

Reactive Power Compensation for Reliability Improvement of Power Systems

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Abstract—This paper investigates the effects of reactive power support limits on power system reliability. In evaluating the reliability of power systems, several load curtailments are caused by voltage limit violations which can be alleviated using reactive power support. The existing methods of estimating the effects of voltage limit violations and reactive power limits do not provide the amount of reactive power support that is required to alleviate the violations. The presented work provides a quantitative measure of reactive power compensation for reliability improvement. This measure is based on constructing a probability distribution function for the required reactive power compensation. Reliability indices of Loss of Load Expectation and Expected Unserved Energy are used to estimate the lack of reactive power support. A non-linear, AC power flow based model is used to accurately represent system load curtailment remedial actions. A state space reduction technique is utilized to reduce the computation time. The proposed method is applied on the modified IEEE RTS and results show the improvement of power system reliability due to reactive power compensation.

Index Terms—Reliability, reactive power, compensation.

I. INTRODUCTION

Reactive power shortage and voltage limits have significant contributions on failures of power systems to meet the demand [1]–[4]. In most of the present methods that evaluate the reliability of composite systems, if the voltage limits are violated, the models shed the loads as remedial actions. In some scenarios, these violations could be mitigated by reactive power rescheduling or compensation. Mathematically, voltage violations are included in calculating the loss of load probability and loss of load expectation disregarding the amount of reactive power that is need to alleviate the violation. One of the main problems of not including the lack of reactive power support and voltage limit violations in most power system reliability studies is the computation burden and time. This work quantifies the requirements of reactive power support based on reliability improvement and uses state space pruning technique to reduce the computation burden and time.

Evaluation of the effects of the voltage and reactive power limits has been introduced in [1], [2]. The methods presented in [1], [2] are based on using the DC power flow model in two steps and linearizing the relationship between the reactive power injections and the voltages at the nodes. Using the AC power flow model in *two steps* to study the aspects of the reactive power on the composite system reliability has

been introduced in [5]–[8]. An AC optimal power flow based method with an objective of minimum load curtailment was proposed in [3], [4]. The method presented in [3], [4] uses state space pruning technique to reduce the computation time.

Methodologies for calculating the sensitivity of some reliability indices with respect to the variations in component's parameters and system operating limits have been introduced in [9]–[12]. Sensitivity of reliability indices with respect to voltage and reactive power limits is proposed in [3], [4]. One of the advantages of using sensitivity analysis is that it allows planners to enhance the overall system reliability by improving the reliability parameters and available capacity of each component in a separate manner. However, sensitivity analysis alone does not provide a quantitative measure for required improvement. Also, the sensitivity analysis is based on linearizing the relationship between the objective function and constraints which does not provide the range of validity of the solution.

Several methods have been introduced to decrease the time and computational effort in evaluating power system reliability indices. The concept of the state space pruning has been introduced in [13], [14]. A search space reduction using the particle swarm optimization (PSO) technique was introduced in [15], [16]. A comparative study of using PIS methods specifically genetic algorithms, repulsive binary particle swarm optimization and binary ant colony optimization has been introduced in [17].

This paper investigates the effects of reactive power limits on composite system reliability indices and provides a quantitative measure for reactive power support for power system reliability improvement. While several methods have been presented in the literature to study the effects of the reactive power shortage on system reliability by providing reactive power related indices, the method presented in this paper calculates the amount of the required reactive power support to alleviate voltage violations. The reactive power support is quantified in terms of probability distribution functions at system buses. Further, contributions of reactive power and voltages limits on reliability indices are provided. The full, non-linear AC power flow model is used to incorporate these constraints in the reliability evaluation. A state space pruning technique is utilized to reduce the computation time. Although

the presented method is applied on a 24 bus test system, it can be applied on different systems for planning studies.

