete simulation. For 10 areas, p_0 less than 0.01 is not used, as the CPU time becomes exorbitant.

Selecting a suitable mix of decomposition and simulation enables the computation of reliability indices of fairly large systems which would otherwise be extremely difficult to analyze by simulation alone. There is, however, a limit to the extent to which de-

composition can be performed. This is because the decomposition of each U -set generates $N_a + N_t - n_1$ more U -sets to be decomposed, unless some of these sets have probabilities lower than p_0 , and are set aside for simulation; consequently, lower threshold probabilities require generation of larger numbers of U -sets, and for very low values of p_0 the numbers of undecomposed U -sets tend to become unmanageably large.

TABLE IV shows the reliability indices obtained for CASE 3 and CASE 4 by using the method described, with $p_0 = 10^{-5}$. In both cases, the system indices are significantly affected by the presence of the cross-ties; so are indices of the areas at which the cross-ties terminate. In CASE 3, due to the symmetry of the configuration, areas 2 and 5 have identical indices, and so have areas 3 and 4. Similarly, the symmetry of the configuration in CASE 4 results in three groups of areas, given in TABLE IV, with the areas in each group having identical indices.

5 Conclusion

This paper presented a methodology for circumventing the problem of noncoherency in multi-area re-

¹CPU time on a DEC Alpha; a "****" entry implies that the time was inordinately long. The reported times correspond to a coefficient of variation of 2.5%, converging on AREA 1 LOLE in all cases.

liability models which take account of Kirchhoff's voltage law. The method comprises a decomposition phase and a simulation phase. The problem of non-coherency is solved by keeping the tie-lines at their maximum capacity during decomposition, but allowing them to assume all other states during sampling.

The effectiveness of the method can be attributed to the following factors:

1. Probabilities of tie-line failures are much lower than those of generation failures; this allows the removal of acceptable states of large probability, during the decomposition phase.
2. Sampling is performed selectively over those parts of the state space where failure states are more likely to occur; this increases the efficiency of simulation.
3. Since all the failure states remain available for sampling, there is no loss of accuracy accompanying the reduction in computational effort.

The method was tested on 5-area and 10-area cases, and was shown to be several times faster than a purely simulation based approach. Even though the network model used in the reported implementation was the DC flow model, the philosophy of the approach applies as well to AC flow models. The computational overheads would, of course, be larger.

The work reported in this paper demonstrated the reduction in computational effort brought about by the proposed method alone. Further reduction may be achieved by using variance reduction techniques [9, 12, 13] such as control variables and importance sampling.

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