

Use of Homotopy-based Approaches in Finding Controlling Unstable Equilibrium Points in Transient Stability Analysis

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Abstract—This paper introduces the use of homotopy-based approaches in computing the Controlling Unstable Equilibrium Points (controlling UEPs) in transient stability analysis using direct methods. It is well known that the regions of convergence of the controlling UEPs are very sensitive to the initial guesses, and traditional iterative methods fail to find the correct controlling UEPs if the initial guesses lie outside their regions of convergence. On the other hand, homotopy-based approaches are very reliable in finding solutions because they are globally convergent. However, homotopy-based approaches are intrinsically slow if the initial point is far from the desired solution because these methods map the trajectory of the solution from an easy and known solution to the desired solution. This paper proposes an algorithm that uses a homotopy-based approach with the exit point as an initial point to reliably find the correct controlling UEP. To reduce computational effort, the proposed method uses an approximate exit point rather than computing an accurate exit point as is common practice in finding controlling UEPs. Further, this method eliminates the necessity of computing the Minimum Gradient Point (MGP), which makes the homotopy-based approaches comparable with the other iterative methods in terms of the speed of computation. An explicit characterization of the region of convergence of a controlling UEP and its boundary starting from an exit point for a typical power system is derived. The method is applied on the WECC and the NE 39 test systems to demonstrate its effectiveness in finding the controlling UEPs.

Index Terms—Region of convergence, controlling UEPs, homotopy-based approaches, Newton methods.

I. INTRODUCTION

Power systems are currently operated close to their stability limits and transient stability analysis is one of the important tools in determining the operating limits. Direct methods of transient stability analysis have been successfully applied in evaluating the stability of power systems [1]–[7]. However, the accuracy of these methods strongly relies on the determination of the controlling UEPs. Despite significant advances in the application of the direct methods in transient stability analysis,

the problem of precise determination of the controlling UEPs and the speed of computation are still of concern. The presence of fractal shapes of the convergence regions of the controlling UEPs is the main reason of failure of several numerical methods in finding the correct controlling UEPs [7]–[9]. The Newton method and its variations, for example, converge only if the initial guess lies within the region of convergence of the desired solution; to find the correct controlling UEPs, the initial guesses have to be within a certain neighborhood of the desired solutions. This requirement makes it difficult to find the best guesses and it is computationally expensive.

Several methods have been proposed to compute the controlling UEPs including the fault-dependent controlling UEP method [1] and Boundary of stability region based Controlling Unstable equilibrium point method (BCU method) [10]. Although these methods are effective, problems may still occur [7], [11] if initial points are not determined precisely. Several techniques and procedures are usually performed along with these methods to ensure convergence to the correct controlling UEP. For example, a stability-boundary-following procedure is utilized with the BCU method to prevent the solution algorithm from converging to an undesired equilibrium point [7]. An alternative approach for computing the controlling UEPs is to use homotopy-based approaches.

Homotopy-based approaches have been successfully applied in computing the *closest* UEPs [12], [13]. However, applications of the homotopy-based approaches in computing the controlling UEPs have not been reported yet in the literature. The approaches reported in [12], [13] utilize the strategy of singular fixed-point and the concept of bifurcation to locate the closest UEPs. These algorithms proceed by choosing a set of initial points that by using homotopy-based approaches converge to a set of type-1 UEPs. By choosing a set of appropriate initial points, the resulting set of the type-1 UEPs will include the closest UEP which can be distinguished by having lowest Lyapunov function value. A continuation-based method that approximates the stability boundary locally and does not use the energy function to compute the controlling UEPs has been proposed in [14].

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This paper proposes the use of a homotopy-based approach in computing the controlling UEPs. The proposed method uses exit points as initial solutions to the homotopy-based approach and maps the solutions from the exit points to the controlling UEPs. In using the homotopy-based approach to compute the controlling UEPs, the necessity of computing the MGP is eliminated. Also, this method does not require an accurate exit point as is required in other iterative methods such as Newton method and its variations. Therefore, by completely avoiding the need to determine the MGP, and by reducing the dependence on the accuracy of the exit point, the homotopy method may offer advantages over other iterative methods. The additional burden on the homotopy method is to ensure that it seeks out the controlling UEPs over other UEPs; the device for ensuring this is explained in section III-B.

The rest of the paper is organized as follows. Section II presents the power system transient stability model and the associated energy function, section III explains the calculation and characterization of the region of convergence of the controlling UEPs including the homotopy-based approach, section IV shows the application of the proposed method, and section V provides conclusion remarks.

II. SYSTEM MODEL AND ENERGY FUNCTION

This section presents the dynamical model of power systems and the associated energy function for transient stability analysis.