II. NETWORK MODELING

In composite system reliability studies, power flow analysis is usually carried out in solving optimization problems for minimum load curtailment. In this paper, the AC power flow model is used. This section describes the formulation and incorporation of the objective function of minimum load curtailment in the nonlinear programming problem. This objective function is subject to equality and inequality constraints of the power system operation limits. The equality constraints include the power balance at each bus and the inequality constraints are the capacity limits of generating units, power carrying capabilities of transmission lines and voltage limits at the nodes. Reactive power constraints are relaxed to determine the amount of reactive power support that is required to alleviate voltage limit violations. The required reactive power support is determined by calculating the amount of reactive power that is used beyond the limits. The minimization problem is formulated as follows [18],

$$\text{Loss of Load} = \min \left(\sum_{i=1}^{N_b} C_i \right). \quad (1)$$

Subject to

$$\begin{aligned} P(V, \delta) - P_D + C &= 0, \\ Q(V, \delta) - Q_D + C_Q &= 0, \\ P_G^{\min} \leq P(V, \delta) \leq P_G^{\max}, \\ Q_G^{\min} \leq Q(V, \delta) \leq Q_G^{\max} \quad (\text{relaxed}), \\ V^{\min} \leq V \leq V^{\max}, \\ S(V, \delta) \leq S^{\max}, \\ 0 \leq C \leq P_D, \\ \delta \text{ unrestricted.} \end{aligned} \quad (2)$$

In (1) and (2), C_i is the load curtailment at bus i , C is the vector of load curtailments ($N_d \times 1$), C_Q is the vector of reactive load curtailments ($N_d \times 1$), V is the vector of bus voltage magnitudes ($N_b \times 1$), δ is the vector of bus voltage angles ($N_b \times 1$), P_D and Q_D are the vectors of real and reactive power loads ($N_d \times 1$), $P_G^{\min}$, $P_G^{\max}$, $Q_G^{\min}$ and $Q_G^{\max}$ are the vectors of real and reactive power limits of the generators ($N_g \times 1$), $V^{\max}$ and $V^{\min}$ are the vectors of maximum and minimum allowed voltage magnitudes ($N_b \times 1$), $S(V, \delta)$ is the vector of power flows in the lines ($N_t \times 1$), $S^{\max}$ is the vector of power rating limits of the transmission lines ($N_t \times 1$) and $P(V, \delta)$ and $Q(V, \delta)$ are the vectors of real and reactive power injections ($N_b \times 1$). Also, N_b is the number of buses, N_d is the number of load buses, N_t is the number of transmission lines and N_g is the number of generators.

III. STATE SPACE PRUNING

In this work, the state space pruning technique which has been introduced in [13] is adapted to reduce the search space.

This technique is based on pruning out the success (no load curtailment) subspace and performing Monte Carlo state next event on the remaining (unclassified) subspaces. In this work, only the first stage of state space pruning [13] was used; all system components (generation and transmission) are assumed in the working states. Using this assumption, the algorithm solves for minimum load curtailment. The vectors of real and reactive power generation for the minimum load curtailment at the buses are used as a minimum threshold for the system to meet the demand for the states that have all the transmission lines are in the up state. This threshold is used as a pruning device and can be justified in the following points: (1) if the real and reactive power generation at all the buses are larger than the threshold, system performance cannot be deteriorated, and (2) due to the fact that power system components are very reliable, this pruned subspace has a high probability in comparison with the other subspaces. Therefore, pruning this subspace will reduce the computational time significantly.

IV. CALCULATION OF RELIABILITY INDICES

In this work, the Monte Carlo next event method [19] is used for the following reasons: (a) the analytical methods are not practical due to the complexity of the composite power system reliability modeling and the computation speed and burden, (b) Monte Carlo state duration technique (sequential) requires large memory storage and computational burden, (c) Monte Carlo state sampling technique (non-sequential) has a disadvantage of difficulty associated with calculating frequency and duration indices due to the requirement that the system should be coherent which cannot be assumed in case of using AC power flow model.

In this work, the well-known composite power system reliability indices were evaluated which are loss of load probability index (q), loss of load frequency index (ϕ), loss of load duration index (τ) and Severity Index (ρ);

A. Calculation of Probability Indices

Failure probability indices evaluate the probability of failure of the system to meet the demand. Through the simulation process, if the state under consideration is a failure state, the probability of this state is added to the failure probability index, q . The probability of system failure to meet the demand is given by,

$$q = \sum_{i=1}^{n_f} p \{x_i : x_i \in X_f\}, \quad (3)$$

where X is the set of all states, X_f is the set of failure states ($X_f \subset X$), x_i is the system state i , $p\{\cdot\}$ is the probability of the state and n_f is the number of failure states.

