

Optimal Location and Size of Distributed Energy Resources Using Sensitivity Analysis-Based Approaches

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Abstract—This paper introduces an analytical approach based on sensitivity analyses of various objective functions with respect to load constraints to determine optimum locations and sizes of distributed energy resources (DERs). This method is based on sequentially calculating Lagrange multipliers of the dual solution of an optimization problem for various load buses. Determining the best candidate locations based on the sensitivity analyses with the assumption that an active constraint would remain active for all source sizes could produce inaccurate results. The reason is that buses that are ranked as the best candidates based on Lagrange multipliers may not be valid for large DERs since Lagrange multipliers change with the change in the system loading. In this work, locations and sizes are jointly determined in a sequential manner based on the validity of the active constraints. The proposed method can be applied with any objective function; however, in this paper, minimum generation cost is used as an objective function in the optimization problem. The method is demonstrated on several test systems including the IEEE RTS, IEEE 14, 30, 57, 118 and 300 bus test systems and the results showed the effectiveness of the proposed method against the traditional sensitivity analysis methods. Also, the results of the proposed method are validated using genetic algorithm.

Index Terms—Size, location, sensitivity analysis, analytical approach, distributed energy resources.

I. INTRODUCTION

In recent years, the drive to bring about technological changes that concern the integration of distributed energy resources (DERs) has gathered significant momentum. Determination of the optimal placement and sizes of these devices in terms of operation and ancillary services and participation in the electricity market has been of concern. Several methods have been introduced to solve this type of problems including analytical and population-based intelligent search methods. In this work, an analytical method that is based on the sensitivity analysis concept is proposed to jointly determine the optimal location and size of these devices with respect to the desired objective function.

Sensitivity analysis has numerous applications in different areas due to its multi-facet nature. It has been amply used to determine the change in an objective function with respect to the problem constraints. For instance, it has been used in [1] to forecast the short-term transmission congestion. Lagrange multipliers have been used in enhancing power

system reliability in [2]. Several definitions have been reported in the literature for Lagrange multipliers among them is the rate of change in the objective function for an infinitesimal change in the right-hand side of the constraints of linear/non-linear programming problems. Therefore, sensitivity analysis can be used in any instance where maximizing/minimizing an objective function with respect to the problem constraints is desired. However, using Lagrange multipliers without determining their range of validity could produce inaccurate results. This can be related in part to the fact that Lagrange multipliers change with the change in system conditions.

Several methods have been introduced in the literature on the optimal sizing and siting of distributed storages and distributed generators with reference to various goals. A near-optimal approach for siting and sizing of distributed storage in transmission networks has been proposed in [3]. The method presented in [3] uses three stages in determining the size and location of the distributed storages. The first stage determines the optimal locations by assuming unlimited storage sizes and located at each bus. The second stage determines the sizes based on the locations that are suggested by the first stage. An optimal siting and sizing of distributed energy storage systems has been introduced in [4] which is based on the alternating direction method of multipliers. A methodology based on genetic algorithm for sizing energy storage devices in microgrids has been presented in [5]. The benders decomposition procedure has been used in [6] for optimal placement of distributed storage systems for voltage control. An analytical approach for optimal placement of distributed generation in networked power systems with an objective of minimum losses has been proposed in [7]. This method simplifies the analysis by assuming one DG is placed at a time. An analytical approach for placement and sizing of distributed generation on distribution systems for losses minimization based on a linearized power flow model and linear programming has been proposed in [8]. A multi-objective optimization framework for sizing and siting of distributed generation based on genetic algorithm and an ϵ -constrained method has been proposed in [9]. An optimal siting and sizing of distributed generators considering uncertainties Monte Carlo simulation-embedded genetic-algorithm-based approach and chance constrained programming has been

proposed in [10]. A reliability-based method for sizing of storage devices has been proposed in [11], [12]. An approach for sizing and siting of distributed generation for optimal microgrid architecture has been proposed in [13].