A. The Dynamical Model

Given an n -generator system and assuming uniform damping, the dynamical model of the equations of motion of the generators with respect to the center of inertia (COI) can be described as follows [5].

$$\ddot{\delta}_i = \tilde{\omega}_i, \quad (1)$$

$$\dot{\tilde{\omega}}_i = \frac{1}{M_i} (P_{mi} - P_{ei}) - \frac{1}{M_T} P_{COI} - \lambda \tilde{\omega}_i, \quad (2)$$

where P_{mi} is the mechanical input of machine i , P_{ei} is the electrical power output of machine i , M_i is the inertia constant of machine i , $\tilde{\delta}_i = \delta_i - \delta_o$, $\tilde{\omega}_i = \omega_i - \omega_o$, $\delta_o = \frac{1}{M_T} \sum_{i=1}^n M_i \delta_i$, $\omega_o = \frac{1}{M_T} \sum_{i=1}^n M_i \omega_i$, $M_T = \sum_{i=1}^n M_i$ and λ is the uniform damping constant.

The electrical power of machine i is given as follows.

$$P_{ei} = \sum_{j=1}^n E_i E_j \left[G_{ij} \cos(\tilde{\delta}_i - \tilde{\delta}_j) + B_{ij} \sin(\tilde{\delta}_i - \tilde{\delta}_j) \right]. \quad (3)$$

The P_{COI} is computed as follows.

$$P_{COI} = \sum_{i=1}^n P_{mi} - \sum_{i=1}^n \sum_{j=1}^n E_i E_j \left[G_{ij} \cos(\tilde{\delta}_i - \tilde{\delta}_j) + B_{ij} \sin(\tilde{\delta}_i - \tilde{\delta}_j) \right], \quad (4)$$

where δ_i and ω_i are power angle and speed of machine i respectively, E_i is the internal voltage magnitude of machine

i and G_{ij} and B_{ij} are the conductance and susceptance of the of the admittance matrix of the network-reduction model.

The compact form of the system of (1) and (2) can be expressed as follows.

$$\dot{x} = F(x). \quad (5)$$

B. The Energy Function

The energy function is used in transient stability analysis for fast screening and to compute the exit point to generate a sequence of steps to calculate the controlling UEP. The energy function associated with the model of (5) is expressed as follows [1], [2].

$$V = \frac{1}{2} \sum_{i=1}^n M_i \tilde{\omega}_i^2 - \sum_{i=1}^n P_i (\tilde{\delta}_i - \tilde{\delta}_i^s) - \sum_{i=1}^{n-1} \sum_{j=i+1}^n \left[C_{ij} (\cos \tilde{\delta}_{ij} - \cos \tilde{\delta}_{ij}^s) - I_{ij} \right], \quad (6)$$

where

$$P_i = P_{mi} - E_i^2 G_{ii},$$

and I_{ij} is the energy dissipated in the network transfer conductances. The term I_{ij} can be expressed as follows.

$$I_{ij} = \int_{\tilde{\delta}_i^s + \tilde{\delta}_j^s}^{\tilde{\delta}_i + \tilde{\delta}_j} D_{ij} \cos \tilde{\delta}_{ij} d(\tilde{\delta}_i + \tilde{\delta}_j).$$

This term is path dependent and can be calculated only if the system trajectory is known. However, the system trajectory is not known in advance. Several methods have been suggested in the literature to approximate this term. In this paper we will use the method suggested by [1] which can be given as,

$$I_{ij} = D_{ij} \frac{\tilde{\delta}_i + \tilde{\delta}_j - \tilde{\delta}_i^s - \tilde{\delta}_j^s}{\tilde{\delta}_i - \tilde{\delta}_j - \tilde{\delta}_i^s + \tilde{\delta}_j^s} \left[\sin \tilde{\delta}_{ij} - \sin \tilde{\delta}_{ij}^s \right]. \quad (7)$$

The first term of (6) represents the kinetic energy and the last two terms represent the potential energy. Since the energy function is used in determining exit points, the approximation in the path dependent term I_{ij} may cause difficulties in finding accurate exit points. However, in the method proposed in this paper, accurate exit points are not necessary in finding the controlling UEPs. Therefore, this approximation will not cause problems in calculating the controlling UEPs.

III. CALCULATION OF THE CONTROLLING UEPs

Computing the controlling UEPs is crucial because the energy at a controlling UEP is used in computing the critical energy to assess the stability of the system. A state vector x is called an equilibrium point x^* of the dynamic system represented in (5) if $F(x^*) = 0$. A controlling UEP is one of the unstable equilibrium points, but it is not an easy task to determine and distinguish it from the other unstable equilibrium points. Mathematically, a controlling UEP is an unstable equilibrium point whose stable manifold, $W^s(X_{co})$,