The estimated loss of load probability index ($\hat{q}$) can be calculated as follows.

$$\hat{q} = E[q], \quad (4)$$

where $E[\bullet]$ is the expectation operator and q can be evaluated as follows.

$$q = \frac{1}{T} \sum_{i=1}^N \vartheta_i, \quad (5)$$

where N is the number of samples, T is the sum of the durations of all sampled system states and ϑ_i is an indicator function that can be expressed as,

$$\vartheta_i = \begin{cases} \tau_i, & \text{If } x_i \in X_f, \\ 0, & \text{otherwise,} \end{cases} \quad (6)$$

where τ_i is the duration of system state i .

B. Calculation of the Severity Index

Severity index is one of the well-known reliability indices that measures the expected demand not supplied. Let ρ denote the severity index. For every tested state, if the state is a failure state, the product of the probability of this state and the amount of load curtailment is added to the ρ index. The Severity Index is given by,

$$\rho = \sum_{i=1}^{n_f} p \{x_i : x_i \in X_f\} \times C \{x_i : x_i \in X_f\}, \quad (7)$$

where C is the amount of load curtailment of state x_i .

The estimated Severity Index, $\hat{\rho}$, can be calculated using Monte Carlo state next event sampling approach as follows.

$$\hat{\rho} = E[\rho], \quad (8)$$

where ρ can be evaluated as follows.

$$\rho = \frac{1}{T} \sum_{i=1}^N \psi_i, \quad (9)$$

and ψ_i is an indicator function for the state curtailment that can be expressed as,

$$\psi_i = \begin{cases} \tau_i \times C_i, & \text{If } x_i \in X_f, \\ 0, & \text{otherwise.} \end{cases} \quad (10)$$

C. Calculation of Frequency and Duration Indices

Calculation of frequency and duration indices is generally more difficult than calculation of probability and severity indices. An approach based on Monte Carlo state sampling approach has been developed in [14], [20]–[22] which requires that the system is coherent. In this work, the Monte Carlo state next event method has been used to calculate frequency and duration indices which does not necessitate the coherency condition. Also, the Monte Carlo state next event method is comparable with the Monte Carlo state sampling approach in terms of memory storage requirements and speed of computation.

The estimated loss of load frequency index ($\hat{\phi}$) can be calculated using Monte Carlo state next event method as follows.

$$\hat{\phi} = E[\phi], \quad (11)$$

where ϕ can be evaluated as follows.

$$\phi = \frac{1}{T} \sum_{i=1}^N \varphi_i \times 8760, \quad (12)$$

and φ_i is an indicator function for the state frequency that can be expressed as,

$$\varphi_i = \begin{cases} 1, & \text{If } x_i \in X_s \text{ and } x_{i-1} \in X_f, \\ 0, & \text{otherwise,} \end{cases} \quad (13)$$

where X_s is the set of success states ($X_s \subset X$).

The estimated loss of load duration index ($\hat{\tau}$) can be calculated as follows.

$$\hat{\tau} = \frac{\hat{q}}{\hat{\phi}}. \quad (14)$$

D. Indices of Voltage and Reactive Power Limits

In this paper, four indices were used to represent the contributions of violations of the voltage and reactive power constraints on the loss of load probability index. These indices are defined as follows: (a) vi_{min} represents the contributions of the minimum voltage level constraints, (b) vi_{max} represents the contributions of the maximum voltage level constraints, (c) ϱ_{min} represents the contributions of the minimum reactive power level constraints and (d) ϱ_{max} represents the contributions of the maximum reactive power level constraints. For every sampled state, if any of these indices is involved in the load curtailment, this index is updated. These indices are related to the failure subspace not to the system state space; these indices reflect the contributions of the violations of the voltage and reactive power constraints on the loss of load probability index. Therefore, the state space of these indices is X_f .

The estimated values of these indices can be calculated using Monte Carlo state next event method as follows.

$$f_k = E[q_k], \quad (15)$$

where f_k is the index under consideration, (f_1 is for the vi_{min} , f_2 is for the vi_{max} , f_3 is for the ϱ_{min} and f_4 is for the ϱ_{max}), and q_k can be evaluated as follows.

$$q_k = \frac{1}{T_f} \sum_{i=1}^N \xi_{ki}, \quad (16)$$

where T_f is the sum of the durations of all sampled failure states and ξ_{ki} is an indicator function for state violation that can be expressed as,

$$\xi_{ki} = \begin{cases} \tau_i, & \text{If } x_i \in X_k, \\ 0, & \text{otherwise,} \end{cases} \quad (17)$$

where X_k is the set in which the index k has a contribution in the load curtailment and $X_k \subset X_f$.

