

A Diakoptic Formulation of the Economic Power Flow Problem

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Abstract

This paper brings out an interesting feature of the economic power flow problem, and illustrates a formulation which is not only amenable to solutions using diakoptic or parallel computation techniques, but may also enable individual areas in interconnected systems to locally obtain power flow solutions which approach the global optimum. A generalized nonlinear objective function is assumed, and a simple two-area system is used to illustrate the concept.

1 Introduction

The economic despatch problem has been addressed in various forms over the last seven decades. Perhaps the most significant stage in its development was its integration with the load flow problem, resulting in the evolution of the economic power flow problem, in the 1960s. In due course, several other constraints, such as generator power limits, transmission capacity limits, and security constraints, have been incorporated into the framework of the optimal power flow problem.

While the optimal power flow problem has been formulated in several forms [1–9], some with the total generation cost as objective function, others seeking to minimize the total transmission loss, the economic power flow problem, which minimizes the total generation cost, possesses the interesting feature that the cost function decouples into several local area cost functions. This decoupling does not occur in the transmission loss function, so only the generation cost function will be addressed in this paper. A very generalized, nonlinear model is assumed, to show that the decoupling is a natural, generalized phenomenon. This feature is illustrated in this paper using a simple two-area example.

2 Model Description

The model assumed in this paper is identical to the one described in [1], and is presented briefly in this section.

The problem is defined as that of minimizing the total gen-

eration cost function

$$F = \sum_{i \in G}^N F_i(x, u) \quad (1)$$

where G is the set of generator buses

N is the number of buses in the system

F_i is the cost function of the i th generator bus

x is the set of unknown state variables

u is the set of control variables

subject to the equality constraints

$$g_n(x, u) = 0 \quad (2)$$

where g_n is the real and reactive power supply and demand function at the n th bus

and the inequality constraints

$$h_k(u) \leq 0 \quad (3)$$

where h_k is the constraint violation at the k th bus.

The equality constraints (3) are dealt with using the Kuhn-Tucker formulation:

$$\begin{aligned} [u] - [u^{max}] &\leq 0 \\ [u^{min}] - [u] &\leq 0 \end{aligned} \quad (4)$$

The nonlinear optimization is therefore equivalent to the minimization of the Lagrangian

$$\begin{aligned} \mathcal{L} = & F(x, u) + [\lambda]^T [g(x, u)] \\ & + [\mu^{max}]^T ([u] - [u^{max}]) \\ & + [\mu^{min}]^T ([u^{min}] - [u]) \end{aligned} \quad (5)$$

with respect to x , u , λ and μ .

In (5), $[\lambda]$ are the Lagrangian multipliers for the equality constraints and $[\mu^{max}]$ and $[\mu^{min}]$ are auxiliary variables similar to Lagrangian multipliers.

This minimization problem requires the simultaneous solution of the following matrix equations:

$$\left[\frac{\partial \mathcal{L}}{\partial \lambda} \right] = [g(x, u)] = 0 \quad (6)$$

$$\left[\frac{\partial \mathcal{L}}{\partial x} \right] = \left[\frac{\partial F}{\partial x} \right] + \left[\frac{\partial g}{\partial x} \right]^T [\lambda] = 0 \quad (7)$$

$$\left[\frac{\partial \mathcal{L}}{\partial u} \right] = \left[\frac{\partial F}{\partial u} \right] + \left[\frac{\partial g}{\partial u} \right]^T [\lambda] + [\mu] = 0 \quad (8)$$

where $\mu_i = \mu_i^{max}$, if $\mu_i > 0$
 $\mu_i = -\mu_i^{min}$, if $\mu_i < 0$

In illustrating the central concept of this paper, it is convenient leave out the inequality constraints, without loss of generality.

The problem therefore reduces to that of minimizing

$$\mathcal{L} = F(x, u) + [\lambda]^T [g(x, u)] \quad (9)$$

with respect to x , u and λ .

This involves the simultaneous solution of the matrix equations

$$\left[\frac{\partial \mathcal{L}}{\partial \lambda} \right] = [g(x, u)] = 0 \quad (10)$$

$$\left[\frac{\partial \mathcal{L}}{\partial x} \right] = \left[\frac{\partial F}{\partial x} \right] + \left[\frac{\partial g}{\partial x} \right]^T [\lambda] = 0 \quad (11)$$

$$\left[\frac{\partial \mathcal{L}}{\partial u} \right] = \left[\frac{\partial F}{\partial u} \right] + \left[\frac{\partial g}{\partial u} \right]^T [\lambda] = 0 \quad (12)$$

The algorithm proposed in [1] for the solution of the above equations consists of the following steps:

1. Assume a set of control parameters $[u]$.
2. Solve equation (10) by Newton's method to yield a set of values for $[x]$.
3. Using the current values of $[u]$ and $[x]$ solve equation (11) and obtain a set of values for

$$[\lambda] = -[\partial g / \partial x]^{T^{-1}} \cdot [\partial F / \partial x] \quad (13)$$