In this paper, an analytical method to determine the sizes and locations of DERs is introduced. The analytical method is based on sequential sensitivity analysis of the objective function with respect to the loading level of the system. It should be noted here that any objective function can be used; however, in this work, the generation cost function is used to determine the sizes and locations of these devices. The sensitivity analysis along with the range of validity of the Lagrange multipliers are used to analytically determine the sizes and locations of the DERs. The range of validity of the Lagrange multipliers is determined from the range at which the binding constraints remain active.

The remainder of this paper is organized as follows. Section II describes the concept of the sensitivity analysis. Section III presents the modeling of power system networks. Section IV presents the proposed method. Section V discusses the solution algorithm. Section VI provides application for the proposed method. Section VII provides concluding remarks.

II. SENSITIVITY ANALYSIS

Lagrange multipliers in terms of shadow prices have been initially proposed by the former Soviet Union economist Conte Petrovic in the late 1930's when he was applying linear programming technique to maximize the output of some products [14]. Several definitions have been reported in the literature for Lagrange multipliers. For instance, from a primal-dual perspective, it can be defined as the primal (x) and dual (y) solutions of the linear/nonlinear programming problem. From the optimization point of view; however, Lagrange multipliers can be interpreted as the rate of change in the objective function for an infinitesimal change in the right-hand side of the linear/nonlinear programming problem. From a geometric perspective, Lagrange multipliers can be understood as the sub-gradients of the objective function along the dimension of resource provisioning changes. Lagrange multipliers approach has numerous applications in different areas. For example, it has been used in [1] to forecast the short-term transmission congestion. It has also been used in the evaluation of some construction projects and management [15]. Lagrange multipliers have been used in enhancing power system reliability in [2], [16]. However, as was mentioned earlier, Lagrange multipliers has been used here as a decision-making tool for determination of sizes and locations of DERs.

Consider a standard *maximum* linear/nonlinear programming problem which can be formulated as follows.

$$\max [Z] = \sum_{j=1}^n c_j x_j \quad (1)$$

Subject to the following constraints,

$$\sum_{i=1}^m a_{ij} x_j \leq b_i \quad \text{for } j = 1 \text{ to } n \quad (2)$$

$$x_j \geq 0 \quad (3)$$

where m is number of constraints, n is the number of variables, x is the problem variables, and a is the matrix of the coefficients of the constraints which can be expressed as follows.

$$a = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad (4)$$

The right hand side vector b of the constraints consists of m constants and can be expressed as follows.

$$b = (b_1, b_2, \dots, b_m)^T \quad (5)$$

The row vector of the objective function c consists of n coefficients as follows.

$$c = (c_1, c_2, \dots, c_n)^T \quad (6)$$

The dual of this standard *maximum* problem is a standard *minimum* problem, that is,

$$\min [W] = \sum_{i=1}^m b_i y_i \quad (7)$$

Subject to the following constraints,

$$\sum_{j=1}^n a_{ij} y_j \leq c_j \quad \text{for } i = 1 \text{ to } m \quad (8)$$

$$y_i \geq 0 \quad (9)$$

where y is the vector of dual variables. If the slack variables of (2) are imported, the standard linear/nonlinear programming problem can be expressed as follows.

$$\max [Z] = CX. \quad (10)$$

With the equality constraints,

$$AX = b \quad (11)$$

$$X \geq 0 \quad (12)$$

The *optimal* and *feasible* solution of (10) can be expressed as,

$$X_b = B^{-1}b \quad (13)$$

where B is the optimal basis at the optimal and feasible solution and is the basic variable sub-vector. In the standard problem (7), the significance of dual variables is described as the i -th Lagrange multiplier. In other words, the y_i of the dual problem are interpreted here as the Lagrange multipliers of the constraints.

