

Pruning and Simulation for Determination of Frequency and Duration Indices of Composite Power Systems

J. Mitra, *Member, IEEE*

LCG Consulting
4962 El Camino Real, Suite 112
Los Altos, CA 94022

C. Singh, *Fellow, IEEE*

Department of Electrical Engineering
Texas A&M University
College Station, TX 77843

Abstract

This paper introduces a method for computing the frequency and duration indices of a composite power system, using random Monte Carlo sampling selectively on those parts of the state space where failure states are more likely to occur. The implementation uses a DC flow model to represent network power flows. The method is tested using a modified form of the IEEE Reliability Test System and is shown to be significantly faster than conventional Monte Carlo simulation, while retaining the same level of accuracy.

Keywords: composite power systems, reliability indices, Monte Carlo simulation, state space pruning

1 Introduction

Reliability analysis forms an important part of almost all power system planning and operation procedures. Applications of reliability analysis range from determination of capital resource allocation for maximization of investment benefits to planning long term and short term operating reserves and maintenance schedules. In the light of the ongoing restructuring of the energy industry and the evolution of new technologies in terms of both equipment and control methodology, system reliability is becoming an increasingly important and complex issue. The problem becomes even more involved due to the extent of the interconnections and the complexity of the interchanges and operating policies. These developments call for the evolution of reliability analysis techniques which allow greater flexibility in system modeling and are computationally more efficient.

At the same time, interest has been shifting from overall system reliability to zonal and even nodal reliability. As the industry deregulates, and as the energy market becomes more competitive, regional indices relating to cost and reliability are coming into focus. The public awareness, too, is growing. The concern regarding reliability and cost of energy is no longer confined to investors and operators. The consumer is getting increasingly aware of these issues.

These circumstances motivate the development of tools and

techniques that permit the determination of indices which are more detailed and easier to interpret. While the most commonly used reliability indices are the LOLE (loss of load expectation, hours per year) and EUE (expected unserved energy, GWh per year), information about the frequency and duration of failures is easier to interpret and at the same time provides a more complete understanding of the reliability of the system or subsystem.

The work described in this paper consists of the development of a method that determines reliability indices, including frequency and duration (F&D) indices, of a composite power system by performing Monte Carlo simulation over the system state space. The method is rendered more efficient by means of a device that allows it to perform sampling over those parts of the state space where the likelihood of occurrence of failure states is higher.

This work uses the pruning and simulation method. This method has been used earlier [1, 2] to determine LOLE and EUE of interconnected systems. The work described here further extends the pruning and simulation method to determine F&D indices, with negligible increase in computational burden.

2 Theory

In this section, we will describe the theory of computation of F&D indices, the pruning and simulation technique, and the conditions under which the computation of F&D indices may be integrated with the framework of the pruning and simulation method. The modeling and implementation issues will be discussed in the next section.

2.1 Probability, Frequency and Duration of System Failure

For the purpose of the treatment presented in this section, a system is assumed to be coherent, i.e., the failure or degradation of a component cannot improve system performance, and, likewise, the improvement or restoration of a component cannot deteriorate system performance. The consequences of relaxing this condition will be discussed later.

For explaining the analysis given in this section, two more conditions will be imposed on the system being analyzed:

- Every component can be represented by a two-state Markov model. We will show later that this approach can be extended to multi-state components.
- The system can transit in one step from one state to another only by failure or restoration of a single component.

A transition that results from the repair of a failed component is said to be an *upward* transition; similarly, a transition that results from the failure of a functional component is said to be a *downward* transition.

We now define the following abbreviations:

a-component $\Rightarrow$ functional (available) component
f-component $\Rightarrow$ failed component
A-state $\Rightarrow$ functional (acceptable) state
F-state $\Rightarrow$ failed state

The probability of system failure is given by

$$P_F = \sum_{i=1}^{n_F} P\{x_i : x_i \in X_F\} \quad (1)$$

where X = set of all states

X_F = set of failed states; $X_F \subset X$

n_F = number of failed states

The frequency of system failure is more difficult to determine. The approach used in this work has been derived and described in detail in [3, 4]. Reference [5] describes a method of determining F&D indices of composite systems, using an approach based on the concepts derived in [3, 4]. The rigorous derivation will not be reproduced here; rather, the expressions will be developed and explained by means of simple arguments.

