

Power Flow Analysis of Radial and Weakly Meshed Distribution Networks

S. Elsaiah, *Student Member*, M. Ben-Idris, *Student Member*, and J. Mitra, *Senior Member, IEEE*

Electrical and Computer Engineering Department- Michigan State University

East Lansing, Michigan 48824, USA

email: {elsaiahs@msu.edu}

Abstract—With increasing penetration of distributed generation, and the evolution of predominantly radial distribution systems into weakly meshed networks, it has become necessary to suitably update power flow solution techniques. This paper presents an unbalanced three-phase power flow solution method that is effective in the analysis of modern active distribution networks. The proposed approach is based on the development of a Branch-Current Matrix (*BCM*), and uses it in conjunction with the line Primitive Impedance Matrix (*PIM*) to obtain the power flow solution. This approach is initially applied to radial networks and then modified to include the case of weakly-meshed networks. The presented approach takes into account most of the special features of the distribution networks and uses neither LU-decomposition, nor Y-bus formulation/factorization and hence it is computationally efficient. The proposed method is tested on 13-node and 76-node unbalanced networks. Results of the proposed method are compared with some other methods presented in the literature.

Index Terms—active distribution networks, power flow, radial system, weakly-meshed system.

I. INTRODUCTION

In recent years, several real-time engineering applications that include operational and planning stages require fast, flexible, and robust repeated power flow solutions. The power flow solution determines the steady-state voltage phase and magnitude at all buses, real and reactive power flows in each line, power losses, and reactive power required by the PV buses for specified loading conditions. The two common methods which have been widely used in solving the power flow problem are the Gauss-Seidel (GS) and the Newton-Raphson (NR) method [1]. The GS method is known as a slow-iterative problem-solving technique as it requires a full-formulation of the bus-admittance matrix (Y_{bus}) and usually requires the solution of a set of nonlinear equations whose cardinality approximately equals the number of system buses. The NR method is basically a tangential approximation technique in which the line parameters are stored in the Jacobian matrix. The formulation of the Jacobian matrix, however, turns to be computationally cumbersome in terms of execution time and storage requirement. Moreover, it was observed that these two classical power flow methods, which

were essentially developed for solving power flow problems at the transmission level, can encounter convergence problems when directly applied to power distribution networks. The Fast-Decoupled (FD) method [2, 3], which is a modification of the NR method, uses an approximate and constant Jacobian that ignores the dependencies between (a) real power and voltage magnitude, and (b) reactive power and voltage angle. This renders the FD method fast and effective for transmission systems that have high reactance to resistance ratios. However, these methods are not very suitable for distribution networks, because of the following characteristics of such systems [4-6].

- Radial structure with sometimes weakly-meshed topology.
- High resistance to reactance (R/X) ratio which sometimes causes the NR and the FD methods to diverge.
- Untransposed or rarely transposed lines where it is often inappropriate to neglect the mutual coupling between phases.
- Unbalanced loads along with single-phase and double-phase laterals.
- Unbalanced distributed loads.
- Dispersed generation.

These characteristics, combined with the large number of nodes and branches of distribution networks make the direct use of the aforementioned techniques unsuitable and inefficient for power flow studies of unbalanced distribution systems. In other words, distribution networks can be considered as ill-conditioned power networks in which three-phase basis should be used rather than single-phase basis. However, it is worthwhile to point out here that the unbalanced three-phase power flow algorithms cannot be developed directly by extending balanced single-phase methods to unbalanced three-phase ones. This means that balanced three-phase line model, for instance, will not be useful for unbalanced power flow studies. Therefore, for unbalanced power flow analysis, various distribution network components should be accurately modeled according to three-phase basis.

Several power flow solution algorithms especially designed for distribution networks are proposed in the literature. Some methods used modified versions of NR method and its decoupled form while others are based on the backward/forward sweeping technique, which can be classified as [7],

- Current summation methods (CSM),
- Admittance summation methods (ASM),
- Power summation methods (PSM),

The CSM only uses V and I instead of P and Q , therefore it is more convenient and faster than the ASM and PSM [8] and was adapted in this paper.

In [9] a three-phase fast-decoupled power flow method is developed. The method is based on NR method but the Jacobian matrix was decoupled on phases and on real and imaginary parts. The proposed method was successfully able to compensate for the assumptions of the traditional FD method with minimum data preparation. Zimmerman and Chiang presented a fast decoupled load flow method in [10]. In this approach, a set of nonlinear power mismatch equations are formulated and solved by Newton's method. This approach utilized the laterals instead of buses, so the problem size was reduced to the number of system laterals. Use of laterals as variables makes this algorithm more efficient for a given system topology, however it may add some difficulties if the network topology is changed.

Shirmohammadi et al. [11] presented a compensation based method for power flow analysis of 'balanced' distribution systems. Basically, the method used KCL and KVL to obtain branch currents and bus voltages and then a forward/backward sweep is applied to obtain the power flow solution. The same method was extended to include the weakly meshed networks by breaking the given system to a number of points- "breakpoints"- and hence a simple radial network can be obtained. The radial network was then solved by the direct application of KCL and KVL laws. The effectiveness of this method, however, diminishes as the number of breakpoints goes up. As a result, the application of this method to the weakly-meshed networks was practically restricted. An algorithm for the power flow solution of unbalanced distribution networks was developed in [12] by Cheng and Shirmohammadi. This three-phase method can be considered as an extension to the work done in [11], but it has dealt with the modeling of the PV buses and emphasized on the modeling of various distribution system components and it was successfully applied for real-time distribution systems.