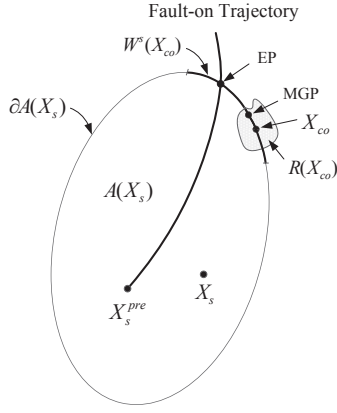


Figure 1. Region of convergence of controlling UEPs under the use of Newton methods [7].

contains the exit point of the fault-on (sustained fault) trajectory $(\delta(t), \omega(t))$ [1], [7], [10], [11] as shown in Fig. 1. The terms used in Fig. 1 are defined as following:

EP	is the exit point,
MGP	is the minimum gradient point,
X_{co}	is a controlling UEP,
$A(X_s)$	is the region of attraction (stability) of the post-fault stable equilibrium point (X_s),
$\partial A(X_s)$	is the boundary of the region of attraction,
$R(X_{co})$	is the region of convergence of a controlling UEP under Newton method,
$W^s(X_{co})$	is the stable manifold of a controlling UEP,
X_s^{pre}	is the pre-fault stable equilibrium point.

The region of convergence of an equilibrium point can be defined as following: starting from an initial guess, a numerical method succeeds in finding the desired solution if the starting point lies inside the region of convergence of the solution and it fails if the initial guess lies outside the region of convergence. The size and shape of the region of convergence of a controlling UEP can be very fractal and different for different numerical methods [7]. Therefore, if the initial guess is not in the region of convergence of the controlling UEP, numerical methods such as Newton methods fail to find the correct controlling UEP. In other words, to find the correct controlling UEP, the initial guess has to be sufficiently close to the controlling UEP.

A popular method to compute controlling UEPs is to use time-domain simulations to simulate the projected fault-on trajectory to obtain the exit point and the MGP, and then use the MGP as an initial guess to generate a sequence of solution steps towards the controlling UEP [10], [11], [14]–[18]. The exit point is the point at which the projected fault-on trajectory exits the stability boundary of the post-fault stable equilibrium point (SEP). Computationally, the exit point is characterized by the first local maximum of the potential energy (the last two terms of (6)) of the post-fault network along the projected fault-on trajectory. Another method to detect the exit point

is detecting the change in the sign of the dot product of the post-fault power mismatch vector and the fault-on speed vector [7]. The MGP is numerically characterized by the first local minimum value of the norm of the vector field of the post-fault trajectory of (1) and (2) [7]. The robustness of finding the controlling UEP depends strongly on the quality of the calculated of MGP [11], [14], [15], [18], [19]. An inaccuracy in detecting the exit point may cause a difficulty in computing the MGP. However, detecting an accurate exit point is computationally involved and sometimes it requires the use of interpolation methods after bounding the exit point in a certain range. Therefore, a numerical inaccuracy in computing the exit point will probably cause failure of numerical methods to calculate the controlling UEP.

Most of the methods described in the literature integrate the fault-on trajectory with large time step until the algorithm locates the exit point between two time points. The algorithm then bounds these points and starts searching for the exit point using some tools, such as the golden section, linear or quadratic interpolation. Once a more accurate exit point is discovered, the algorithm uses this point as an initial point to compute the MGP and then the controlling UEP. Therefore, we can see that it requires significant efforts to compute the exit point.

To overcome the difficulty of finding an initial point inside the region of convergence, we propose the use of a homotopy-based approach in computing the controlling UEP. Homotopy-based approaches are known to be reliable in finding the solutions and they are globally convergent [20]–[23]. In this paper, a homotopy-based approach is used to eliminate the requirement of computing the MGPs and accurate exit points. The work in this paper shows that a controlling UEP can be obtained by using the homotopy-based approach with an approximate exit point as an initial guess. This section presents the proposed homotopy-based approach and the well known BCU method for the seek of comparison.

A. BCU Based Approach

The BCU method is a systematic method that uses the concept of reduced-state model for computing controlling UEPs [4], [7]. This method defines an artificial model that can be solved to find all the equilibrium points of the original model. For each power system stability model there is a corresponding reduced-state model such that static and dynamic properties of the original model are captured to compute the controlling UEPs on the stability boundary of the original model. Computing controlling UEPs using the BCU method on the reduced-state model is much easier than computing the controlling UEP of the original model [7]. For example, consider the following generic model [7]:

$$T\dot{x} = -\frac{\partial U}{\partial x}(x, y) + g_1(x, y), \quad (8)$$

$$\dot{y} = z, \quad (9)$$

$$M\dot{z} = -Dz - \frac{\partial U}{\partial y}(x, y) + g_2(x, y), \quad (10)$$

where x , y , and z are state variables, T is a positive definite matrix, M and D are diagonal positive definite matrices, and $g_1(x, y)$ and $g_2(x, y)$ represent the transfer conductances.