V. CASE STUDIES

It is well-known that the transmission lines of the IEEE RTS [23] are very reliable with respect to the generation. Also, the power carrying capability limits of the transmission lines are much higher than the normal loading level even in the case of the peak load. Therefore, the contributions of the transmission lines on the system reliability of this test system are very small and can be ignored. For this reason, several studies have

suggested to use the modified version of the IEEE RTS which is the same as the original system except that the generation is multiplied by 2 and the load at the buses is multiplied by 1.8. The detailed data such as the capacities of generating units, the line carrying capabilities of the transmission lines, the failure and repair rates of system components and load profile of the IEEE RTS are given in [23]. IEEE RTS consists of 24 buses, 38 transmission lines/transformers (33 transmission lines and 5 transformers) and 32 generating units on 10 buses. The total generation of this system is 3405 MW and total peak load is 2850 MW.

Reliability assessments have been performed on the IEEE RTS and the modified IEEE RTS at the peak load (Annualized Indices). As a comparison, the results of using DC and AC power flow models for the IEEE RTS and the Modified IEEE RTS are shown in Table I and Table II respectively. As it is obvious from Table I and Table II, estimates from the DC power flow model produce optimistic results in comparison with the AC power flow model counterpart.

TABLE I
SYSTEM ANNUALIZED INDICES OF THE IEEE RTS

Power Flow Model	$\hat{q}$	$\hat{p}$ MW	$\hat{\phi}$ occ./yr	$\hat{\tau}$ hour
AC	0.10427	17.37479	22.76247	40.018
DC	0.08455	14.72996	19.76867	37.364

TABLE II
SYSTEM ANNUALIZED INDICES OF THE MODIFIED IEEE RTS

Power Flow Model	$\hat{q}$	$\hat{p}$ MW	$\hat{\phi}$ occ./yr	$\hat{\tau}$ hour
AC	0.44337	20.13860	63.82885	60.682
DC	0.07141	10.52347	17.76162	35.123

During the simulation, if the voltage or reactive power limits contributes to the load curtailment, the related index is updated according to (16). The results of these indices of the Modified IEEE RTS are shown in Table III. From Table III, it is clear that the maximum voltage limit at bus 2 has contributed to almost all the failure states (the probability of the contribution of the maximum voltage limit at bus 2 is 0.99643 or 99.64%). Maximum voltage limits at buses 7, 13, 21 and 23 have contributed in around 85.30%, 78.75%, 82.10% and 82.51% of the failure states, respectively. Minimum voltage limit at bus 6 has contributed in around 83.35% of the failure states. Maximum reactive power limit at buses 2, 3 and 15 have contributed in around 84.20%, 96.75% and 91.01% of the failure states, respectively. The minimum reactive power limit at bus 6 has contributed in around 15.43% of the failure states.

One of the possible solutions to alleviate voltage limit violations is to install reactive power compensator. These compensators will provide reactive power support to the system so that voltages at the buses will be maintained within the specified limits. Columns 6 and 7 of Table III show the amounts of reactive power supports that are required at the buses. Table IV shows system reliability indices after the compensation.

TABLE III
CONTRIBUTIONS OF VOLTAGE AND REACTIVE POWER CONSTRAINTS AND THE REQUIRED REACTIVE POWER COMPENSATION

Bus No.	$v_{i_{max}}$	$v_{i_{min}}$	$q_{i_{max}}$	$q_{i_{min}}$	Q_{max} MVar	Q_{min} MVar
1	0.1544	0.0000	0.0068	0.0000	0.29	-88.76
2	0.9964	0.0000	0.8420	0.0000	107.10	-56.38
3	0.0006	0.0094	0.9675	0.0000	0	0
4	0.0000	0.0002	—	—	0	0
5	0.0000	0.0001	—	—	0	0
6	0.0001	0.8335	0.0031	0.1543	0	0
7	0.8530	0.0000	0.0338	0.0000	6.37	0
8	0.0000	0.0480	—	—	0	0
9	0.0001	0.0079	—	—	0	0
10	0.0477	0.0000	—	—	0	0
11	0.0000	0.0003	—	—	108.95	-11.18
12	0.0000	0.0060	—	—	135.62	-0.21
13	0.7875	0.0000	0.1279	0.0000	15.69	-4.13
14	0.0026	0.0044	0.0303	0.0000	0	0
15	0.1867	0.0000	0.9101	0.0002	1282.44	-1040.86
16	0.0758	0.0005	0.1986	0.0894	1.03	-109.32
17	0.0001	0.0000	—	—	11.91	-26.35
18	0.2013	0.0000	0.0802	0.0000	9.73	-104.62
19	0.0002	0.0000	—	—	0	0
20	0.0005	0.0000	—	—	0	0
21	0.8210	0.0000	0.3231	0.0000	122.56	-0.07
22	0.2935	0.0001	0.0000	0.0000	0.49	-4.72
23	0.8251	0.0040	0.0223	0.0001	2013.65	-2020.91
24	0.0003	0.0410	—	—	195.46	-0.01