A direct approach to obtain the distribution power flow solution was presented in [13] by Teng. Two matrices, the bus injection to branch current $BIBC$ and the branch current to bus voltage $BCBV$ and direct matrix multiplication are used to obtain the distribution power flow solution. This method uses direct matrix multiplication; therefore the computational burden will be increased. Further, these matrices contain many zero elements, so the memory space is not economically utilized, and large CPU time is required.

This paper introduces a method for power flow solution based on the work done in [13]. While ref. [13] provided a

formulation of three matrices, $BIBC$, $BCBV$, and DLF , our work provides a detailed formulation and implementation of the Branch Current Matrix BCM which is directly used in conjunction with the primitive impedance Matrix PIM to obtain the solution. The method described in this paper neither use (Y_{bus}) formulation as it is necessary in GS method, nor include the development of the Jacobian matrix that is needed in the standard NR method. In our method, instead of calculating the voltage drop with respect to the root bus, it was calculated between any two seceding buses which help in reducing the matrix size and more importantly minimize the zero elements in the resultant matrix. Compared to [13], the resultant power flow solution matrix is less-sparse, which means that the LU-decomposition might not be necessary especially for small-scale distribution systems and considerable amount of memory-space is saved. For instance, for a system with n - buses and m - branches, the method of [13] involves the formulation of a matrix whose dimensions of $3m \times 3(n - 1)$ to obtain the distribution power flow solution. However, the product of the BCM and the PIM in the presented method results in a matrix with dimensions of $3m \times 3$ to obtain the solution. This means that the saving in memory space memory will be $3m \times 3(n - 2)$ as it will be given later. In the presented work, the BCM has to be formulated first and then the term by term product of BCM and PIM (.* in Matlab) gives the solution. To account for the weakly-meshed networks, we need to include the new existing branch in the BCM and correspondingly its impedance in the PIM . The algorithm which was used for the radial systems is modified to account for the weakly-meshed networks based on Kirchhoff's current and voltage laws. In this manner, the branches that contribute in existing of the loop have to be introduced in the original BCM while those outside the loop will retain the same mathematical expressions. A flexible subroutine has been integrated with the main algorithm to account for this purpose. Results of 13-node and 76-node unbalanced distribution Networks show that the method converges even for higher R/X ratios with savings in the computational burden.

This work is organized as follows: in section II various distribution network components are modeled. Solution procedures are described in section III. Study cases are given in section IV. While test results and some discussions are given in section V, concluding remarks and future work are drawn in section VI.

II. MODELING ASPECTS

Due to the special features of the distribution networks, much attention has to be paid in dealing with the modeling of the various system components. This section discusses the network components that were used in the presented power flow solution.

A. Modeling of Unbalanced Three-Phase Line Section

For unbalanced distribution power flow analysis it is important to accurately model three-phase, two-phase, and single-phase line sections. A four-wire distribution network is

assumed in this paper as this type of systems is widely used worldwide. Fig.1 shows a three-phase line section connected between two buses m and n .

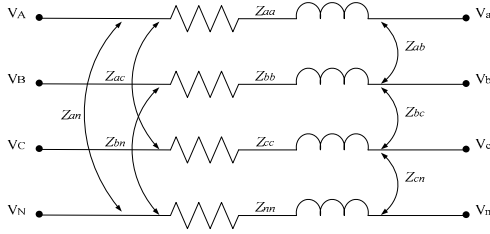


Fig. 1 Three-phase line section model

The line parameters can be found by the method developed by Carson and Lewis [14]. A 4×4 sized primitive matrix which takes the effect of the self-and-mutual coupling between phases can be expressed as,

$$[Z_{abcn}] = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{an} \\ Z_{ba} & Z_{bb} & Z_{bc} & Z_{bn} \\ Z_{ca} & Z_{cb} & Z_{cc} & Z_{cn} \\ Z_{na} & Z_{nb} & Z_{nc} & Z_{nn} \end{bmatrix} \quad (1)$$

However, it is convenient to represent (1) as a 3×3 matrix instead of the 4×4 matrix by using Kron's method. The effect of the ground conductor is still included in the resultant matrix, that is

$$[Z_{abc}] = \begin{bmatrix} Z_{aa-n} & Z_{ab-n} & Z_{ac-n} \\ Z_{ba-n} & Z_{bb-n} & Z_{bc-n} \\ Z_{ca-n} & Z_{cb-n} & Z_{cc-n} \end{bmatrix} \quad (2)$$

Now by applying the KVL to the circuit model of Fig.1, the relationships between the bus voltages and branch currents can be simply written as,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} - \begin{bmatrix} Z_{aa-n} & Z_{ab-n} & Z_{ac-n} \\ Z_{ba-n} & Z_{bb-n} & Z_{bc-n} \\ Z_{ca-n} & Z_{cb-n} & Z_{cc-n} \end{bmatrix} \begin{bmatrix} I_{Aa} \\ I_{Ba} \\ I_{Ca} \end{bmatrix} \quad (3)$$

It should be noted, however, that single-phase and two-phase line sections are most common in distribution networks. Hence, in this research for any phase fails to present, the corresponding row and column in (3) will have zeros entries. For example, for two phase line section with a and b phase, equation (3) will be reduced to,

$$[Z_{ab}] = \begin{bmatrix} Z_{aa-n} & Z_{ab-n} \\ Z_{ba-n} & Z_{bb-n} \end{bmatrix} \quad (4)$$

Also, for single phase line section, equation (3) will be reduced to,

$$V_a = V_A - Z_{aa-n} I_{Aa} \quad (5)$$

B. Modeling of Loads

Loads in distribution network can be represented as spot loads or distributed loads. The following section represents the modeling of both types of loads.