The associated artificial, reduced-state model is given as follows [7].

$$T\dot{x} = -\frac{\partial U}{\partial x}(x, y) + g_1(x, y), \quad (11)$$

$$\dot{y} = -\frac{\partial U}{\partial y}(x, y) + g_2(x, y). \quad (12)$$

The general procedure of finding controlling UEPs using the BCU method is summarized as follows—the definitions are referred to Fig. 1 [10].

- 1) From the fault-on trajectory, determine the exit point.
- 2) Use the point exit point as an initial point and integrate the post-fault reduced-state model to find the MGP.
- 3) Use the MGP as an initial point to calculate the desired controlling UEP of the reduced-state model.
- 4) Determine the controlling UEP of the original system.

In implementing the above procedure several numerical issues may arise [7]. For example, an inaccuracy in calculating the exit point may cause an inaccuracy in computing the MGP. If the computed MGP is not sufficiently close to the controlling UEP, the iterative method may diverge or converge to another equilibrium point. Two situations where the exit points are not accurate (either inside the stability region or outside the stability region) are shown in Fig. 2.

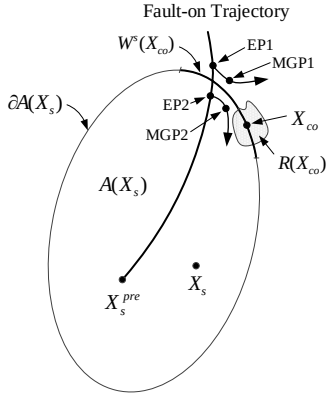


Figure 2. The exit point either lies inside the stability region or outside the stability region where MGP1 and MGP2 are the minimum gradient points corresponding to the exit points EP1 and EP2.

From Fig. 2, it can be seen that an inaccurate exit point can cause numerical problems in calculating the controlling UEP. An approach that can be used to solve this problem is called stability-boundary-following procedure [7]. The procedure can be summarized as follows [7]: starting from the given exit point, integrate the post-fault model for a few iterations and check if the MGP is detected. If it is detected, check if the controlling UEP can be computed using this MGP. If the MGP is not detected or the controlling UEP cannot be computed (either converges to the post-fault SEP or the distance between the computed controlling UEP and the MGP is larger than a

threshold), draw a ray connecting the current point on the post-fault trajectory with the post-fault SEP and search along this trajectory for a new MGP. This procedure is depicted in Fig. 3 for the case where the exit point lies inside the stability region. For more detail about this procedure, readers are referred to [7].

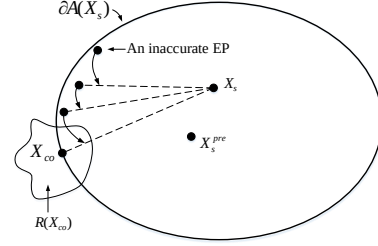


Figure 3. A stability-boundary-following procedure [7].

B. Homotopy-Based Method

Traditional methods of solving (5) include the use of Newton methods. Newton methods, although computationally sound, will only converge to a solution if the initial guess is within the neighborhood of the convergence region of the desired solution. Homotopy methods have been developed to overcome the local convergence of Newton methods since they are globally convergent [20]–[23]. That is, these methods are guaranteed to converge to a solution regardless of an initial starting point provided that there are no singularities, bifurcation points, or turning points in its path. In the case where there may be multiple equilibrium points, the homotopy method has the ability to find those solutions. However, when multiple solutions exist, a better initial condition can help to obtain the solution of interest first. The logic behind these methods in computing the controlling UEP is that with an exit point as the initial guess, the homotopy-based approach can find the controlling UEP first since its location is the closest to the exit point in terms of the energy value.

Homotopy is a numerical method to solve and find the roots of non-linear systems ($F(x^*) = 0$). The homotopy method traces the solution trajectory by a predictor-corrector algorithm to get a solution of the original equation. The basic concept of the homotopy-based methods is that they find the solutions by path continuation, starting at a known solution x^0 that satisfies $G(x^0) = 0$ as shown in (13). The most widely used homotopy function is expressed as follows.

$$H(x, t) = tF(x) + (1 - t)G(x) = 0, \quad (13)$$

where t changes from 0 to 1 with an incremental step-size through the mapping process, i.e., $H(x, 0) = G(x)$ and $H(x, 1) = F(x)$.

$G(x)$ can be chosen arbitrary as long as it has a known solution. However, the most widely used homotopy function is Newton Homotopy which can be expressed as follows.

$$G(x) = F(x) - F(x^0), \quad (14)$$

where x^0 can be any starting point. Therefore, the homotopy function becomes

$$H(x, t) = F(x) - (1 - t)F(x^0) = 0. \quad (15)$$

In this paper, we used Newton homotopy and it has succeeded to compute the controlling UEPs of the tested systems. The $F(x)$ function represents the dynamical model of the system as given in (5) and x^0 is the exit point.

