

Impacts of Transient Instability on Power System Reliability

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Abstract—This paper addresses the effects of transient instability on power system reliability. Composite system reliability evaluation has been performed based on steady-state estimation of load curtailments. In composite reliability evaluation, after each contingency, faulted components are assumed to be isolated from the rest of the system immediately and the system is assumed to return to a stable state with proper generation rescheduling for minimum load curtailments. In this context, minimum load curtailments are usually performed by solving linear/non-linear programming optimization problems. Although the optimization problem with minimum load curtailment may find a steady-state feasible solution, a stable transition to a post-fault stable equilibrium point is not guaranteed. In this paper, three probabilistic transient stability indices are proposed to assess system robustness against transient contingencies and update the reliability indices. Transient stability direct methods are used in assessing system stability and determining the probabilistic stability indices. This method is applied on the reduced WECC (Western Electricity Coordinating Council) system and the results showed that the effect of transient instability should not be ignored.

Index Terms—Direct methods, reliability, transient stability.

I. INTRODUCTION

As a result of market forces, increased renewable generation, and recent advances in power flow control technologies, power systems are increasingly being operated closer to their stability limits. Conventional power system reliability evaluation tools assume that power systems transit to a post-fault stable equilibrium point (PES) instantaneously after an occurrence of a fault. Although this is generally accepted, in practice, the system may encounter instability problems when dynamically transiting from one equilibrium point to another. Therefore, transient stability assessment is an important factor that should be considered in evaluating the reliability of a given system.

The effect of transient instability on power system reliability has been introduced in [1]–[3]. The work in [1] evaluates the effect of transient instability on power system reliability using time-domain simulation. The work presented in [2] evaluates the transient stability of the system based on transient energy function utilizing the potential energy boundary surface (PEBS) method. However, the work presented in [1], [2] do not consider system robustness against fault events.

Transient stability methods are known to be computationally intensive, and in recent times several efforts have been directed

toward developing methods that execute close to real-time speeds. Transient stability screening tools based on direct methods [4]–[9] selectively omit the non-severe contingencies and perform detailed simulations on the severe contingencies. A transient stability screening tool based on three levels of filtering has been proposed in [4], [5]. The filters screen the contingencies along the solution trajectory toward the controlling unstable equilibrium point (controlling UEP). Another on-line dynamic contingency screening tool based on the BCU method (Boundary of stability region based Controlling Unstable equilibrium point method) has been proposed in [6]. This tool uses six classifiers to screen out a small number of critical contingencies for detailed simulations. Improvements for the screening tool of [6] have been proposed in [7]. In the improved screening tool, the authors have added another level of classification which is the detection of the islanding mode and improvements in the six classifiers of [6]. A fast transient screening tool that utilizes a homotopy-based approach in finding the controlling UEP was proposed in [10], [11].

The work presented in this paper evaluates both the effect of transient instability on power system reliability and the degree of system stability using a direct method (Lyapunov function). The proposed investigation of the impact of transient instability on power system reliability adds another dimension toward realistic modeling of power system reliability evaluation. Some indicators to test the convergence of the solution are adapted from [6]. Three probabilistic transient stability indices are introduced: (1) expected transient instability index which provides an evaluation for the system instability, (2) expected transient stability robustness index which measures the ability of a system to withstand against fault events, and (3) the expected system risk of instability index which measures the risk of a system being unstable against fault events. Also, four reliability indices are evaluated, namely, loss of load probability, loss of load frequency, loss of load duration and expected power not supplied. The results of effect of transient instability on power system reliability as well as the robustness of power systems against disturbances are provided.

II. NETWORK MODELING AND RELIABILITY ANALYSIS

Monte Carlo simulation with linear programming for minimum load curtailment has been extensively used in calculating the reliability indices of composite power systems. If

under any scenario the curtailment is unavoidable, the linear programming minimizes the amount of load to be shed.