1) Lumped Loads

Equivalent current injection technique was used in this research to represent the distribution network loads. This is attributed in the first place to the nature of the loads in the distribution networks which are inherently unbalanced loads. It is assumed that all three-phase loads are connected Y or Δ , and all single-phase and two-phase loads have connections between line and neutral and line-to-line, respectively. Further, a constant power model at each bus was assumed during the realization of this work.

a) Y-connected loads

Fig. 2 shows a three-phase Y-connected unbalanced load model.

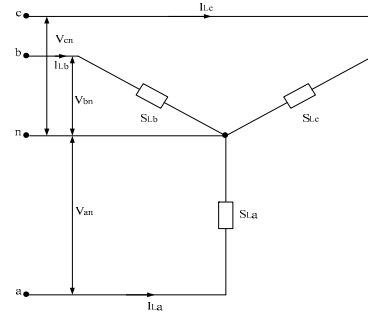


Fig.2 Unbalanced Y-connected load

The load current injections at the k^{th} bus for three-phase Y-connected or single-phase connected line-to-neutral can be expressed as,

$$\begin{bmatrix} I_a^k \\ I_b^k \\ I_c^k \end{bmatrix} = \begin{bmatrix} \left(\frac{S_{La}^k}{V_a^k} \right)^* \\ \left(\frac{S_{Lb}^k}{V_b^k} \right)^* \\ \left(\frac{S_{Lc}^k}{V_c^k} \right)^* \end{bmatrix} = \begin{bmatrix} \frac{P_{La}^k - jQ_{La}^k}{V_a^{k*}} \\ \frac{P_{Lb}^k - jQ_{Lb}^k}{V_b^{k*}} \\ \frac{P_{Lc}^k - jQ_{Lc}^k}{V_c^{k*}} \end{bmatrix} \quad (6)$$

b) Δ -connected loads

Referring to Fig. 3 shown below, the current injections at the k^{th} bus for three-phase loads connected in Δ or single-phase connected line-to-line can be expressed as,

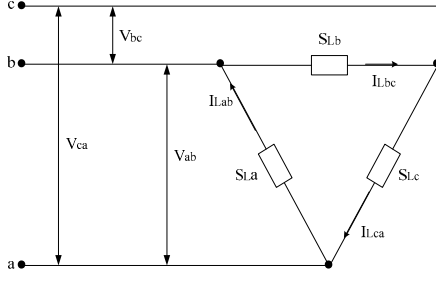


Fig.3 Unbalanced Δ -connected load

$$\begin{bmatrix} I_a^k \\ I_b^k \\ I_c^k \end{bmatrix} = \begin{bmatrix} \frac{P_{Lab}^k - jQ_{Lab}^k}{V_a^{k*} - V_b^k} - \frac{P_{Lca}^k - jQ_{Lca}^k}{V_c^{k*} - V_a^k} \\ \frac{P_{Lbc}^k - jQ_{Lbc}^k}{V_b^{k*} - V_c^k} - \frac{P_{Lab}^k - jQ_{Lab}^k}{V_a^{k*} - V_b^k} \\ \frac{P_{Lca}^k - jQ_{Lca}^k}{V_c^{k*} - V_a^k} - \frac{P_{Lbc}^k - jQ_{Lbc}^k}{V_b^{k*} - V_c^k} \end{bmatrix} \quad (7)$$

2) Distributed Loads

Distributed loads are modeled in a similar way to that developed by Cheng and Shirmohammadi [12]

C. Distribution Transformers

For unbalanced networks, the conventional models which were derived on balanced three-phase assumptions will not be useful for power flow analysis. Ref. [15] developed a transformer model for short circuit, contingency, and power flow studies which can be used here for the power flow analysis.

D. Modeling of Capacitors

Capacitors are assumed of Y-connected with ground conductor. The current injections are given as [16],

$$\begin{aligned} I_{cap}^a &= -\frac{jQ_{nom}}{|V_{nom}|^2} V_a^k \\ I_{cap}^b &= -\frac{jQ_{nom}}{|V_{nom}|^2} V_b^k \\ I_{cap}^c &= -\frac{jQ_{nom}}{|V_{nom}|^2} V_c^k \end{aligned} \quad (8)$$

E. Modeling of Shunt Capacitances

Charging capacitors have a noticeable influence on system's voltage profile. In section A, a model for unbalanced three-phase line section is introduced. This model can be further improved by including the line shunt (charging) capacitors. A model takes into account the effect of the self and mutual capacitance is shown in Fig. 4.