It should be noted that in applying homotopy methods, the direction of the search (forward or backward) for the solution is an important factor in determining the correct controlling UEPs. In backward direction, the mapping factor is considered negative. Therefore, equation (13) can be expressed as follows.

$$H(x, t) = tF(x) + (1 + t)G(x) = 0. \quad (16)$$

The direction of the search depends on the location of the exit point with respect to the controlling UEP on the stability boundary. If this direction is unknown and if the forward direction (for example) is performed, the solution trajectory may converge to an unstable equilibrium point (UEP) but not the controlling UEP. Fig. 4 shows a case where the controlling UEP is located in the direction of forward mapping from the exit point. If the backward mapping were used, the solution trajectory would converge to another UEP but not the controlling UEP. Also, Fig. 5 shows a case where the controlling UEP is located in the direction of backward mapping from the exit point. If the forward mapping were used, the solution trajectory would converge to a different UEP. During the calculation process, the algorithm proceeds by assuming forward mapping for one homotopy iteration; if the solution diverges or converges to a point far from the exit point, the algorithm uses backward mapping.

The speed of calculation depends on the closeness of the initial points to the the controlling UEP. For the studied systems in this paper, the number of the homotopy iterations to arrive to the controlling UEP starting from the exit point ranges from one to three iterations where each homotopy iteration is equivalent to a full Newton (or other method) solution. This can be justified by the fact that if the initial point is at or close to the MGP, the algorithm takes one iteration which is equivalent to the BCU method. However, if the initial point is the exit point, the proposed method takes two or more iterations.

IV. CASE STUDIES

To demonstrate the application of the proposed homotopy-based approach in computing controlling UEPs using exit points as initial points, we have applied this method on several systems including the Southern California Edison system, the equivalent WECC (Western Electricity Coordinating Council) system and the New England 39 bus system. Due to space limitation, we present the results for the WECC and NE 39 bus systems. Intel Fortran is used to generate an executable file that calculates the exit points and then the controlling UEPs. The program has succeeded to find all the exit points and controlling UEPs.

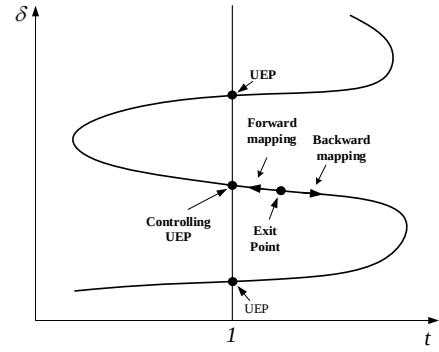


Figure 4. Forward and Backward search for the controlling UEP - the backward mapping will converge to a UEP not the controlling UEP

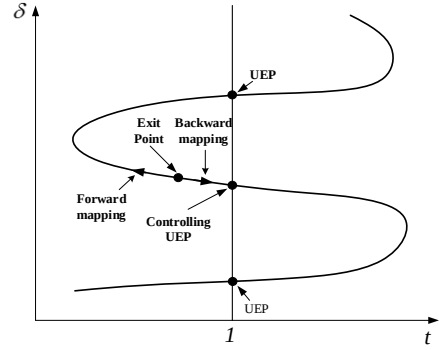


Figure 5. Forward and Backward search for the controlling UEP - the forward mapping will converge to a UEP not the controlling UEP

A. WECC Test System

This system consists of 3 generators, 9 buses and six transmission lines [7]. The exit points associated with the designated 12 contingencies are shown in Table I.

The exit points are used as initial conditions to calculate the controlling UEPs. To show the solution trajectories of the Newton-homotopy method for these contingencies, we have selected generator #1 to be a reference for plotting purposes only in order for the trajectories to appear in three-dimension plots. That is, generator #1 was taken as a reference for the plotting only and the calculations were carried out using the

TABLE I
CONTINGENCY LIST DETAILS AND THE CORRESPONDING EXIT POINTS

No.	Fault at Bus	Line Trip		Exit point, (rad)		
		From	To	$\tilde{\delta}_1$	$\tilde{\delta}_2$	$\tilde{\delta}_3$
1	4	4	5	-0.83157	2.05223	2.16748
2	5	4	5	-0.85604	2.24468	1.95048
3	4	4	6	-0.82573	2.00656	2.22575
4	6	4	6	-0.82663	2.00656	2.22575
5	5	5	7	-0.77351	2.07331	1.66662
6	7	5	7	-0.84828	2.68764	0.94769
7	6	6	9	-0.75797	1.83370	2.05405
8	9	6	9	-0.50293	0.51129	2.86278
9	7	7	8	-0.73581	2.28572	0.91888
10	8	7	8	-0.77159	1.74633	2.34681
11	8	8	9	-0.76930	1.74136	2.33940
12	9	8	9	-0.45872	0.48828	2.56452

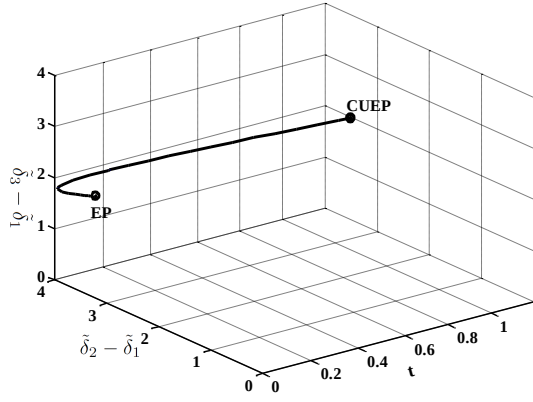


Figure 6. Solution trajectory using Newton-homotopy—Contingency #1.