A. Linear Programming for Minimum Load Curtailment

Using DC power flow, there are three main constraints which are power balance equation, generation capacity limits and power carrying capabilities of transmission lines. The linear programming is used here to minimize load curtailments similar to the formulation of the DC power flow model given in [12].

$$LC = \min \left(\sum_{i=1}^{N_b} C_i \right). \quad (1)$$

Subject to:

$$\begin{aligned} \hat{B}\theta + G + C &= D \\ G &\leq G^{max} \\ C &\leq D \\ b\hat{A}\theta &\leq F_f^{max} \\ -b\hat{A}\theta &\leq F_r^{max} \\ G, C &\geq 0 \\ \theta &\text{ unrestricted} \end{aligned} \quad (2)$$

where LC is the amount of load curtailment, N_b is number of buses, N_t is number of transmission lines, $\hat{B}$ is the augmented node susceptance matrix ($N_b \times N_b$), b is the transmission line susceptance matrix ($N_t \times N_t$), $\hat{A}$ is the element-node incidence matrix ($N_t \times N_b$), θ is the vector of node voltage angles ($N_b \times 1$), C is the vector of bus load curtailments ($N_b \times 1$), D is the vector of bus demand ($N_b \times 1$), G^{max} is the vector of maximum available generation ($N_b \times 1$), F_f^{max} is the vector of forward flow capacities of lines ($N_t \times 1$), F_r^{max} is the vector of reverse flow capacities of lines ($N_t \times 1$), and G is the solution vector of the generation at buses ($N_b \times 1$).

B. Calculation of Reliability Indices

In the literature, both analytical and simulation-based approaches have been used for composite system reliability evaluation. In this work, the Monte Carlo next event method [13] is used to generate a sequence of events to evaluate the reliability and the probabilistic transient stability indices.

In this work, the well-known composite power system reliability indices are evaluated namely loss of load probability index (LOLP), loss of load frequency index (LOLF), loss of load duration index (LOLD) and expected power not supplied (EPNS). In estimating system indices, using Monte Carlo simulation, the expected values are used to evaluate the indices. If an index is denoted by η , the expected value of the index is calculated as follows.

$$\hat{\eta} = E[\eta], \quad (3)$$

where $E[\bullet]$ is the expectation operator.

1) *Calculation of Probability Indices:* Failure probability indices evaluate the probability of failure of the system to meet the demand. Let q denote the LOLP index. The probability of system failure to meet the demand is given in (4).

$$q = \sum_{i=1}^{n_f} p\{x_i : x_i \in X_f\}, \quad (4)$$

where X_f is the set of failure states ($X_f \subset X$), X is the set of all states, x_i is the system state i , $p\{x_i : x_i \in X_f\}$ is the probability of the state and n_f is the number of failure states.

Through the simulation process, if the state under consideration is a failure state, the probability of this state is added to the failure probability index, q . The estimated LOLP index ($\hat{q}$) can be calculated using Monte Carlo next event approach as follows.

$$\hat{q} = \frac{1}{T} \sum_{i=1}^N \vartheta_i, \quad (5)$$

where N is the number of samples, T is the sum of the duration hours of all sampled system states and ϑ_i can be expressed as follows.

$$\vartheta_i = \begin{cases} \tau_i, & \text{if } x_i \in X_f \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

where τ_i is the duration of system state i .

2) *Calculation of the Expected Power Not Supplied Index:* The EPSN is one of the well-known reliability indices that measures the expected demand not supplied. Let ρ denote the EPNS index which can be calculated using (7).

$$\rho = \sum_{i=1}^{n_f} p\{x_i : x_i \in X_f\} \times LC\{x_i : x_i \in X_f\}, \quad (7)$$

where $LC\{x_i : x_i \in X_f\}$ is the amount of load curtailment of state x_i .

For every sampled state, if the state is a failure state, the product of the probability of this state and the amount of load curtailment is added to the ρ index. The estimated EPNS index, $\hat{\rho}$, can be calculated using Monte Carlo next event sampling approach as follows.

$$\hat{\rho} = \frac{1}{T} \sum_{i=1}^N \psi_i, \quad (8)$$

where ψ_i is a function for the load curtailment that can be expressed as,

$$\psi_i = \begin{cases} \tau_i \times LC_i, & \text{if } x_i \in X_f \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