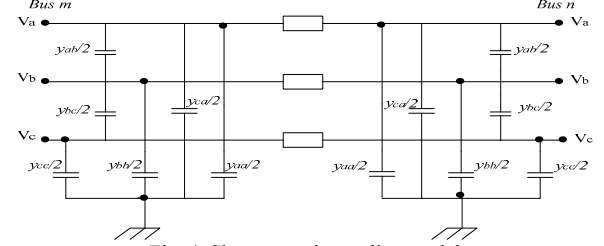


Fig. 4. Shunt capacitance line model

The current injections at bus n can be written as,

$$\begin{bmatrix} I_{chg}^a \\ I_{chg}^b \\ I_{chg}^c \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -Y_a & y_{ab} & y_{ac} \\ y_{ba} & -Y_b & y_{bc} \\ y_{ca} & y_{cb} & -Y_c \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (9)$$

Where,

$$\begin{aligned} Y_a &= y_{aa} + y_{ab} + y_{ac} \\ Y_b &= y_{ba} + y_{bb} + y_{bc} \\ Y_c &= y_{ca} + y_{cb} + y_{cc} \end{aligned} \quad (10)$$

III. APPROACH AND IMPLEMENTATION

The proposed approach is based on the development of a Branch-Current Matrix (BCM), and use it in conjunction with the line Primitive Impedance Matrix (PIM) obtain the power flow solution. This section describes the procedures of the proposed method.

A. Equation Development

Let us consider the 7-bus, 6-branch radial distribution network shown below in Fig. 5.

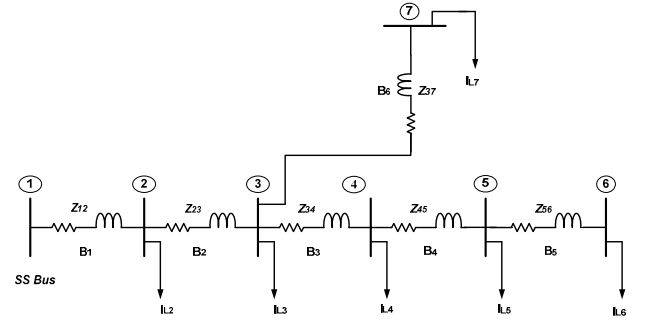


Fig. 5. 7-bus distribution system

The complex load at bus i can be expressed in terms of the real and reactive power as,

$$S_i = P_i + jQ_i \quad (11)$$

Then, the corresponding current injected at the same node can be further described as,

$$I_i = \left(\frac{S_i}{V_i} \right)^* \quad (12)$$

The relationships between the bus current injections, branch currents, and voltage at the various system buses can be obtained by applying Kirchhoff's current law *KCL* and Kirchhoff's voltage law *KVL*, respectively. The branch currents can be expressed by equivalent current injections as,

$$\begin{aligned} B_1 &= I_{L2} + I_{L3} + I_{L4} + I_{L5} + I_{L6} + I_{L7} \\ B_2 &= I_{L3} + I_{L4} + I_{L5} + I_{L6} + I_{L7} \\ B_3 &= I_{L4} + I_{L5} + I_{L6} \\ B_4 &= I_{L5} + I_{L6} \\ B_5 &= I_{L6} \\ B_6 &= I_{L7} \end{aligned} \quad (13)$$

The branch currents can generally be described in compact form as,

$$[BCM] = [B_1 \ B_2 \ B_3 \ B_4 \ B_5 \ B_6]^T \quad (14)$$

Similarly, the branches' primitive impedance vector between buses i and j can be given as,

$$[Z] = [Z_{12} \ Z_{23} \ Z_{34} \ Z_{45} \ Z_{56} \ Z_{37}] \quad (15)$$

Now, the relationship between branch currents and bus voltages can be determined by applying the *KVL*, as will be illustrated in the algorithm. The step-by-step development of the Branch-Current-Matrix *BCM* is explained in section B.

Algorithm Development

BCM Formulation

1. Number the system buses from 1 to n where n is the number of system buses and let 1 denotes the substation bus.
2. Label the branch currents such that the branch between buses i and j is B_{j-1} .
3. Form the branch currents matrix, $BCM \in \mathbb{R}^{3(n-1) \times 3(n-1)}$, as follows:
 - i. Start from bus n phase C , fill the element $BCM_{3(n-1),3(n-1)}$ in the branch current matrix with 1.
 - ii. Search for bus n phase C in the line section data in the receiving end buses column and determine the number of the bus that is connected to bus n phase C in the sending end buses column, e.g., j .
 - iii. If $j \neq 1$, fill the element $B_{3(j-1),3(n-1)}$ in the branch current matrix with 1 and let $n = j$ and go to ii, otherwise go to iv.

- iv. Reduce n by one and check if $n - 1 \geq 1$, repeat sub-steps i, ii and iii, otherwise stop and go to sub-step v
- v. Repeat for phases A and B such that for phase B use $BCM_{3(n-1-1),3(n-1-1)}$ and $B_{3(j-1-1),3(n-1-1)}$; for phase A use $BCM_{3(n-1-2),3(n-1-2)}$ and $B_{3(j-1-2),3(n-1-2)}$.

Power Flow Solution

1. Form the branch currents matrix, $BCM \in \mathbb{R}^{3(n-1) \times 3(n-1)}$
2. For flat start, all nodes' voltages are assumed to be $1.0 + j0.0$ except for the voltages of the substation nodes which are specified.
3. Calculate node currents from the following equation,

$$I_j = \frac{S_j^*}{V_j^*} + Y_j V_j$$

Where j is the node number, S is load complex power and Y_j is the node shunt admittance

4. Calculate the branch currents vector as follows:

$$B = [BCM] [I]$$

where $[I]$ is the nodes injected currents vector,

$$[I] = \begin{bmatrix} I_{2A} \\ I_{2B} \\ \vdots \\ I_{nB} \\ I_{nC} \end{bmatrix}$$