III. NETWORK MODELING WITH NON-LINEAR PROGRAMMING AND AC POWER FLOW

In economic dispatch studies, power flow analyses are usually carried out in solving optimization problems for minimum generation cost. In this paper, the AC power flow model is used to solve for minimum generation cost and determine the values of Lagrange multipliers. This section describes the formulation and incorporation of the objective function of minimum generation cost in the non-linear programming problem similar to the formulation presented in section II. This objective function is subject to equality and inequality constraints of the power system operation limits. The equality constraints include the power balance at each bus and the inequality constraints are the capacity limits of generating units, power carrying capabilities of transmission lines, voltage limits at the nodes and reactive power capability limits. The minimization problem is formulated as follows [17].

$$F = \min \sum_{i=1}^{N_g} F_i(P_{Gi}) \quad (14)$$

Subject to

$$\begin{aligned} P(V, \delta) - P_D &= 0 \\ Q(V, \delta) - Q_D &= 0 \\ P_G^{min} &\leq P(V, \delta) \leq P_G^{max} \\ Q_G^{min} &\leq Q(V, \delta) \leq Q_G^{max} \\ V^{min} &\leq V \leq V^{max} \\ S(V, \delta) &\leq S^{max} \\ \delta &\text{ unrestricted.} \end{aligned} \quad (15)$$

In (14) and (15), F is the generation cost function, P_{Gi} is the generated power of unit i , V is the vector of bus voltage magnitudes ($N_b \times 1$), δ is the vector of bus voltage angles ($N_b \times 1$), P_D and Q_D are the vectors of real and reactive power loads ($N_d \times 1$), P_G^{min} , P_G^{max} , Q_G^{min} and Q_G^{max} are the vectors of real and reactive power limits of the generators ($N_g \times 1$), V^{max} and V^{min} are the vectors of maximum and minimum allowed voltage magnitudes ($N_b \times 1$), $S(V, \delta)$ is the vector of power flows in the lines ($N_t \times 1$), S^{max} is the vector of power rating limits of the transmission lines ($N_t \times 1$) and $P(V, \delta)$ and $Q(V, \delta)$ are the vectors of real and reactive power injections ($N_b \times 1$). Also, N_b is the number of buses, N_d is the number of load buses, N_t is the number of transmission lines and N_g is the number of generators.

In the standard minimization problem given by (14) and (15), all generation and network constraints have been taken into consideration. Also, it has been assumed that one of the bus angles is zero in the constraints (15) to work as a reference bus.

IV. THE PROPOSED METHOD

The proposed method tracks the change in the Lagrange multipliers and their region of validities of the optimization problem that is depicted in (14) and (15). For each operating point, there will be different values for the Lagrange multipliers depending on the problem constraints. The Lagrange multipliers used in this paper are those associated with the power balance equation which is in turn limited from the right hand side by the loading level of the system. To illustrate this concept, consider a linear system consists of two variables (x_1 and x_2) and three constraints. The graphical representation of this system is shown in Fig. 1.

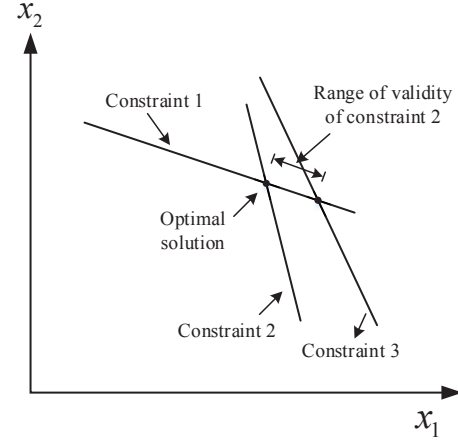


Fig. 1. The flow chart of the solution process.