Each term in the summation described by (1) denotes an F-state, and each F-state describes a combination of a-components and f-components. It is possible for an F-state to transit to a higher state by an upward transition, and the resulting state may be an A-state or an F-state. If the resulting state is an A-state, then the transition is said to have crossed the boundary separating the F-states from the A-states. Now the frequency of encountering A-states is the sum of all the individual frequencies associated with those transitions of the F-states which cross the boundary.

Consider the sum described by equation (2), given below:

$$F_1 = \sum_{i=1}^{n_F} \left[P\{x_i : x_i \in X_F\} \sum_{j \in Z_i} m_j \right] \quad (2)$$

where Z_i = set of f-components in the i -th state

m_j = repair rate of j -th component

F_1 is the sum of the frequencies of all upward transitions from the failed states. Some of these transitions will cross the boundary, while others will not.

Now consider the sum described by equation (3), given below:

$$F_2 = \sum_{i=1}^{n_F} \left[P\{x_i : x_i \in X_F\} \sum_{k \in \bar{Z}_i} l_k \right] \quad (3)$$

where $\bar{Z}_i$ = set of a-components in the i -th state

l_k = failure rate of k -th component

F_2 represents the sum of the frequencies of all downward transitions of the failed states. However, because the system is coherent, none of these transitions will cross the boundary.

Now consider any transition included in F_2 , from state l to state m , by failure of the r -th component. Since all components are two-state Markov components, the following relationship holds:

$$P\{x_l\} l_r = P\{x_m\} m_r \quad (4)$$

Note that $P\{x_l\} l_r$ is included in F_2 while $P\{x_m\} m_r$ is included in F_1 . Observe also that this property extends to every transition included in F_2 .

Now all those transitions included in F_1 which do not cross the boundary are balanced by corresponding transitions included in F_2 ; all the remaining transitions included in F_1 cross the boundary, and the sum of their frequencies gives F_S , the frequency of system success. At the same time, every transition included in F_2 is balanced by an upward transition which fails to cross the boundary. Consequently, we have

$$F_S = F_1 - F_2 \quad (5)$$

In the steady state, $F_F = F_S$, where F_F is the frequency of system failure. So we can conclude that

$$F_F = \sum_{i=1}^{n_F} \left[P\{x_i : x_i \in X_F\} \left(\sum_{j \in Z_i} m_j - \sum_{k \in \bar{Z}_i} l_k \right) \right] \quad (6)$$

Observe that we could have used a similar approach to compute F_F directly from the A-states, using

$$F_F = \sum_{i=1}^{n_S} \left[P\{x_i : x_i \in X_S\} \left(\sum_{k \in \bar{Z}_i} l_k - \sum_{j \in Z_i} m_j \right) \right] \quad (7)$$

where X_S = set of success states

and n_S = number of success states

but (6) is in general easier to compute than (7), since for most systems $n_F < n_S$.

Notice also that (6) is contingent upon (4) being true. For two-state Markov components, the frequency balance holds. In cases where the components assume multiple states, frequency balance does not always apply. However, in such cases, forced frequency balance [6] can be imposed. This is achieved by the following device. For every downward transition, say from system state l to state m , by degradation of the r -th component from component state s to state t , the actual downward transition rate l_{st} is replaced by the fictitious transition rate l'_{st} , given by

$$l'_{st} = \frac{p_t}{p_s} l_{ts} \quad (8)$$

where p_t = probability of component r assuming state t

l_{ts} = actual upward transition rate of component r from state t to state s

If we now apply the same arguments as before and use fictitious downward transition rates from the F-states to balance all those upward transitions which do not cross the boundary, we then have the following equivalent for (6), for the case of multi-state components.

$$F_F = \sum_{i=1}^{n_F} \left[P\{x_i : x_i \in X_F\} \sum_{j=1}^{n_C} \left(l_{ji}^+ - l_{ji}^- \right) \right] \quad (9)$$

where n_C = number of components in the system

l_{ji}^+ = transition rate of component j from its state in system state i to higher states

l_{ji}^- = frequency balanced transition rate of component j from its state in system state i to lower states

The transition rates l_{ji}^+ and l_{ji}^- are given by

$$l_{ji}^+ = \sum_{s=r+1}^{n_j} l_{rs} \quad \text{and} \quad l_{ji}^- = \sum_{t=1}^{r-1} l'_{tr} \quad (10)$$

where r = state of component j in system state i
 n_j = number of states component j can assume

It is possible to construct the frequency distributions of the components in such a manner that forced frequency balance applies, without having to individually compute all the fictitious transition rates. This will be described in section 3. Using these distributions, it is possible to apply (9) in a simple and direct manner.