5. Calculate the branches' voltage drops,

$$[\Delta V_i] = [B_i][Z_i]$$

Where i is the branch number, $[B_i]$ is current matrix of branch i and $[Z_i]$ is primitive impedance matrix of branch i

6. Starting from the substation nodes towards the subsequent nodes, calculate the new node voltages as follows: for nodes i and j which are connected through branch B_{j-1} ,

$$V_j = V_i - (\Delta V)_{j-1}$$

7. After calculating all node voltages, calculate the maximum error between the new voltages and the voltages of the previous step.

$$e_{max} = \max\{|V_{2A}^i - V_{2A}^{i+1}|, \dots, |V_{nC}^i - V_{nC}^{i+1}|, |angle(V_{2A}^i) - angle(V_{2A}^{i+1})|, \dots, |angle(V_{nC}^i) - angle(V_{nC}^{i+1})|\}$$

If the maximum error is less than or equal to the specified tolerance, go to step 9, otherwise continue to step 8.

8. Use new voltage values and repeat steps 3 to 7.

9. Print out the power flow results.

Solution for Weakly-Meshed Networks

It is well known that, loops can exist in distribution networks due to the opening/closing of the tie-switches. Our approach here is to account only for weakly-meshed networks. Now, let us consider the same system given before and repeated here in Fig. 6 when some loops took place. One loop, for instance, can be formulated by connecting a line between buses, ($q = 6, r = 7$). This branch connected a lateral l with the main feeder m , and hence it is denoted as B_{ml} . Back to the set of equations (13), it can be observed that this branch will affect only the branches in the loop i.e. $B_3 - B_6$. The rest

will have the same mathematical expressions. Another loop can exist if a line is connected between buses, ($q = 7, r = 5$). In this case, branches which are going to be modified in the (BCM_{old}) are $B3, B4$, and $B6$. So, instead of having new factious bus which could contribute in increasing the computational burden, this principle has been considered and a brief/general procedure is given below for this purpose,

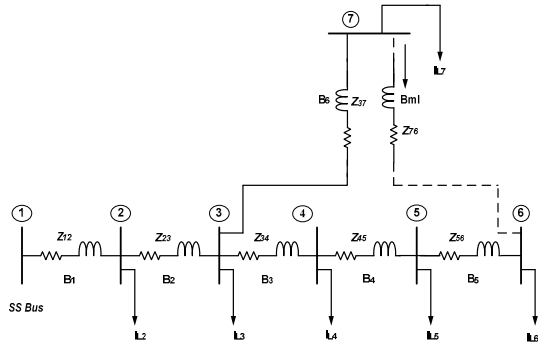


Fig. 6. 7-bus distribution system

$$B_{ml} = \frac{V_r - V_q}{Z_{qr}} + V_r Y_r - V_q Y_q$$

- iv. If a lateral originates at bus s and ends at q , include $-B_{ml}$ to all the B 's between s , and r (i.e. to the B 's on the main feeder) and $+B_{ml}$ to the B 's between s , q (i.e. to the B 's on the lateral itself) in (13).

IV. NUMERICAL EXAMPLES

To check the convergence characteristics and the accuracy of the proposed method, it was implemented on an 8-bus (equivalent to 13-node) network [13]. This test system consists of a main feeder trunk with 3x90 phase buses and all single-phase and double-phase laterals have been lumped to form unbalanced loads for testing purposes. This system is characterized by its high R/X ratio (approximately 1.889), which makes it an ill-conditioned distribution network. The base values for this system are chosen to be 14.4 KV and 100 KVA. More details about the system can be found in [13]. The second test system is a modified 33-bus (equivalent to 76-node) unbalanced network [17] and is depicted in Fig.8 for explanation purposes. System data are modified to increase the amount of the unbalance between the system's phases and loads. Test results and discussion are provided in the following section.

- i. For a distribution line connected between buses q and r , $r > q$, denote the new branch current as B_{ml} .
- ii. For flat start, all nodes' voltages are assumed to be $1.0 + j0.0$ except for the voltages of the substation nodes, which are specified.
- iii. The branch current B_{ml} can be calculated from the following relation,