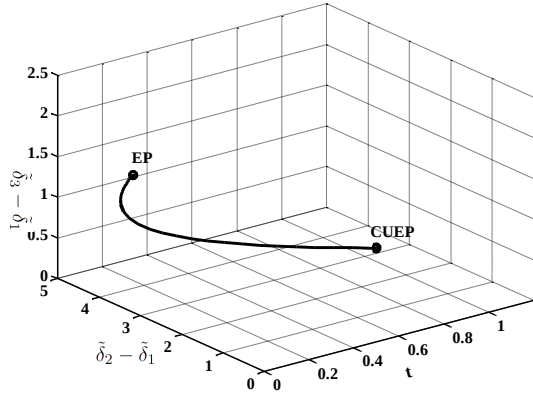


Figure 7. Solution trajectory using Newton-homotopy—Contingency #2.

COI as a reference frame. Fig. 6, Fig. 7 and Fig. 8 show the trajectories of the solutions for the first three contingencies where CUEP denotes the controlling UEP. The trajectories show the solution process starting from the exit points to the controlling UEPs while changing the homotopy factor t .

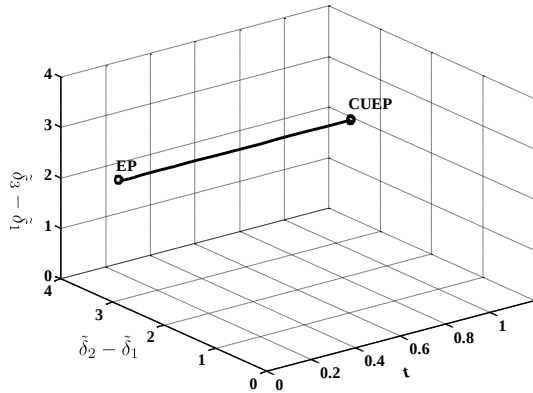


Figure 8. Solution trajectory using Newton-homotopy—Contingency #3.

The controlling UEPs of the 12 designated contingencies are shown in Table II. Starting from the exit points of Table I, the algorithm converged to these controlling UEPs. The controlling UEPs using the BCU method, which were taken from [7], are shown in Table III. It can be seen from Table

II and Table III that the computed controlling UEPs using the homotopy-based approach are very close to the results provided in [7] for the same test system.

TABLE II
THE COMPUTED CONTROLLING UEPs USING HOMOTOPY METHOD

No.	Fault near Bus	Line Trip		Controlling UEPs, (rad)		
		From	To	$\tilde{\delta}_1$	$\tilde{\delta}_2$	$\tilde{\delta}_3$
1	4	4	5	-0.8323	2.0742	2.1447
2	5	4	5	-0.8323	2.0742	2.1447
3	4	4	6	-0.8266	2.0821	2.0540
4	6	4	6	-0.8266	2.0821	2.0540
5	5	5	7	-0.7598	1.9521	1.8071
6	7	5	7	-0.7598	1.9521	1.8071
7	6	6	9	-0.7586	1.8576	1.9979
8	9	6	9	-0.7586	1.8576	1.9979
9	7	7	8	-0.5430	2.1797	-0.3764
10	8	7	8	-0.3500	0.0738	2.5861
11	8	8	9	-0.2915	-0.1017	2.5004
12	9	8	9	-0.2915	-0.1017	2.5004

TABLE III
THE COMPUTED CONTROLLING UEPs USING BCU METHOD [7]

No.	Fault near Bus	Line Trip		Controlling UEPs, (rad)		
		From	To	$\tilde{\delta}_1$	$\tilde{\delta}_2$	$\tilde{\delta}_3$
1	4	4	5	-0.8364	2.0797	2.1466
2	5	4	5	-0.8364	2.0797	2.1466
3	4	4	6	-0.8256	2.0830	2.0549
4	6	4	6	-0.8256	2.0830	2.0549
5	5	5	7	-0.7589	1.9528	1.8079
6	7	5	7	-0.7589	1.9528	1.8079
7	6	6	9	-0.7576	1.8583	1.9986
8	9	6	9	-0.7576	1.8583	1.9986
9	7	7	8	-0.5424	2.1802	-0.3755
10	8	7	8	-0.3495	0.0745	2.5864
11	8	8	9	-0.2910	-0.1011	2.5008
12	9	8	9	-0.2910	-0.1011	2.5008

To test the performance of the proposed approach, less accurate exit points (by utilizing large step size in calculating the exit points), have been used in calculating the controlling UEPs. Less accurate exit points are easy to calculate but they are expected to cause numerical problems when attempting to use conventional iterative methods. The homotopy-based approach has succeeded in calculating all the controlling UEPs providing that the exit points are not very accurate.