Given P_F and F_F , it is trivial to calculate T_F , the expected duration of system failure.

$$T_F = \frac{P_F}{F_F} \quad (11)$$

In the rest of this paper, therefore, we will only deal with failure probability and frequency.

2.2 Determination of Failure Indices using Monte Carlo Simulation

Equations (1) and (6) can be applied when all the failure states are known. For a real system with a large number of states, it is often impossible to identify and enumerate all the failure states. In such a situation, it is expedient to use other techniques that avoid exhaustive enumeration of failure states. Monte Carlo simulation is one such technique that affords ample modeling flexibility, and has been used for calculation of LOLE, EUE and F&D indices of composite systems [5, 7]. The equations used in [5] are adaptations of equations (1) and (9).

In this work, too, we use Monte Carlo simulation, but our approach differs from that given in [5] in two major aspects:

- (i) We use a hybrid technique, pruning and simulation, which will be described shortly.
- (ii) Equation (6) assumes that the component models are frequency balanced. If the models are not frequency balanced, forced frequency balancing can be employed as shown in [6]. The techniques described in section 3 of this paper, however, automatically ensure that the frequency balance is achieved.

These features make our approach significantly more efficient and powerful than a purely simulation based method using equations (1) and (8).

We first describe how Monte Carlo simulation can be used to determine failure probability and frequency.

The aggregate of all possible states the system can assume (defined in section 2.1 as the set X) constitutes the system state space. From this state space X , a state x_i is picked, using proportional sampling. Details of proportional sampling are available in [8]; simply stated, it means that states are sampled in such a manner that the likelihood of x_i being picked is equal to the actual probability of the system assuming the state x_i . Based on the sampled x_i , the values of the following random variables are determined.

$$p_i = \begin{cases} 1 & \text{if } x_i \in X_F \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

$$f_i = \begin{cases} \sum_{j=1}^{n_C} (l_{ji}^+ - l_{ji}^-) & \text{if } x_i \in X_F \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

A sufficiently large number, N , (we will soon describe how large is sufficiently large) of such samples are drawn, and the values of the following random variables are computed.

$$\Pi_N = \frac{1}{N} \sum_{i=1}^N p_i \quad (14)$$

$$\Phi_N = \frac{1}{N} \sum_{i=1}^N f_i \quad (15)$$

Π_N and Φ_N are the estimates of P_F and F_F , respectively, and the failure indices are given by

$$P_F = E[\Pi_N] \quad (16)$$

$$F_F = E[\Phi_N] \quad (17)$$

where $E[\bullet]$ is the expectation operator. The estimates Π_N and Φ_N are said to have converged to the statistics P_F and F_F when the sample size N is large enough that the coefficients of variation of the estimates are arbitrarily small, i.e., when the following relationships hold.

$$\frac{\sqrt{\text{Var}(\Pi_N)}}{E[\Pi_N]} \leq d \quad (18)$$

$$\frac{\sqrt{\text{Var}(\Phi_N)}}{E[\Phi_N]} \leq e \quad (19)$$

where $\text{Var}(\bullet)$ is the variance function and d and e are prespecified tolerances.

2.3 The Method of Pruning and Simulation

For a system of realistic size, the state space would be very large, and if the system is highly reliable, as most systems are designed to be, the sample size would have to be quite large before the indices converge to reasonably small tolerances. Depending on the complexity of the system model, the process of testing a large number of samples may turn out to be extremely time consuming. We have therefore developed a method whereby some arbitrarily large parts of the state space are identified where failure states do not occur, and these sets of acceptable states (A-sets) are pruned off prior to performing simulation on the remaining state space. This increases the likelihood of a sampled state being an F-state, since the proportional probability of every state with respect to the sample space is increased. Naturally, the simulation converges much faster. This is the essence of the pruning and simulation technique. This has been described in considerable detail in [1, 2, 9]. It has also been shown in [1, 2, 9] that if N samples are required from the complete state space for Π_N to converge to some value of d then N' samples are required from the residual state space for $\Pi_{N'}$ to converge to the same value of d where N' is given by

$$\frac{N'}{N} = a \frac{1 - P_F/a}{1 - P_F} \quad (20)$$

where a is the probability of the residual state space. Observe that for very small values of P_F , $N' \gg a N$.