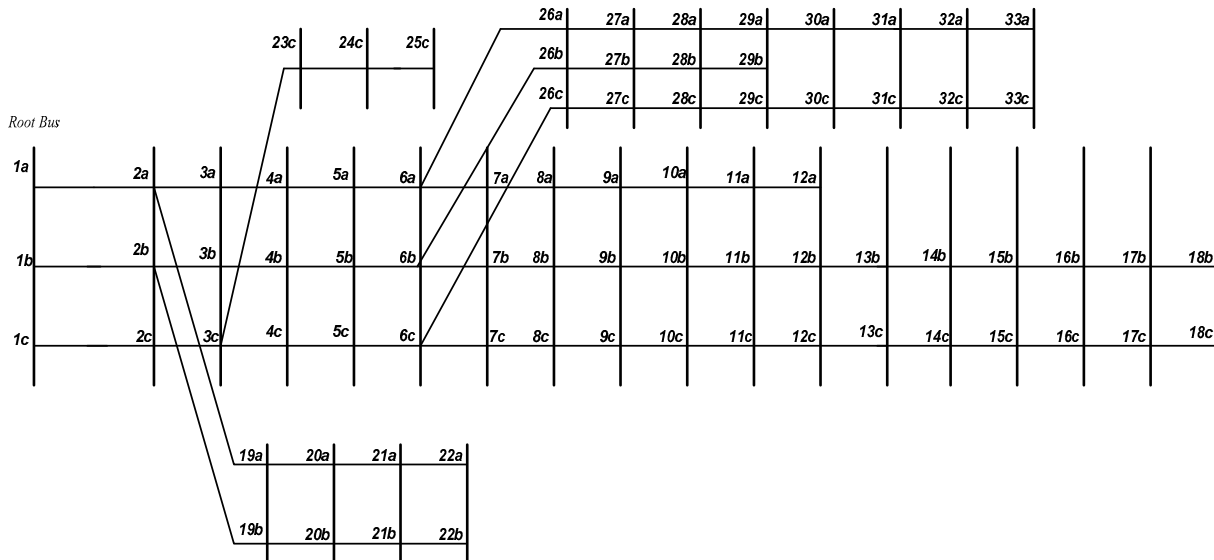


Fig. 8 a 76-node Unbalanced distribution network

V. TEST RESULTS AND DISCUSSION

The proposed three-phase power flow solution is carried out using Matlab 7.9® code with a tolerance of $1e-4$ on a Centrino Intel® Core™ 2 Duo Processor T5800 computer. Two methods have been used in the tests for comparison purposes. These methods are,

Method I: The proposed method.

Method II: The method reported by Jen-Hao Teng [13].

The above two methods were compared on the basis of their performance parameters.

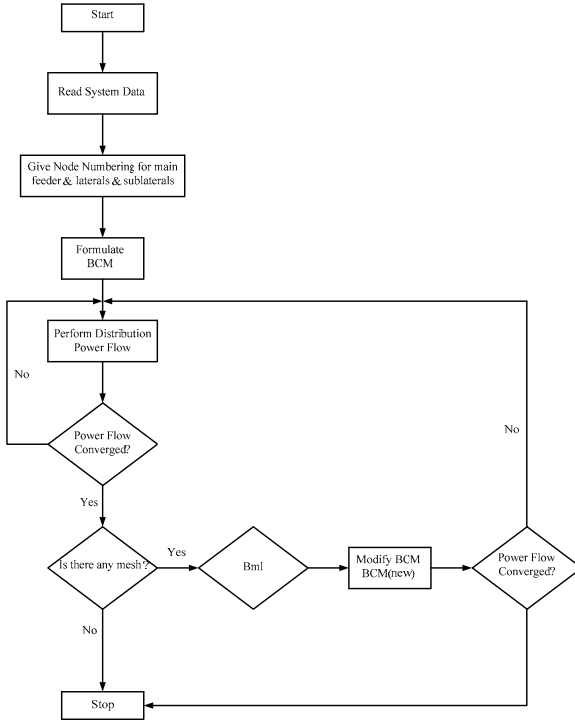


Fig. 7. Flow chart of power flow calculation

The power flow results of the 13-node system are given in Table I. The method converged in three iterations. Method II took three iterations to converge too. The maximum deviations in both voltage magnitude and angle from Method II are 0.0001 and 0.0019, respectively. Though Method II has the same number of iterations, the memory space and, hence, the computational time taken by method II is higher than the proposed method since Method II involves the formulation of a matrix whose dimensions of $3m \times 3(n-1)$ to obtain the distribution power flow solution, where n is the number of system buses and m is the number of system branches. The product of the *BCM* and the *PIM* in the presented method, however, results in a matrix with dimensions of $3m \times 3$ to obtain the distribution power flow solution. This means that the saving in memory space memory will be $3m \times 3(n-2)$. Fig.9 and 10 show the saving in memory space by using the proposed method. For the weakly-meshed system, a

branch of $Z = (2.5 + 1.5i) \Omega/m$ was added between some nodes of the 13-node network where the method took five iterations to converge. Also, another branch was introduced between 6c and 8c with different lengths to have the case of multi-loop system. In this case the method took also five iterations to converge when the impedance was high and gave similar results to the radial system, which indicates that, the method well-converged. Table II shows the results of this case. The results of the 76-node system are only drawn in Fig.11-13 for a-phase, b-phase, and c-phase respectively due to space limitations. For the 76-node meshed network, the number of iterations and the average execution time are given in Table III and from which we can see that the number of iterations is stable. Table III shows also that the average execution time was increased as the number of meshes increased. This can be partly attributed to the increase of the BCM and PIM elements. We also examined the convergence characteristics of the proposed method by changing the *R/X* ratio up certain higher values and acceptable results are obtained.

VI. CONVERGENCE TEST OF THE PROPOSED METHOD

Divergence of the solution of the power flow problem usually occurs when certain ill-conditioned are presented in the system. As it is mentioned earlier in Section I, distribution networks with high *R/X* ratio are inherently ill-conditioned power networks. Ill-condition often takes place when the system involves short-lines, which is the case in this paper, or even very long lines. In order to examine the convergence ability of the proposed method, a convergence test was done on the 13-node distribution network. Firstly, selected primitive impedances were taken from each section of the system sections at random basis. Secondly, among the selected impedances, one is divided by a factor of 8 to represent a short-line while the other is multiplied by 8 to have a long line representation. The method did converge, yet the computational speed was decreased.