It can be seen from Fig. 1, under the current conditions, that the optimum operating point of the system is constrained by two constraints (constraint 1 and constraint 2). These two constraints are called active constraints (or binding constraints). The third constraint (constraint 3) is an inactive constraint. In other words, the Lagrange multiplier associated with constraint 3 has zero value but Lagrange multipliers associated with constraints 1 and 2 have some values. If constraint 2 is relaxed, for example, the optimum point can be moved along constraint 1 until constraint 3 becomes an active constraint. The amount of the change in the optimum value between these two points is what we call the region of validity of constraint 2.

In the proposed method, locations and sizes of the new sources are deployed sequentially according to the range of validity of the Lagrange multipliers of the power balance equation. The power balance equation is constrained by the loading level of the system. In other words, the Lagrange multipliers associated with this equation represent the amount of load that can be compensated by the new sources to maximize/minimize the objective function. From the solution of the optimization problem, Lagrange multipliers are determined and ranked. Then, the range of validity is determined by increasing the capacities of the new sources at the selected locations. The locations are selected based on the values of the Lagrange multipliers which in turn are changing for each value of the

new sources. Therefore, in this manner, the locations and sizes are determined jointly and sequentially.

V. SOLUTION ALGORITHM

The solution process starts by solving the optimization problem. In this work, an optimal power flow with an objective function of generation cost is solved for the base case (no new DER is added). From the dual solution of the optimization problem, the Lagrange multipliers are determined. The Lagrange multipliers are then ranked in a descending order. According to the number of new devices to be added and their sizes and the ranked Lagrange multipliers, the locations and sizes are determined. For example, if there are three sources, then the highest three Lagrange multipliers are used in determining the locations and sizes. In other words, the source with the highest capacity is located at the bust with the highest value of the Lagrange multipliers. Then, the optimal power flow problem is solved for the new case, after adding the devices, to determine the range of validity of the Lagrange multipliers. The sizes and locations are arranged again according the range of validity. The process continues until all range of validity becomes larger than the largest device. The flowchart of the solution process is shown in Fig. 2.

VI. CASE STUDIES

The proposed method is demonstrated on several test systems including IEEE RTS [18], IEEE 14, 30, 57, 118 and 300 bus test systems [19], [20]. These systems have been extensively tested on several research objectives. The simulation results are compared with those obtained using genetic algorithm and with those obtained using traditional sensitivity analysis methods.

The traditional sensitivity analysis methods are based on determining the Lagrange multiplier values from the current operating point. In other words, they do not consider the range of validity of the Lagrange multipliers. For example, after solving an optimization problem, the Lagrange multipliers are determined and ranked and the new resources are located at the highest Lagrange multiplier values according to their capacities. As it was mentioned above, this type of selection depends on the current operating limits which can be used in the operating environment to reschedule the generation for example. The locations and sizes of the new sources according to the traditional sensitivity analysis methods are shown in the last three columns of Table I and Table II.

The solution of the optimization problems is performed using MATPOWER (an open source) [20]. We have modified these programs to provide an access to the Lagrange multipliers of the constraint of the balance equation which are not provided in the original programs. The MATPOWER is performed in the blocks associated with solving an optimization problem of Fig. 2.

The optimal locations and locations using the proposed method are presented in Table I and Table II. From these tables we can see how the traditional sensitivity analysis methods fail in determining the best locations and sizes.