Pruning is performed by identifying an acceptable state (partition vector) and discarding this state and all higher states. This may be performed as many times as desired, or deemed suitable. The manner in which the partition vector is determined depends on how the system is modeled, and on how the network flows are represented. In this work, the network is represented by nodes, representing the buses, interconnected by arcs, representing the transmission lines. The generation and load at each node and the transmission capacity of each arc is modeled as a set of discrete capacity levels, each level associated with a probability and frequency. These probability and frequency distributions are constructed as cumulative distributions, so that frequency balance is enforced. We will elaborate on this in the next section. The network flow assumed in this work is the DC flow model [1, 2, 9, 10, 13]. Reference [9] provides a discussion on the different power flow models used and their appropriateness in various reliability analysis applications. The DC flow model, which is widely considered appropriate for composite system analysis, is used in this work.

For a given load scenario, the available bus generations and the transmission line capacities will determine whether the bus loads will be satisfied. We can therefore define the state space as the set of all possible combinations of generation levels and transmission line capacities. The capacities of the generation and transmission components in each state comprise the elements of the vector describing that state. The treatment of temporal load variations and planned outages of generators can be accommodated using load clustering. This involves clustering the multi-node hourly loads into a relatively small number (10–20) of multi-area load vectors, each with an associated probability and frequency. Then a reference load state is defined, using which the generation model is modified for every load cluster, and these modified models are interleaved to construct an integrated generation model. Similar modification and subsequent integration is used to accommodate planned outages. The resulting generation model needs to use only one load state, which is the reference state. The method is described in detail in [11, 12]. Since clustering finally results in a model that uses a single load level, our method will be described in this paper using a single load level. The idea can be easily extended to include multiple load levels and planned maintenance. In general, therefore, a system of N_A areas and N_T transmission lines will have a discrete state space of dimension $(N_A + N_T)$, each axis consisting of area generation or transmission line capacity levels, zero levels included.

The method of pruning used in this work uses an approach similar to state space decomposition [9, 10, 13, 14]. The original state space is first treated as an unclassified set (U-set). Based on the maximum capacity levels available in this U-set, the system load curtailment is minimized; then the combination of the lowest capacity states which yield zero curtailment constitutes a partition vector which will be called the u-vector. The u-vector has the property that all states between and including the u-vector and the upper boundary of the U-set will be acceptable states and will constitute an A-set. Using the u-vector, the original U-set is now decomposed into an A-set, and $N_A + N_T$ disjoint U-sets, some of which may be empty. The A-set is discarded, and the remaining U-sets are submitted to simulation. If more A-sets are desired to be removed, more U-sets are decomposed. Now as the pruning progresses, the size of the A-sets generated diminishes, but the

effort expended on determination of a u-vector (which requires solution of a constrained optimization problem) remains the same. So it is prudent to stop pruning after a point, and perform sampling over the residual state space.

From the undecomposed U-sets (comprising the residual space) a U-set is randomly selected in such a manner that the likelihood of selecting that state equals the ratio of the probability of the U-set to that of the residual space. Within the selected U-set, proportional sampling is used to select a generation level at every node, and a transmission capacity for every arc. If multiple load levels are used, a load level is also sampled. The generation-transmission-load scenario thus selected constitutes the sampled state, which is tested for acceptability, and equations (10) – (13) are used to update the estimates of the system indices. Similar criteria are used to estimate the failure indices at the buses. It is important to state at this point that equations (10) – (13) estimate indices with respect to the sample space, so these estimates must be divided by the residual space probability α to get the correct estimates of P_F and F_F .