TABLE IV
SYSTEM ANNUALIZED INDICES OF THE MODIFIED IEEE RTS AFTER COMPENSATION

Power Flow Model	$\hat{q}$	$\hat{p}$ MW	$\hat{\phi}$ occ./yr	$\hat{\tau}$ hour
AC	0.13867	15.14576	27.41871	44.304

Fig. 1, Fig. 2 and Fig. 3 show the probability distributions of the reactive power requirement at buses 2, 12 and 15 respectively. From these distributions, a planner can choose the range of reactive power that has the most effect on the reliability of the system.

VI. CONCLUSION

In this paper, we have investigated the effects of the voltage and reactive power constraints on power system reliability and provided a quantitative measure for the required reactive power support. The well-known reliability indices as well as the voltage and reactive power indices have been used to estimate the effects of voltage violations that can be alleviated by reactive power compensation. Also, four indices have been used to address the contributions of the voltage and reactive power constraints on system load curtailments. Extensive studies on the Modified IEEE RTS have been conducted to investigate the effects of the voltage and reactive power constraints on the power system reliability indices. Also, a state space pruning technique has been used to reduce the computation time and burden. The amount of reactive power support that is required

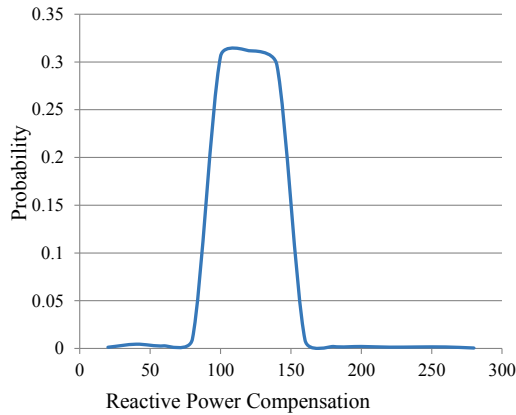


Fig. 1. Probability distribution of the positive reactive power (capacitive) requirement at Bus 2.

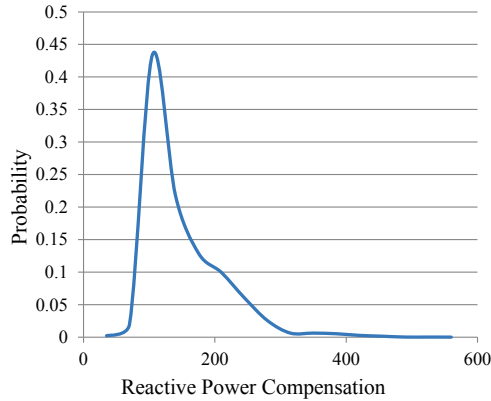


Fig. 2. Probability distribution of the positive reactive power (capacitive) requirement at bus 12.

to alleviate voltage violations at each bus has been provided in terms of probability distribution functions.

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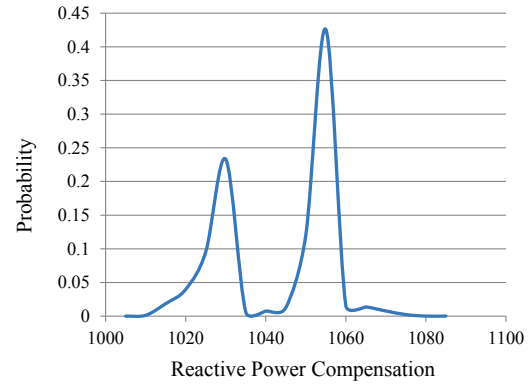


Fig. 3. Probability distribution of the negative reactive power (inductive) requirement at bus 15.