4. Insert $[\lambda]$ from (13) into (12) and compute the gradient

$$[\nabla F] = [\partial g / \partial u] + [\partial g / \partial u]^T [\lambda] \quad (14)$$

5. If $[\nabla F]$ is sufficiently small, the minimum has been reached.
6. Otherwise find a new set of control parameters from

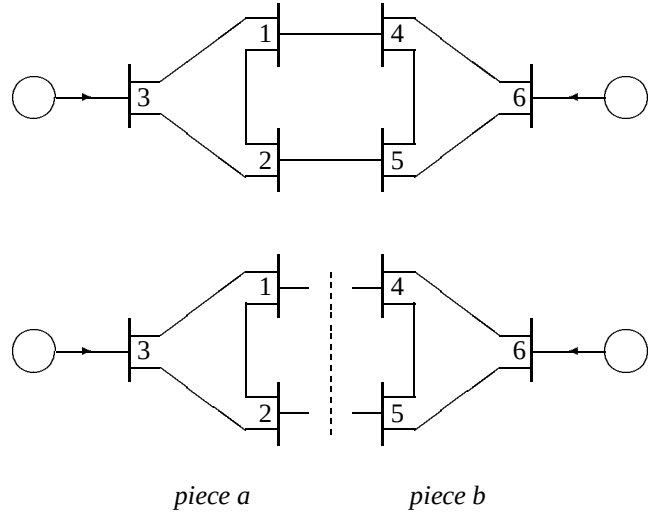
$$\begin{aligned} [u^{new}] &= [u^{old}] + [\Delta u] \\ \text{with } [\Delta u] &= -c \cdot [\nabla F] \end{aligned} \quad (15)$$

This is a fairly generalized formulation, and has been implemented in various forms. The formulation proposed in this paper, which involves decoupling into local sub-problems, is described in the next section.

3 Diakoptic Formulation

Diakoptic approaches begin with tearing of a network. Considerable research has gone into the idea of network tearing [11, 12]. Several methods of tearing have been proposed, some with a single common bus, some with multiple common buses, some without any common buses (line tearing), etc..

To achieve decoupling of the generation cost function, it is necessary to use line tearing in such a manner that not one of the torn lines is connected directly to a generator bus. The figure below shows a simple network that has been torn in this manner.



The pieces so generated can correspond to areas in an interconnected system, or geographically separated areas, or just any arbitrary pieces of convenient size to facilitate computation.

The piecewise solution of equations (10), (11) and (12), applied to the above network, can now be demonstrated.

3.1 Piecewise Solution of

$$[g(x, u)] = 0 \quad \text{and} \quad \left[\frac{\partial F}{\partial x} \right] + \left[\frac{\partial g}{\partial x} \right]^T [\lambda] = 0$$

These equations are basically the same as (10) and (11). Of these, (10) is identical to the load flow equations, constructed only for the non-generator buses. The variables corresponding to the generator buses are basically the control variables u , whose values we initially assume at the beginning of the iterations, and then update using equation (15). If (10) is solved using the Newton-Raphson method, then we get the equation

$$[\Delta g] = \left[\frac{\partial g}{\partial x} \right] [\Delta x] \quad (16)$$

which for the example system expands to

$$\begin{bmatrix} \Delta g_{V1} \\ \Delta g_{\theta1} \\ \Delta g_{V2} \\ \Delta g_{\theta2} \\ \Delta g_{V4} \\ \Delta g_{\theta4} \\ \Delta g_{V5} \\ \Delta g_{\theta5} \end{bmatrix} = \begin{bmatrix} x & x & x & x & x & x & o & o \\ x & x & x & x & x & x & o & o \\ x & x & x & x & o & o & x & x \\ x & x & x & x & o & o & x & x \\ x & x & o & o & x & x & x & x \\ x & x & o & o & x & x & x & x \\ o & o & x & x & x & x & x & x \\ o & o & x & x & x & x & x & x \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \Delta \theta_1 \\ \Delta V_2 \\ \Delta \theta_2 \\ \Delta V_4 \\ \Delta \theta_4 \\ \Delta V_5 \\ \Delta \theta_5 \end{bmatrix} \quad (17)$$

where the x -s denote non-zero elements and the o -s denote zero elements.