B. New England 39 Bus Test System

In this section, analysis of the New England 39 bus system [24] is provided. This system consists of 10 generators, 39 buses and 46 transmission lines. Considering two possible locations for faults on each line (one near to each end), the number of contingencies is 92. The proposed method succeeded in calculating the controlling UEPs of all contingencies. Owing to space constraints, we provide only the results of 15 contingencies. Table IV presents the results of these contingencies using the homotopy-based approach.

TABLE IV
CONTROLLING UEPs FOR THE NE 39 BUS SYSTEM.

Opened Line		Controlling UEPs ($\delta_1, \delta_2, \dots, \delta_{10}$)				
From	To	(rad)				
1*	2	-1.3013	2.7573	2.6174	2.2003	2.2820
		2.21707	2.2400	2.2316	2.3452	1.8976
1	2*	-1.3013	2.7573	2.6174	2.2003	2.28205
		2.2171	2.2400	2.2316	2.3452	1.89760
1*	39	-1.3214	2.7956	2.6579	2.2356	2.3163
		2.2522	2.2748	2.2653	2.3758	1.9345
1	39*	-1.3214	2.7956	2.6579	2.2356	2.3163
		2.2522	2.2748	2.2653	2.3758	1.9345
2*	3	-0.1708	-0.1564	-0.1575	-0.1872	-0.1021
		-0.1707	-0.1474	2.5571	1.81520	-0.2649
2	3*	-0.9547	2.22144	2.1303	1.85222	1.96781
		1.8677	1.89528	0.7979	2.75037	0.00816
2*	25	-0.9795	2.58858	2.2507	1.71826	1.8343
		1.7308	1.75901	1.87089	2.00382	0.30096
2	25*	-0.93940	1.35430	1.41388	1.70457	1.83813
		1.71461	1.74534	2.73216	3.07945	0.07443
3*	4	-0.9903	2.00471	1.92278	1.61030	1.72400
		1.62467	1.65246	1.41327	3.15929	0.74249
3	4*	-0.0259	2.35525	0.17086	-0.3027	-0.2265
		-0.2867	-0.2647	-0.2244	-0.1602	-0.5248
3*	18	-0.0092	2.31206	0.01459	-0.3639	-0.2894
		-0.3467	-0.3248	-0.1515	-0.1371	-0.4528
3	18*	-0.0092	2.31206	0.01459	-0.3639	-0.2894
		-0.3467	-0.3248	-0.1515	-0.1371	-0.4528
4*	5	0.0104	2.33362	0.03656	-0.3865	-0.3129
		-0.3705	-0.3490	-0.2470	-0.1928	-0.5536
4	5*	0.0104	2.33362	0.03656	-0.3865	-0.3129
		-0.3705	-0.3490	-0.2470	-0.1928	-0.5536
4*	14	-0.0288	2.31479	0.01472	-0.2831	-0.2086
		-0.2665	-0.2448	-0.1601	-0.0940	-0.4726

*: Fault near this bus

V. CONCLUSION

This paper introduced the concept of using a homotopy-based approach in calculating controlling UEPs in transient stability analysis—direct methods. The potential use of the proposed homotopy-based approach in computing the controlling UEP was demonstrated through mathematical explanation and examples. The method is intended to overcome the problem of irregularity (fractal shape) of the region of convergence of the controlling UEPs and their sensitivities to the initial points that are encountered in iterative methods such as Newton methods. To test the performance of the proposed method, approximate exit points (rather than accurate) were utilized. The intrinsic low speed of mapping an easy and known solution to the desired solution of the homotopy-based approach is justified by using approximate exit points as initial points and eliminating the necessity of computing the MGP. This method was applied on several test systems and the results of the WECC and NE 39 test systems are provided.

REFERENCES

- [1] T. Athay, R. Podmore, and S. Virmani, "A Practical Method for the Direct Analysis of Transient Stability," *IEEE Trans. Power App. Syst.*, vol. PAS-98, no. 2, pp. 573–584, Mar. 1979.
- [2] A. Fouad and S. Stanton, "Transient Stability of a Multi-Machine Power System Part I: Investigation of System Trajectories," *IEEE Trans. Power App. Syst.*, vol. PAS-100, no. 7, pp. 3408–3416, July 1981.