VII. INCLUSION OF DG SOURCES

To include the effect of the Distributed Energy Resources DER's into the proposed power flow algorithm, some PV panels operated at the MPP and interfaced with high frequency converters are installed at certain buses. Most of these resources are connected at the buses at which the voltage magnitude is mostly violated. For phase a, the PV's were installed at 7 nodes and were totaling of 1040 KW. For phase's b and c, these PV's are assumed to be installed at 5 nodes and 10 nodes and were totaling 660KW and 1850 KW, respectively. It is observed that most voltages are now becoming with the specified limit of 5% and the total real power losses flowing in the distribution feeders have been tremendously decreased as depicted in Fig. 11-13 and Fig. 14, respectively.

TABLE I CASE OF RADIAL 13 NODE POWER FLOW SOLUTION

Bus Number & Phase	Method 1		Method 2	
	V (p.u.)	Angle (Deg.)	V (p.u.)	Angle (Deg.)
1 - A	1.0000	0.0000	1.0000	0.0000
1 - B	1.0000	-120.0	1.0000	-120.0
1 - C	1.0000	120.0	1.0000	120.0
2 - A	0.9839	0.1830	0.9839	0.1830
2 - B	0.9711	-119.76	0.9712	119.76
2 - C	0.9697	119.97	0.9697	119.97
3 - A	0.9832	0.1790	0.9832	0.1780
4 - B	0.9652	-119.73	0.9652	-119.73
4 - C	0.9668	119.93	0.9669	119.93
5 - B	0.9640	-119.74	0.9640	-119.74
6 - C	0.9649	119.92	0.9650	119.92
7 - C	0.9683	119.96	0.9683	119.96
8 - C	0.9671	119.96	0.9671	119.95

TABLE II CASE OF 13 NODE POWER FLOW SOL. WITH SOME LOOPS

Bus Number & Phase	$Z_{mm} = Z_{2c} + Z_{8c}$ (2.5 + 1.5i) × l		$Z_{ml} = Z_{6c} + Z_{8c}$ (2.5 + 1.5i) × l	
	V (p.u.)	Angle (Deg.)	V (p.u.)	Angle (Deg.)
1 - A	1.0000	0.0000	1.0000	0.0000
1 - B	1.0000	-120.0	1.0000	-120.0
1 - C	1.0000	120.0	1.0000	120.0
2 - A	0.9839	0.1837	0.9839	0.1830
2 - B	0.9711	-119.76	0.9712	119.76
2 - C	0.9697	119.97	0.9697	119.97
3 - A	0.9832	0.1802	0.9832	0.1780
4 - B	0.9652	-119.73	0.9652	-119.73
4 - C	0.9669	119.93	0.9669	119.93
5 - B	0.9640	-119.74	0.9640	-119.74
6 - C	0.9649	119.92	0.9650	119.92
7 - C	0.9685	119.85	0.9683	119.96
8 - C	0.9676	119.64	0.9671	119.95

TABLE III CASE OF WEAKLY-MESHED 76 NODE SYSTEM

No. of created loops	Average Execution Time in sec.	N0. Of iterations taken
0	3.1503	5
1	3.2200	5
2	3.2000	8
3	3.3860	8
4	3.3800	8