3) *Calculation of Frequency and Duration Indices:* Calculation of frequency and duration indices is generally more difficult than calculation of probability and energy indices. An approach based on Monte Carlo state sampling has been developed in [14]–[16]. In this work, the Monte Carlo next event method is used to calculate frequency and duration

indices. The estimated loss of load frequency index ($\hat{\phi}$) can be calculated using Monte Carlo next event method as follows.

$$\hat{\phi} = \frac{1}{T} \sum_{i=1}^N \varphi_i \times 8760, \quad (10)$$

where φ_i is a function to account for the failure of transitions which can be expressed as,

$$\varphi_i = \begin{cases} 1, & \text{if } x_{i-1} \in X_s \text{ and } x_i \in X_f \\ 0, & \text{otherwise} \end{cases} \quad (11)$$

where X_s is the set of success states ($X_s \subset X$).

The estimated loss of load duration index ($\hat{\tau}$) can be calculated as follows,

$$\hat{\tau} = \frac{\hat{q}}{\hat{\phi}}. \quad (12)$$

III. TRANSIENT STABILITY INDICES

To incorporate transient stability in power system reliability evaluation, this paper proposes three indices that can measure system robustness against fault events. Direct methods of transient stability analysis in the form of the energy function are used to evaluate system stability for each event.

A. Expected Transient Instability Index

The expected transient instability (ETI) index provides probability of the system being in an unstable state. Let α denote the ETI index which can be calculated as follows.

$$\alpha = \sum_{i=1}^{n_u} p \{x_{i-1,i} : x_{i-1,i} \in X_u\}, \quad (13)$$

where $p \{x_{i-1,i} : x_{i-1,i} \in X_u\}$ is the probability of the system being *unstable* while transitioning from state x_{i-1} to state x_i , x_i is the system new state, X_u is the set of unstable transitions ($X_u \subset X$), X is the set of all system states, and n_u is the number of unstable transitions.

A transition is considered unstable if the energy margin (EM) is less than zero, that is $EM < 0$. Using Monte Carlo next event sampling approach, the estimated ETI index, $\hat{\alpha}$, can be estimated as follows.

$$\hat{\alpha} = \frac{1}{T} \sum_{i=1}^N \gamma_i, \quad (14)$$

where γ_i can be expressed as,

$$\gamma_i = \begin{cases} \tau_i, & \text{if } x_{i-1,i} \in X_u \\ 0, & \text{otherwise.} \end{cases} \quad (15)$$

B. Expected Transient Stability Robustness Index

Expected transient stability robustness (ETSR) index measures the ability of a system to withstand against fault events. Let β denote the ETSR index which can be calculated as follows.

$$\beta = \sum_{i=1}^{n_{st}} p \{x_{i-1,i} : x_{i-1,i} \in X_{st}\} \cdot EM \{x_{i-1,i} : x_{i-1,i} \in X_{st}\} \quad (16)$$

where $p \{x_{i-1,i} : x_{i-1,i} \in X_{st}\}$ is the probability of the system being *stable* while transitioning from state x_{i-1} to state x_i , X_{st} is the set of stable transitions ($X_{st} \subset X$), n_{st} is the number of stable transitions and $EM \{x_{i-1,i} : x_{i-1,i} \in X_{st}\}$ is the energy margin of the transition from state x_{i-1} to state x_i for the set $x_{i-1,i} \in X_{st}$.

Using Monte Carlo next event sampling approach, the estimated ETSR index, $\hat{\beta}$, can be calculated as follows.

$$\hat{\beta} = \frac{1}{T} \sum_{i=1}^N \varrho_i, \quad (17)$$

where ϱ_i can be expressed as,

$$\varrho_i = \begin{cases} EM, & \text{if } x_{i-1,i} \in X_{st} \\ 0, & \text{otherwise.} \end{cases} \quad (18)$$

C. Expected System Risk of Instability Index

Expected system risk of instability (ESRI) index measures the risk of a system being unstable against fault events. Let ξ denote the ESRI index which can be calculated as follows.

$$\xi = \sum_{i=1}^{n_u} p \{x_{i-1,i} : x_{i-1,i} \in X_u\} \cdot |EM \{x_{i-1,i} : x_{i-1,i} \in X_u\}| \quad (19)$$

Using Monte Carlo next event sampling approach, the estimated ESRI index, $\hat{\xi}$, can be calculated as follows.

$$\hat{\xi} = \frac{1}{T} \sum_{i=1}^N \sigma_i, \quad (20)$$

where σ_i can be expressed as,

$$\sigma_i = \begin{cases} |EM|, & \text{if } x_{i-1,i} \in X_u \\ 0, & \text{otherwise.} \end{cases} \quad (21)$$

IV. TRANSIENT STABILITY ASSESSMENT

This section presents the dynamical model of power systems and the associated energy function for transient stability analysis.