The equation (11) can be similarly expanded to

$$\begin{bmatrix} F'_{V1} \\ F'_{\theta1} \\ F'_{V2} \\ F'_{\theta2} \\ F'_{V4} \\ F'_{\theta4} \\ F'_{V5} \\ F'_{\theta5} \end{bmatrix} = - \begin{bmatrix} x & x & x & x & x & x & o & o \\ x & x & x & x & x & x & o & o \\ x & x & x & x & o & o & x & x \\ x & x & x & x & o & o & x & x \\ x & x & o & o & x & x & x & x \\ x & x & o & o & x & x & x & x \\ o & o & x & x & x & x & x & x \\ o & o & x & x & x & x & x & x \end{bmatrix} \begin{bmatrix} \lambda_{a1} \\ \lambda_{a2} \\ \lambda_{a3} \\ \lambda_{a4} \\ \lambda_{b1} \\ \lambda_{b2} \\ \lambda_{b3} \\ \lambda_{b4} \end{bmatrix} \quad (18)$$

Notice that both equations (17) and (18) contain the Jacobian $[\partial g / \partial x]$, whose off-diagonal blocks are in general sparse, containing only terms corresponding to the interconnecting ties (the torn branches). If suitable compensation is provided for these terms, then (17) and (18) may be decoupled and solved locally [10]. For the case of an interconnected system, the compensation terms may be obtained from the flows in the inter-area ties, since these flows are monitored and their values under various operating conditions are known.

3.2 Piecewise solution of

$$[\nabla F] = \left[\frac{\partial F}{\partial u} \right] + \left[\frac{\partial g}{\partial u} \right]^T [\lambda]$$

It is in this part of the solution that the manner of tearing becomes relevant.

Due to the very nature of the tearing, $[\partial F / \partial u]$ for each subsystem is independent of the control variables in the other subsystems.

The $[\partial g / \partial u]$ matrix, too, gets decoupled; so the above equa-

tion expands into the form

$$\begin{bmatrix} \mathcal{L}'_{V3} \\ \mathcal{L}'_{\theta3} \\ \mathcal{L}'_{V6} \\ \mathcal{L}'_{\theta6} \end{bmatrix} = \begin{bmatrix} F'_{V3} \\ F'_{\theta3} \\ F'_{V6} \\ F'_{\theta6} \end{bmatrix} + \begin{bmatrix} x & x & x & x & o & o & o & o \\ x & x & x & x & o & o & o & o \\ o & o & o & o & x & x & x & x \\ o & o & o & o & x & x & x & x \end{bmatrix} \begin{bmatrix} \lambda_{a1} \\ \lambda_{a2} \\ \lambda_{a3} \\ \lambda_{a4} \\ \lambda_{b1} \\ \lambda_{b2} \\ \lambda_{b3} \\ \lambda_{b4} \end{bmatrix} \quad (19)$$

Therefore equation (19) completely decouples into

$$[\nabla F_a] = \left[\frac{\partial F}{\partial u_a} \right] + \left[\frac{\partial g}{\partial u_a} \right]^T [\lambda_a] \quad (20)$$

$$[\nabla F_b] = \left[\frac{\partial F}{\partial u_b} \right] + \left[\frac{\partial g}{\partial u_b} \right]^T [\lambda_b] \quad (21)$$

It is now evident how the technique of tie-line tearing has resulted in a complete mathematical decoupling of equation (12).

This decoupling becomes obvious when the cost function F is written out explicitly:

$$F = F_3(x, u) + F_6(x, u) \quad (22)$$

Assuming a quadratic generation cost function of the form

$$F_i = A_i + B_i P_i + C_i P_i^2 \quad (23)$$

where A_i , B_i and C_i are constants for the i -th generator and P_i is the real power generated by the i -th generator, P_3 and P_6 are given by

$$\begin{aligned} P_3 = & V_1 V_3 [g_{13} \cos(\theta_3 - \theta_1) + b_{13} \sin(\theta_3 - \theta_1)] \\ & + V_2 V_3 [g_{23} \cos(\theta_3 - \theta_2) + b_{23} \sin(\theta_3 - \theta_2)] \\ & + V_3^2 g_{33} \end{aligned} \quad (24)$$

$$\begin{aligned} P_6 = & V_4 V_6 [g_{46} \cos(\theta_6 - \theta_4) + b_{46} \sin(\theta_6 - \theta_4)] \\ & + V_5 V_6 [g_{56} \cos(\theta_6 - \theta_5) + b_{56} \sin(\theta_6 - \theta_5)] \\ & + V_6^2 g_{66} \end{aligned} \quad (25)$$

which are well known equations.

Notice that (24) does not contain any term corresponding to the admittance of any of the torn lines or of any element outside area a . The same observation is true of equation (25) and area b . So if (24) is substituted in (23), the expression for F_3 will not contain any term corresponding to elements lying outside area a , and the same applies to F_6 and area b . This explains the zero block diagonals in (19) and its complete decoupling into (20) and (21).

4 Conclusion

This paper has described a diakoptic formulation of the economic power flow problem and has explained the property of the cost function which enables such a formulation. It has also offered speculations as to the areas where the formulation can be applied. While no implementation details or computational experience has been reported, it should be realized that the focus of the paper has been on describing the concept.

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