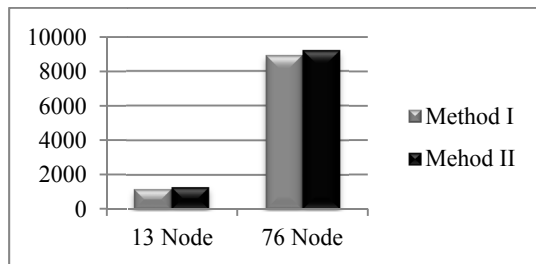


Fig. 9 Memory space comparison in bit

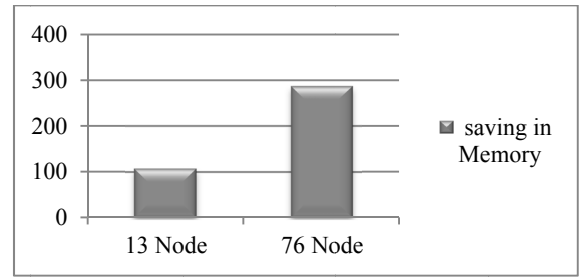


Fig. 10 Memory space comparison in bit

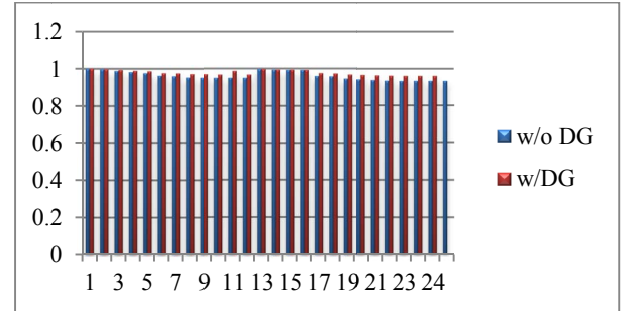


Fig.11 Phase (a) Voltage With and without DG sources

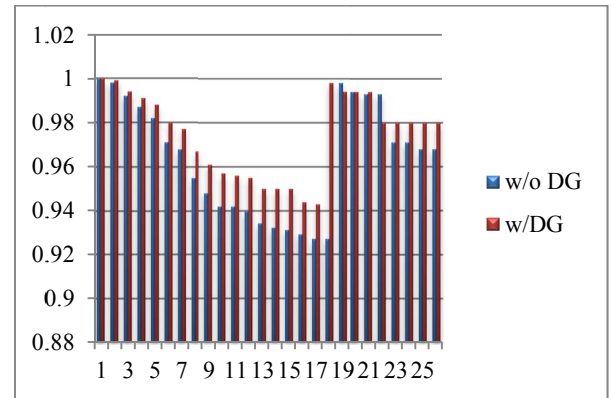


Fig. 12 Phase (b) Voltage With and without DG sources

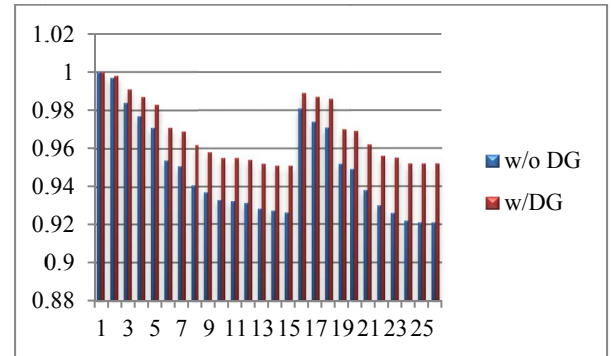


Fig. 13 Phase (c) Voltage With and without DG sources

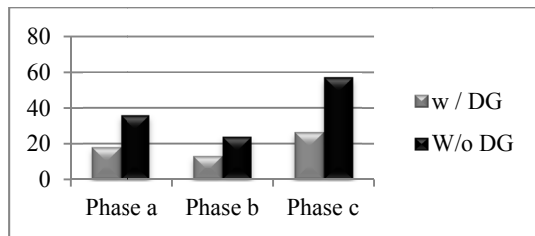


Fig. 14 Real power losses in kw with and without DGs

VIII. CONCLUSION

This work presented a power flow solution method for weakly meshed active distribution networks. The method utilizes basic circuit theory fundamentals. The presented work has not utilized any Y-bus formulation or factorization. Also, the LU-decomposition has not been used in the presented study as the resultant matrix contains a few zero elements. The proposed approach can easily be modified to incorporate constant current, constant impedance or exponential load models. It is also worthwhile to point out here that more work has to be done to improve the mathematical formulation and to check convergence ability of the proposed method for large distribution systems.

IX. REFERENCES

- [1] W.M. Lin and J.H.Teng, "Phase-decoupled load flow method for radial and weakly-meshed distribution networks," *IEE Gener. Transm. Distrib.*, vol.143, No1.1, pp. 39-42 January 1996.
- [2] B. Stott, "Decoupled Newton load flow," *IEEE Trans. Power App. Syst.*, vol. PAS-91, no. 5, pp. 1955-1959, Sep. 1972.
- [3] B. Stott, "Review of load-flow calculation methods," *Proc. IEEE*, vol. 62, no. 7, pp. 916-929, Jul. 1974.
- [4] W. H. Kersting, "Distribution system modeling and analysis," Book, CRC Press 2002.
- [5] Y. Zhu, and K. Tomsovic, "Adaptive power flow method for distribution systems with dispersed generation," *IEEE Trans. Power Del.*, vol.17, no. 3, pp. 822-827, July 2002.
- [6] G.X. Luo, and A. Semlyen, "Efficient load flow for large weakly-meshed networks," *IEEE Trans. Power Syst.*, vol.5, no. 4, pp. 1309-1316, Nov. 1990.
- [7] M.S. Srinivas, "Distribution load flows: A brief review," *IEEE-PES Winter meeting*, Vol. 2, pp.942-945, Jan. 2000.
- [8] S. Khushalani, and N. Schulz, "Unbalanced distribution power flow with distributed generation," *IEEE Transmission and Dist. and Trans. Conf. and Exposition*, 2006.
- [9] J. H. Teng, "A novel and fast three-phase load flow for unbalanced radial distribution systems," *IEEE Trans. Power Syst.*, vol.17, no. 4, pp. 1238-1244, Nov. 2002.
- [10] R. D. Zimmerman and H.D. Chiang, "Fast decoupled load flow for unbalanced radial distribution systems," *IEEE-PES Winter meeting*, Paper no.95, New York, 1995.
- [11] D. Shirmohammadi, H. W. Hong, A. Semlyen, and G. X. Luo, "A compensation-based power flow method for weakly meshed distribution and transmission networks," *IEEE Trans. Power Syst.*, vol. 3, no. 2, pp. 753-762, May 1988.
- [12] C. Cheng, and D. Shirmohammadi, "A Three-Phase power flow method for real-time distribution systems analysis," *IEEE Trans. Power Sys.*, vol.10, no.2, May 1995.
- [13] J. H. Teng, "A direct approach for distribution system load flow solution," *IEEE Trans. Power Del.*, vol. 18, no. 3, pp. 882-887, July 2003.
- [14] IEEE Tutorial Course on Power Distribution Planning.
- [15] Tsai-Hsiang Chen, Mo-Shing Chen, T. Inoue, P. Kotas, and E. A. Chebli, "Three-phase cogenerator and transformers models for distribution system analysis," *IEEE Trans. Power Del.*, vol.6, no. 4, pp. 1671-1678, Oct. 1991.
- [16] J. Vieira Jr, W. Freitas, and A. Morelato, "Phase-decoupled method for three-phase power-flow analysis of unbalanced distribution systems," *IEE Proc.-Gener., Transm., Distrib.*, vol. 151, no. 5, pp. 568-574, Sep. 2004.
- [17] M.E. Baran and F.F. Wu, "Optimal capacitor placement on radial distribution systems," *IEEE Transactions On Power Delivery* Vol.4, No.1, 1989, pp 725-735.