A. The Dynamical Model

Given an n -generator system and assuming uniform damping, the dynamical model of the equations of motion of the generators with respect to the center of inertia (COI) can be described as follows [17].

$$\ddot{\delta}_i = \tilde{\omega}_i, \quad (22)$$

$$\dot{\tilde{\omega}}_i = \frac{1}{M_i} (P_{mi} - P_{ei}) - \frac{1}{M_T} P_{COI} - \lambda \tilde{\omega}_i, \quad (23)$$

where P_{mi} is the mechanical input of machine i , P_{ei} is the electrical power output of machine i , M_i is the inertia constant of machine i , δ_i and ω_i are power angle and speed of machine i respectively, P_{COI} is the power associated with the COI reference frame, $\tilde{\delta}_i = \delta_i - \delta_o$, $\tilde{\omega}_i = \omega_i - \omega_o$, $\delta_o = \frac{1}{M_T} \sum_{i=1}^n M_i \delta_i$, $\omega_o = \frac{1}{M_T} \sum_{i=1}^n M_i \omega_i$, $M_T = \sum_{i=1}^n M_i$ and λ is the uniform damping constant.

The compact form of the system of (22) and (23) can be expressed as follows.

$$\dot{x} = F(x). \quad (24)$$

The electrical power of machine i is given as follows.

$$P_{ei} = \sum_{j=1}^n E_i E_j \left[G_{ij} \cos(\tilde{\delta}_i - \tilde{\delta}_j) + B_{ij} \sin(\tilde{\delta}_i - \tilde{\delta}_j) \right], \quad (25)$$

where E_i is the internal voltage magnitude of machine i and G_{ij} and B_{ij} are the conductance and susceptance of the admittance matrix of the network-reduction model.

The P_{COI} is computed as follows.

$$P_{COI} = \sum_{i=1}^n P_{mi} - \sum_{i=1}^n \sum_{j=1}^n E_i E_j \left[G_{ij} \cos(\tilde{\delta}_i - \tilde{\delta}_j) + B_{ij} \sin(\tilde{\delta}_i - \tilde{\delta}_j) \right]. \quad (26)$$

B. The Energy Function

The energy function is used in transient stability analysis for fast screening and to compute the exit point to generate a sequence of steps to calculate the controlling UEP. The energy function associated with the model of (24) is expressed as follows [18], [19].

$$V = \frac{1}{2} \sum_{i=1}^n M_i \dot{\omega}_i^2 - \sum_{i=1}^n P_i (\tilde{\delta}_i - \tilde{\delta}_i^s) - \sum_{i=1}^{n-1} \sum_{j=i+1}^n \left[C_{ij} (\cos \tilde{\delta}_{ij} - \cos \tilde{\delta}_{ij}^s) - I_{ij} \right], \quad (27)$$

where

$$P_i = P_{mi} - E_i^2 G_{ii},$$

and I_{ij} is the energy dissipated in the network transfer conductances. The term I_{ij} can be expressed as follows.

$$I_{ij} = \int_{\tilde{\delta}_i^s + \tilde{\delta}_j^s}^{\tilde{\delta}_i + \tilde{\delta}_j} D_{ij} \cos \tilde{\delta}_{ij} d(\tilde{\delta}_i + \tilde{\delta}_j).$$

This term is path dependent and can be calculated only if the system trajectory is known. However, the system trajectory is not known in advance. Several methods have been suggested in the literature to approximate this term. In this paper, the method suggested by [18] is used which can be given as follows.

$$I_{ij} = D_{ij} \frac{\tilde{\delta}_i + \tilde{\delta}_j - \tilde{\delta}_i^s - \tilde{\delta}_j^s}{\tilde{\delta}_i - \tilde{\delta}_i^s - \tilde{\delta}_j^s + \tilde{\delta}_j^s} \left[\sin \tilde{\delta}_{ij} - \sin \tilde{\delta}_{ij}^s \right]. \quad (28)$$

The first term of (27) represents the kinetic energy and the last two terms represent the potential energy.