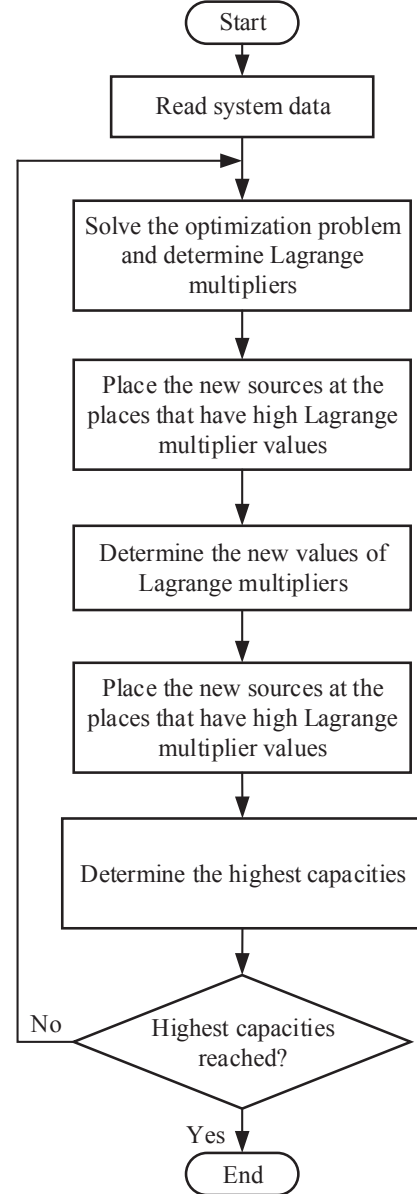


Fig. 2. The flow chart of the solution process.

To validate the results of the proposed method, we performed genetic algorithm on the IEEE 14, 30, 57 and 118 bus test systems. The optimal locations and sizes using the genetic algorithm are shown in Table III. The results of the proposed method confirms very well with those obtained using genetic algorithm.

VII. CONCLUSION

This paper has introduced the use of the sensitivity analysis concept in jointly determining the sizes and locations of DERs. This method is generic in a sense that it can be applied with any objective function and determining the optimal deploy-

TABLE I
OPTIMAL LOCATIONS AND SIZES USING THE PROPOSED METHOD AND
TRADITIONAL SENSITIVITY ANALYSIS METHODS

System		Site and Size					
		Proposed			Traditional		
IEEE 14 bus	Bus	3	14	10	4	10	12
	Size (MW)	30	20	10	30	20	10
IEEE 30 bus	Bus	5	19	30	4	16	24
	Size (MW)	30	20	10	30	20	10
IEEE RTS	Bus	6	7	8	3	10	20
	Size (MW)	40	50	60	60	50	40
IEEE 57 bus	Bus	12	38	36	15	35	55
	Size (MW)	60	50	40	60	50	40
IEEE 118 bus	Bus	41	112	53	30	60	90
	Size (MW)	60	50	40	60	50	40
IEEE 300 bus	Bus	17	251	236	20	120	220
	Size (MW)	60	50	40	60	50	40

TABLE II
COST FUNCTION VALUES USING THE PROPOSED METHOD OF THE TESTED
CASES

System	Cost (\$/hr)		
	Base Case	Traditional	Proposed
IEEE 14 bus	8,171.89	5,724.03	5,693.16
IEEE 30 bus	8,906.15	6,553.00	6,505.47
IEEE RTS	63,352.21	56,428.10	55,902.51
IEEE 57 bus	41,737.79	35,379.46	35,356.85
IEEE 118 bus	129,660.69	123,860.06	123,731.47
IEEE 300 bus	719,725.08	713,709.84	713,446.64

ment of various resources. In performing sensitivity analysis of an objective function with respect to a set of constraints, the proposed method determines optimal locations and sizes of resources based on the range of validity of the subjected constraints. The method was demonstrated on several test systems including the IEEE RTS, IEEE 14, 30, 57, 118 and 300 bus systems and the results were compared with those were calculated using genetic algorithm. The objective function that was used in the tested systems is the generation cost function. Therefore, the best locations and sizes are deployed according to the saving in the generation production cost. However, this method can be applied with various objective functions such as the contribution in the electricity market and providing ancillary services.

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TABLE III
OPTIMAL SIZES AND LOCATIONS USING GENETIC ALGORITHM

System		Location and Size		
		Optimal		
IEEE 14 bus	Bus	3	14	10
	Size (MW)	30	20	10
IEEE 30 bus	Bus	5	19	30
	Size (MW)	30	20	10
IEEE 57 bus	Bus	12	38	36
	Size (MW)	60	50	40
IEEE 118 bus	Bus	41	112	53
	Size (MW)	60	50	40

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