2.4 Dealing with Noncoherence

In the methodology presented so far, we have seen that the state space needs to be coherent, i.e., an upward transition from an A-state should result in an A-state, and a downward transition from an F-state should result in an F-state. If this condition is not satisfied, we have the following problems to deal with.

- The arguments used in developing equations (2) – (8) cease to be valid.
- The process of pruning also depends on all states higher than the u-vector being A-states.

When a DC flow model is assumed, the system is coherent for generation capacity changes, but not under transmission capacity variations. This is because transmission capacity changes are accompanied by changes in line impedances, causing the power flows to be redistributed. So if a system is in an A-state, the redistribution of flows resulting from the restoration of a failed transmission line may cause loss of load in some part of the system; similarly, a line failure may result in a transition from an F-state to an A-state.

We deal with the problem as follows. During the pruning phase, the decomposition is performed over the generation levels, holding the transmission levels at the highest capacity states. In other words, every time the u-vector is determined, the components of the u-vector corresponding to the transmission lines are set at the maximum capacity levels. Since the transmission system is generally far more reliable than the generation system, fixing the transmission levels does not significantly reduce the effectiveness of the pruning. This takes care of problem (b).

To deal with problem (a), we construct a premise, based on the following propositions:

Proposition 1: Dispatcher action precludes the occurrence of those events whereby the restoration of a failed component results in deterioration of system performance.

Proposition 2: In a power system rendered noncoherent solely by inclusion of DC or AC flow constraints, the aggregated probability of states contributing to noncoherence is negligibly small.

Proposition 1 simply means that a dispatcher can choose not to bring a repaired transmission line back into service, if such restoration is deemed detrimental to system performance. We claim that proposition 1 is justifiable from an engineering standpoint, and as such does not represent an approximation or inaccuracy. However, it excludes those events whereby the degradation or failure of a component improves system performance, since these are random events beyond the dispatcher's control. Ignoring these transitions constitutes an approximation which, we claim via proposition 2, may be acceptable. The results reported in [7], as well as our own work reported in this paper, support this assertion. Besides, when pruning is performed, these states always reside in the residual space, so if the indices are estimated only from sampled F-states, these states, if sampled, are appropriately identified as A-states, and contribute to no error at all.

Nevertheless, the above premise represents a mathematical imperfection, and its acceptance or otherwise is essentially a matter of engineering judgement. If, however, we choose to accept it, then problem (a) is solved, because the method described in sections 2.1 and 2.2 computes the F&D indices from sampled F-states, and pruning the state space leaves *all* F-states for simulation. We emphasize the word *all*, since we have chosen to accept proposition 1.

3 Model Description

In this section we will briefly describe the component models, i.e., the generation, transmission and load models used in this work.

We have stated earlier in section 2.3 that the generation, transmission and load are represented by discrete distributions of probability and frequency corresponding to the various capacity levels.

Based on the capacity states and interstate transition rates of generating units available at a given bus, discrete probability and frequency distributions are constructed, at every bus, using the *Unit Addition Algorithm* [11]. It should be noted that when algorithms in [11] are used, frequency balance is automatically preserved. Now it is known [6] that

$$P_j(n) = P_j(n-1) + p_j(n) \quad (21)$$

$$F_j(n) = F_j(n-1) + p_j(n)(I_{jn}^+ - I_{jn}^-) \quad (22)$$

where $P_j(n)$ = cumulative probability of n -th state
 $F_j(n)$ = cumulative frequency of n -th state
 $p_j(n)$ = exact state probability of n -th state
and the subscript j is the component index.

A less general form of equation (22), applicable to two-state generating units, had in fact been derived earlier in [15]. The generalized, multi-state form given by (22) was developed later in [6].

An array of 'incremental transition rates,' L_j , is constructed from the cumulative frequencies, using equations (21) and (22).

$$L_j(n) = \frac{F_j(n) - F_j(n-1)}{P_j(n) - P_j(n-1)} \quad (23)$$

So during the simulation equation (13) is applied in the following form:

$$f_i = \begin{cases} \sum_{j=1}^{n_C} L_j(n) & \text{if } x_n \in X_F \\ 0 & \text{otherwise} \end{cases} \quad (24)$$