- [3] P. Varaiya, F. Wu, and R. Chen, "Direct methods for transient stability analysis of power systems: Recent results," *Proceedings of the IEEE*, vol. 73, no. 12, pp. 1703–1715, Dec. 1985.
- [4] H. Chiang, F. Wu, and P. Varaiya, "Foundations of direct methods for power system transient stability analysis," *IEEE Trans. Circuits Syst.*, vol. 34, no. 2, pp. 160–173, Feb. 1987.
- [5] A. Fouad and V. Vittal, *Power system transient stability analysis using the transient energy function method*. Englewood Cliffs, NJ: Prentice-Hall, 1991.
- [6] H. Chiang, C. Chu, and G. Cauley, "Direct stability analysis of electric power systems using energy functions: theory, applications, and perspective," *Proceedings of the IEEE*, vol. 83, no. 11, pp. 1497–1529, Nov. 1995.
- [7] H. Chiang, *Direct methods for stability analysis of electric power systems: theoretical foundation, BCU methodologies, and applications*. John Wiley & Sons, 2011.
- [8] J. Thorp and S. Naqavi, "Load Flow Fractals," in *Proc. of the 28th IEEE Conference on Decision and Control*, vol. 2, Dec. 1989, pp. 1822–1827.
- [9] J. Thorp, S. Naqavi, and N. Chiang, "More Load Flow Fractals," in *Proc. of the 29th IEEE Conference on Decision and Control*, vol. 6, Dec. 1990, pp. 3028–3030.
- [10] H. Chiang, F. Wu, and P. Varaiya, "A BCU method for direct analysis of power system transient stability," *IEEE Trans. Power Syst.*, vol. 9, no. 3, pp. 1194–1208, Aug. 1994.
- [11] I. Nazareno, L. Alberto, and N. Bretas, "Problems in the precise determination of BCU's controlling unstable equilibrium points and PEBS's exit point method in real-time transient stability analysis," in *Proc. IEEE/PES Trans. and Dist. Conf. Expo.: Latin America*, Nov. 2004, pp. 475–480.
- [12] J. Lee and H. Chiang, "A singular fixed-point homotopy method to locate the closest unstable equilibrium point for transient stability region estimate," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 51, no. 4, pp. 185–189, Apr. 2004.
- [13] J. Lee, "A novel homotopy-based algorithm for the closest unstable equilibrium point method in nonlinear stability analysis," in *Proc. of the International Symp. on Circuits and Syst.*, vol. 3, May 2003, pp. III-8–III-11.
- [14] L. Chen, Y. Min, F. Xu, and K. Wang, "A Continuation-Based Method to Compute the Relevant Unstable Equilibrium Points for Power System Transient Stability Analysis," *IEEE Trans. Power Syst.*, vol. 24, no. 1, pp. 165–172, Feb. 2009.
- [15] R. Treinen, V. Vittal, and W. Kliemann, "An improved technique to determine the controlling unstable equilibrium point in a power system," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 43, no. 4, pp. 313–323, Apr. 1996.
- [16] J. Lee and H. Chiang, "Convergent regions of the Newton homotopy method for nonlinear systems: theory and computational applications," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 48, no. 1, pp. 51–66, Jan. 2001.
- [17] A. Llamas, J. De La Ree Lopez, L. Mili, A. Phadke, and J. Thorp, "Clarifications of the BCU method for transient stability analysis," *IEEE Trans. Power Syst.*, vol. 10, no. 1, pp. 210–219, Feb. 1995.
- [18] A. Xue, S. Mei, and B. Xie, "A comprehensive method to compute the controlling unstable equilibrium point," in *Third International Conference on Electric Utility Deregulation and Restructuring and Power Technologies*, Apr. 2008, pp. 1115–1120.
- [19] J. Lee, "An optimization-driven framework for the computation of the controlling UEP in transient stability analysis," *IEEE Trans. Autom. Control*, vol. 49, no. 1, pp. 115–119, Jan. 2004.
- [20] D. Wolf and S. Sanders, "Multi-parameter homotopy methods for finding DC operating points of nonlinear circuits," in *Proc. of the IEEE International Symposium on Circuits and Systems*, vol. 4, Chicago, IL, USA, May 1993.
- [21] L. Watson, "Globally convergent homotopy methods: a tutorial," *Applied Mathematics and Computation*, vol. 31, pp. 369–396, 1989.
- [22] R. Kalaba and L. Tesfatsion, "Solving nonlinear equations by adaptive homotopy continuation," *Applied Mathematics and Computation*, vol. 41, no. 2, pp. 99–115, 1991.
- [23] J. Alexander and J. Yorke, "The homotopy continuation method: numerically implementable topological procedures," *Transactions of the American Mathematical Society*, vol. 242, pp. 271–284, 1978.
- [24] University of Washington. Power Systems Test Case Archive. [Online]. Available: <http://www.ee.washington.edu/research/pstca>