C. Calculation of the controlling UEP

Computing the controlling UEPs is crucial because the energy at a controlling UEP is used in computing the critical energy to assess the stability of the system. A state vector x is called an equilibrium point x^* of the dynamic system represented in (24) if $F(x^*) = 0$. A controlling UEP is one of the unstable equilibrium points, but it is not an easy task to determine and distinguish it from the other unstable equilibrium points. Mathematically, a controlling UEP is an unstable equilibrium point whose stable manifold, $W^s(X_{co})$, contains the exit point of the fault-on (sustained fault) trajectory $(\delta(t), \omega(t))$ [18], [20]–[22] as shown in Fig. 1.

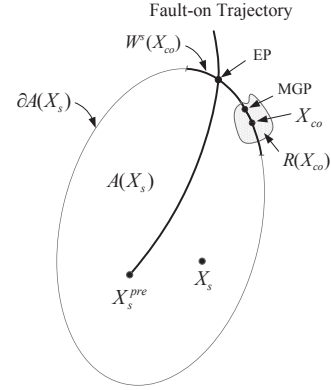


Fig. 1. Region of convergence of controlling UEPs under the use of Newton methods [20].

The terms used in Fig. 1 are defined as following:

EP	is the exit point,
MGP	is the minimum gradient point,
X_{co}	is a controlling UEP,
$A(X_s)$	is the region of attraction (stability) of the post-fault stable equilibrium point (X_s),
$\partial A(X_s)$	is the boundary of the region of attraction,
$R(X_{co})$	is the region of convergence of a controlling UEP under Newton method,
$W^s(X_{co})$	is the stable manifold of a controlling UEP,
X_s^{pre}	is the pre-fault stable equilibrium point.

The region of convergence of an equilibrium point can be defined as following: starting from an initial guess, a numerical method succeeds in finding the desired solution if the starting point lies inside the region of convergence of the solution and it fails if the initial guess lies outside the region of convergence. The size and shape of the region of convergence of a controlling UEP can be fractal and different for different numerical methods [20]. Therefore, if the initial guess is not in the region of convergence of the controlling UEP, numerical methods such as Newton methods fail to find the correct controlling UEP. In other words, to find the correct controlling UEP, the initial guess has to be sufficiently close to the controlling UEP.

A popular method to compute controlling UEPs is to use time-domain simulations to simulate the projected fault-on trajectory to obtain the exit point and the MGP, and then use

the MGP as an initial guess to generate a sequence of solution steps toward the controlling UEP [21]–[27]. The exit point is the point at which the projected fault-on trajectory exits the stability boundary of the post-fault stable equilibrium point (SEP). Computationally, the exit point is characterized by the first local maximum of the potential energy (the last two terms of (27)) of the post-fault network along the projected fault-on trajectory. The MGP is numerically characterized by the first local minimum value of the norm of the vector field of the post-fault trajectory of (22) and (23) [20]. The robustness of finding the controlling UEP depends strongly on the quality of the calculated MGP [21], [23], [24], [27], [28]. An inaccuracy in detecting the exit point may cause difficulty in computing the MGP.

To overcome the difficulty of finding an initial point inside the region of convergence, we use a homotopy-based approach in computing the controlling UEP. Homotopy-based approaches are known to be reliable in finding the solutions and they are globally convergent [29], [30]. In this paper, a homotopy-based approach is used to eliminate the requirement of computing the MGPs and accurate exit points.

D. Homotopy-Based Method

Traditional methods of solving (24) include the use of Newton methods. Due to the difficulties associated with determining the controlling UEPs which in turn are attributed to numerical problems in calculating the EPs and MGPs, we have applied a homotopy based method to calculate the controlling UEPs [31]. We have found that the proposed homotopy based method is robust, reliable, and efficient in calculating the controlling UEP. Also, homotopy methods are not sensitive to the initial points.