The modeling of loads is a little more complex, unless only one load state is used. If a multi-state load model is used, then the frequency indices will be incorrect unless all nodal loads are perfectly correlated, i.e., unless they all increase and decrease together, otherwise the residual state space will be noncoherent. But in real life temporal load profiles are bound to be diverse across geographically extensive systems. However, in composite system analysis we normally use a detailed representation of only a part of a system, and if the load diversity is not too large, then the LOLF can be computed, albeit with reduced accuracy. In constructing the P , F and L distributions for a multi-node, multi-state load model, the clustering approach is used [9, 11, 12]. There are two more points to note in the context of load modeling. One is that in order that system coherence is preserved, load states are sequenced in the reverse order, i.e., a higher load level is regarded as a lower load state, since a decrease in system load improves the system performance from a reliability perspective. The other point to note is that since a multi-state load model is constructed in the form of multi-node vectors, the entire load model is associated with only one set of P , F and L distributions. Consequently, in (22), $n_C = N_A + N_T + 1$ for a multi-state load model, and $n_C = N_A + N_T$ for a single state load model.

The distributions P , F and L need to be constructed only once. The distributions F and L can be constructed simultaneously with the distribution P , and require little additional effort. During simulation the use of (22) also requires very little additional effort. So the same algorithm which was used in [1] to calculate the loss of load expectation (LOLE) and expected unserved energy (EUE) can be used to additionally calculate the loss of load frequency (LOLF) with minimal increase in computational burden.

4 Test Cases and Results

The method described in the preceding sections was tested using the *Modified Reliability Test System* [10]. The MRTS is identical in topology to the IEEE-RTS [16], and differs from the latter in that all generation capacities are doubled and the loads are multiplied by a factor of 1.8, while the transmission line characteristics are the same. The MRTS is preferred over the RTS because it was found [10] that the transmission network of the RTS was too strong, and transmission constraints had little effect on the reliability indices.

The MRTS has 24 buses and 38 lines; 32 generators inject power into 10 buses, and loads are connected to 17 buses. The total generation capacity is 6810 MW, and the peak load is 5130 MW. For the studies reported here, only the peak load levels were used.

The indices calculated were the LOLE, LOLF and EUE for the system and at the individual buses. These indices are reported in TABLE I. In TABLE II we demonstrate the relative decrease in sample size and computation time with increase in the extent of pruning. In all cases, the indices were made to converge to within 2.5% of the LOLE at bus 14.

TABLE I: COMPOSITE RELIABILITY INDICES OF MRTS

Location	Reliability Indices			Location	Reliability Indices		
	LOLE <i>h/y</i>	LOLF <i>f/y</i>	EUE <i>GWh/y</i>		LOLE <i>h/y</i>	LOLF <i>f/y</i>	EUE <i>GWh/y</i>
System	623.847	17.761	91.933	Bus 9	167.717	1.738	17.557
Bus 1	3.688	0.229	0.351	Bus 10	106.057	2.269	12.837
Bus 2	7.546	0.363	0.713	Bus 13	24.451	0.887	4.35
Bus 3	54.373	1.613	7.822	Bus 14	261.341	7.651	24.711
Bus 4	46.629	2.066	4.372	Bus 15	3.256	0.216	0.669
Bus 5	37.409	0.784	2.285	Bus 16	43.291	2.270	4.757
Bus 6	98.863	4.958	5.732	Bus 18	1.733	0.119	0.096
Bus 7	12.241	0.623	0.179	Bus 19	22.271	1.111	2.419
Bus 8	40.426	1.729	2.801	Bus 20	3.798	0.275	0.281

TABLE II: VARIATION OF SAMPLE SIZE AND COMPUTATION TIME WITH PRUNING LEVEL

Residual Probability a	Sample Size ¹ N'	$\frac{N'}{N}$	$a \frac{1-p/a}{1-p}$	Computation Time ² (minutes)		
				Pruning time	Simulation time	Total time
1.0000	51697	1.000	1.000	0.00	60.58	60.58
0.5743	29865	0.578	0.561	0.01	33.50	33.51
0.2655	12359	0.239	0.243	0.65	13.31	13.96
0.1532	6700	0.130	0.127	3.82	7.22	11.04
0.0975	3609	0.070	0.070	18.58	4.47	23.05

The first case in TABLE II corresponds to simulation over the entire state space. The indices obtained in the five cases were about the same, with small deviations that can be attributed to uncertainty and numerical error. This observation validates the method, as well as the premise we constructed in section 2.4.

The value of p used in TABLE II column 4 equals the bus 14 LOLP. Columns 3 and 4 are included with the intent to demonstrate the compliance of the results with equation (20). The data in columns 5, 6 and 7 are shown graphically in Figure 1. This plot indicates that there is an optimal mix of pruning and simulation, with the efficiency of pruning deteriorating as the residual probability approaches the system LOLP. This is due to the fact that as the ‘concentration’ of F-states increases in the residual space, the size of the A-set that can be removed by a u-vector progressively diminishes, while the effort expended on determining the u-vector remains the same.

The pruning process involves removing sets of acceptable states, and since the number of these sets is relatively small, the process is quite efficient. The stopping of pruning can be based on comparing the probability of sets removed at any stage with the previous stages.

5 Discussion and Conclusion

In this paper we have presented a method of calculating the F&D indices of composite systems, using the pruning and simulation technique. Even though the estimation of indices is based on Monte Carlo simulation, the method is much more efficient than techniques that rely on simulation alone. The following aspects result in the method being powerful and efficient.

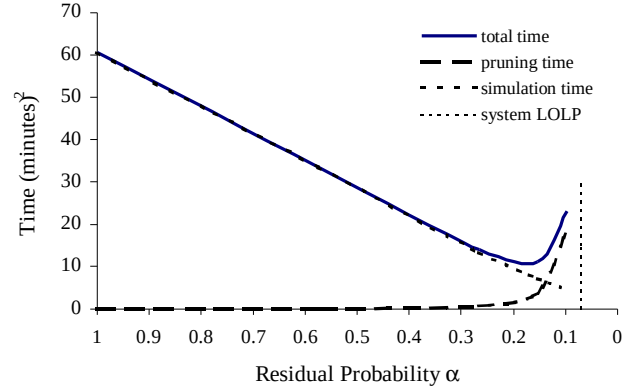


Figure 1. Variation of computation time with pruning level

- (i) The probability and frequency distributions of the component capacities are so constructed that frequency balancing is enforced. This enables the estimates of the frequency indices to be updated quickly and easily during Monte Carlo simulation.
- (ii) The state space is pruned prior to performing simulation. This results in considerable acceleration of convergence of the simulation.

The method also allows the use of other variance reduction techniques [10, 17] to further accelerate the convergence.

¹Converging on bus 14 LOLE, coefficient of variation 2.5%.

²CPU time on a DEC Alpha 2100-4/275.

At the same time, the method retains the ability to include generation and transmission line contingencies of all orders, to handle arbitrary network configurations, to treat arbitrary probability distributions of generation and transmission states, and to capture the correlation between nodal loads by means of clustering techniques.

The pruning and simulation method has been used earlier [1, 2, 9] to determine LOLE and EUE of both composite and multi-area systems. However, using this method (or for that matter any method which uses random sampling) to estimate frequency indices of multi-area systems could introduce error in the frequency estimates due to the large system-wide diversity in temporal load profiles. In the analysis of composite systems, though, it is common and accepted practice to assume nodal loads to be perfectly correlated; then the system remains coherent with respect to load variations.

Another issue that warrants comment is that of the optimization routine. The minimization of load curtailment under resource constraints and DC flow constraints requires solution of a linear programming problem. This optimization takes up the bulk of the solution time, so the computation time depends substantially on the LP package used. The work reported in this paper used the IMSL routine DLPRS. (The work reported in the earlier papers [1, 2, 13] used LP packages developed by the authors.) There are commercially available LP routines that are significantly faster than DLPRS, and the use of such routines would further reduce the solution time. Another relevant observation is that the use of different LP routines results in some variation in nodal indices, though the system indices remain the same, within bounds of numerical error. In any case, the purpose of this work was to evolve a technique that is relatively much faster than conventional Monte Carlo simulation and at the same time permits the determination of F&D indices.

Acknowledgment

The work reported in this paper was supported by the National Science Foundation Grant ECS-9412712 and Energy Resources Grant 95-20.