While homotopy-based approaches are known to be reliable in finding a solution, they are intrinsically slow because these methods map the trajectory of the solution from an easy and known solution to the desired solution. In this paper, the homotopy-based method is used with EPs as the initial points to find controlling UEPs. Using approximate EPs rather than computing accurate EPs, as is common practice in finding the controlling UEP, the intrinsic slow speed of computation of homotopy-based approaches is compensated. Further, homotopy-based approaches eliminate the necessity of computing the MGP, which makes the homotopy-based approach comparable with iterative methods in terms of the speed of computation.

Homotopy is a numerical method to solve and find the roots of non-linear systems ($F(x^*) = 0$). The homotopy method traces the solution trajectory by a predictor-corrector algorithm to get a solution of the original equation. The basic concept of the homotopy-based methods is that they find the solutions by path continuation, starting at a known solution x^0 that satisfies $G(x^0) = 0$ as shown in (29). The most widely used homotopy function is expressed as follows.

$$H(x, t) = tF(x) + (1 - t)G(x) = 0, \quad (29)$$

where t changes from 0 to 1 with an incremental step-size through the mapping process, i.e., $H(x, 0) = G(x)$ and $H(x, 1) = F(x)$.

$G(x)$ can be chosen arbitrary as long as it has a known solution. However, the most widely used homotopy function is Newton Homotopy which can be expressed as follows.

$$G(x) = F(x) - F(x^0), \quad (30)$$

where x^0 can be any starting point. Therefore, the homotopy function becomes

$$H(x, t) = F(x) - (1 - t)F(x^0) = 0. \quad (31)$$

In this paper, Newton homotopy is used and it has succeeded to compute the controlling UEPs of the tested systems. The $F(x)$ function represents the dynamical model of the system as given in (24) and x^0 is the exit point which is calculated using the fault-on trajectory.

In applying homotopy methods, the direction of the search (forward or backward) for the solution is an important factor in determining the correct controlling UEPs. During the calculation process, the algorithm proceeds by assuming forward mapping for one homotopy iteration; if the solution diverges or converges to a point far from the exit point, the algorithm uses backward mapping.

V. CASE STUDY

The proposed method was applied on the reduced WECC 9 bus system [20]. This system consists of 9 buses, 6 transmission lines, 3 generators, 3 transformers and 3 load buses. The reason of choosing this system is because it has been extensively tested from transient stability perspective and also it is appropriate to use a small system to explain and demonstrate the proposed method.

The reliability data of this system were taken from [1]. Table I shows the reliability indices as calculated with and without considering the effect of transient stability. These results provide a measure for static and dynamic estimation of the ability of the system to meet the demand. The amount of increase of these values in comparison with the case of not considering the transient instability reflects the contribution of dynamic instabilities.

TABLE I
STATIC AND DYNAMIC RELIABILITY INDICES

	LOLP	EPNS MW	LOLF occ./yr	LOLD hour
Static	0.01575	1.42170	5.84841	23.59103
Static & dynamic	0.04092	1.93237	12.58263	28.48842

Table II shows the stability indices of the tested system. The ETI index represents the contribution of transient instability on the interruption of the load. The ETSR index represents robustness of the system against fault events which is calculated from the energy margins of the stable contingencies. The ESRI index represents the risk of the being unstable against fault events which is calculated from the energy margins of the unstable contingencies.

TABLE II
THE STABILITY INDICES

ETI	ETSR	ESRI
0.02517	0.25603	0.10411

VI. CONCLUSION

This paper has introduced the effect of power system transient instability on power system reliability. Three indices were introduced to measure the effect of transient stability which are: expected transient instability, expected transient stability robustness and the expected system risk of instability. Transient stability direct methods were used in assessing system stability, determining the energy margins and updating the stability indices. Also, power system reliability indices were evaluated for the cases considering and not considering the effect of transient instability. This method was applied on the reduced WECC test system and the results showed that the effect of transient stability should not be ignored when evaluating reliability of the system. Also, the results of the probabilistic stability indices can be used to measure system robustness against disturbances.

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