References

- [1] C. Singh, J. Mitra, "Composite system reliability evaluation using state space pruning," *IEEE Trans. Power Syst.*, vol. 12, no. 1, pp. 471–479, Feb 1997.
- [2] J. Mitra, C. Singh, "A hybrid approach to addressing the problem of noncoherency in multi-area reliability models," in *Proc. Power System Computation Conference*, (Dresden, Germany), pp. 1011–1017, Aug 1996.
- [3] C. Singh, "Calculating the time-specific frequency of system failure," *IEEE Trans. on Reliability*, vol. 28, no. 2, pp. 124–126, Jun 1979.
- [4] C. Singh, "Rules for calculating the time-specific frequency of system failure," *IEEE Trans. on Reliability*, vol. 30, no. 4, pp. 364–366, Oct 1981.
- [5] A. C. G. Melo, M. V. F. Pereira, A. M. Leite da Silva, "A conditional probability approach to the calculation of frequency and duration indices in composite system reliability evaluation," *IEEE Trans. Power Syst.*, vol. 8, no. 3, pp. 1118–1125, Aug 1993.
- [6] C. Singh, "Forced frequency-balancing technique for discrete capacity systems," *IEEE Trans. on Reliability*, vol. 32, no. 4, pp. 350–353, Oct 1983.
- [7] A. C. G. Melo, M. V. F. Pereira, A. M. Leite da Silva, "Frequency and duration calculations in composite generation and transmission reliability evaluation," *IEEE Trans. Power Syst.*, vol. 7, no. 2, pp. 469–476, May 1992.
- [8] C. Singh, R. Billinton, *System Reliability Modeling and Evaluation*, London, UK: Hutchinson Educational, 1977.
- [9] J. Mitra, "Models for reliability evaluation of multi-area and composite systems," Ph.D. dissertation, Texas A&M University, College Station, TX, 1997.
- [10] EPRI, "Composite system reliability evaluation methods," Report EL-5178, Jun 1987.
- [11] C. Singh, Q. Chen, "Generation system reliability evaluation using a cluster based load model," *IEEE Trans. Power Syst.*, vol. 4, no. 1, pp. 102–107, Feb 1989.
- [12] Z. Deng, C. Singh, "A new approach to reliability evaluation of interconnected power systems including planned outages and frequency calculations," *IEEE Trans. Power Syst.*, vol. 7, no. 2, pp. 734–743, May 1992.
- [13] J. Mitra, C. Singh, "Incorporating the DC load flow model in the decomposition-simulation method of multi-area reliability evaluation," *IEEE Trans. Power Syst.*, vol. 11, no. 3, pp. 1245–1254, Aug 1996.
- [14] D. P. Clancy, G. Gross, F. F. Wu, "Probabilistic flows for reliability evaluation of multiarea power system interconnections," *Electric Power & Energy Systems*, vol. 5, no. 2, pp. 101–114, Apr 1983.
- [15] J. D. Hall, R. J. Ringlee, A. J. Wood, "Frequency and duration methods for power system reliability calculations. Part I: Generation system model," *IEEE Trans. Power App. Syst.*, vol. 87, pp. 1787–1797, Sep 1968.
- [16] IEEE Committee Report, "IEEE reliability test system," *IEEE Trans. Power App. Syst.*, vol. 98, no. 6, pp. 2047–2054, Nov/Dec 1979.
- [17] G. J. Anders, *Probability Concepts in Electric Power Systems*, New York: Wiley, 1990.

Biographies

Joydeep Mitra received his B.Tech.(Hons.) degree in Electrical Engineering from the Indian Institute of Technology, Kharagpur, in May 1989. He then worked for two years with Tata Steel, Jamshedpur, India. In May 1997 he received his Ph.D. degree in Electrical Engineering from Texas A&M University, College Station. Since then he has been working with LCG Consulting, Los Altos. His research interests include reliability, production costing, network power market analysis, and power system planning.

Chanan Singh is Professor and Department Head of Electrical Engineering, Texas A&M University. He received the 1972 Best Paper Award of the Engineering Institute of Canada, the 1986–87 Haliburton Professorship, and the 1992–1993 Dresser Professorship. He also served as Director, NSF Power System Program, for the year 1995–96. Dr. Singh is a senior TEES Fellow at Texas A&M University, Fellow of the IEEE, and Advisory Editor for *Microelectronics and Reliability*, Pergamon Press.