

Incorporating the DC Load Flow Model in the Decomposition-Simulation Method of Multi-Area Reliability Evaluation

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Abstract

This paper presents a computationally viable means of incorporating the DC Load Flow model in the decomposition-simulation method of multi-area reliability evaluation. The implementation described herein uses a multi-state generation model for each area, a cluster-based multi-area load model which accommodates load correlation between areas, a linear programming model with DC Load Flow constraints for determining the partitioning states in the decomposition phase, and a similar LP model for determining acceptability of sampled states in the simulation phase. Formulations have been described which enhance the speed and accuracy of the method. The implementation has been tested on 3-area, 4-area, and 5-area cases, and the results obtained therefrom have been compared with those produced by an explicit Monte-Carlo simulation. While the results obtained from both methods are similar, the current method has been shown to be considerably faster than explicit simulation.

Keywords: interconnected systems, multi-area reliability, reliability evaluation, DC load flow, simultaneous decomposition-simulation

1 Introduction

Power utilities have traditionally operated as power pools, to enhance such operational attributes as reliability, stability, and economy. This has required the development of appropriate multi-area tools for planning and operation of interconnected systems. Some recent and emerging developments like competitive markets, non-utility generation, and open transmission access will necessitate further developments in this area.

Several methods have been proposed [1, 2, 3, 4, 5, 6, 7, 10, 11, 13] for multi-area reliability analysis of looped configura-

tions. Some of the proposed methods are purely analytical [1, 2, 4, 5], some are entirely based on Monte Carlo simulation [6, 7], while others are hybrid in nature [3, 10, 11, 13], with an analytical phase followed by a simulation phase. These methods have used two types of transmission models — the *capacity flow model*, and the *DC flow model*. Models used in multi-area studies typically require macro representation, and thus equivalent models need to be developed for tie-lines [7]. There exists, among the users and developers of these models, a difference in opinion as to which of the two transmission models is more appropriate. For instance, the MARS model [6], which is used by some utilities, incorporates the capacity flow constraints, whereas the one in preliminary use [7, 14] at ERCOT incorporates the DC flow constraints. There is thus a need for both types of models; a preference towards one or the other can be based on the particular application.

So far the analytical and hybrid approaches have almost solely relied on the capacity flow model, and very little significant work has been reported on efforts to include power flow constraints in such studies. The need exists, therefore, for a model which efficiently includes the effect of tie-line admittances and thereby produces more realistic indices.

Perhaps the most straightforward implementation of the DC flow model is in the form of an explicit Monte-Carlo simulation, which randomly samples generation state — tie-line state — load state scenarios from the complete state space, and computes the contribution of each sampled state to the reliability indices, till the mean computed indices converge to stable values. This method affords tremendous flexibility in terms of modeling, and is often used to provide benchmark results against which results obtained by other methods can be tested. However, simulation is often extremely time consuming, especially for highly reliable systems.

The method of state space decomposition was identified as a potential time-efficient alternative. In 1980, Cheong and Dillon [8] successfully tested a decomposition algorithm including DC flow constraints. The method reported in [8] used a specially adapted parametrized linear programming model, and was applied to predict upper and lower bounds for the reliability indices. However, the test system used in [8] consisted of two areas, each with one generator and one load state. The

need remained for a viable methodology which would be capable of handling multi-area systems of realistic dimensions. The work reported in this paper represents a major step towards fulfilling this need. The proposed implementation consists of a hybrid method, using the idea of simultaneous decomposition-simulation incorporating DC load flow constraints.

2 Theoretical Basis

The solution process consists of two phases, decomposition and simulation, which are briefly explained here. A more detailed development will follow in later sections.

2.1 State-Space Decomposition

For any given load scenario, the available area generations and the tie-line capacities will determine whether or not the area loads will be satisfied. We can, therefore, define a state space as the set of all possible combinations of generation levels and tie-line capacities. For a given load scenario, the state space can be recursively decomposed [3] into *A*-sets (acceptable sets, comprising states which do not result in loss of load), *L*-sets (loss of load sets), and *U*-sets (unclassified sets). In the first sweep, the entire state space is regarded as an *U*-set, and, based on its highest and lowest capacity states, is decomposed into an *A*-set, *L*-sets, and *U*-sets. In subsequent sweeps, each *U*-set is further decomposed into an *A*-set, *L*-sets, and more *U*-sets. This is continued till no *U*-set remains. The total probability of all the *L*-sets is the system Loss of Load Probability (*LOLP*). Every *L*-set is also decomposed into *B*-sets (comprising identical area loss of load states) and *W*-sets (of unclassified loss of load states), and every *W*-set is further decomposed into *B*-sets and more *W*-sets, till at the end we have only *A*-sets and *B*-sets. The *B*-sets yield the *area LOLPs* and *area EUEs* (Expected Unserved Energy indices). The *system EUE* equals the sum of the *area EUEs*.

2.2 Simulation

While exhaustive decomposition, being analytical, is extremely accurate, it suffers from the disadvantage that as the decomposition progresses, the sets obtained from decomposition get progressively smaller, and even though the effort expended on every level of decomposition increases, the contribution to the computed indices becomes progressively smaller. It is therefore prudent to stop decomposition at a certain point, and subject the undecomposed state space to Monte Carlo simulation. States are picked randomly from the undecomposed *U*-sets, *L*-sets and *W*-sets, each sampled state is tested for acceptability, and, if found unacceptable, its contribution to the system and area *LOLPs* and *EUEs* is computed. This is continued till the standard errors of the computed indices drop below prespecified tolerances.

2.3 Simultaneous Decomposition

The method of decomposition described in § 2.1 applies to one given load scenario. To perform this for every hour of forecasted load, or even for daily peaks, would be computationally

intractable. This problem is circumvented by using a cluster based load model [9], with an appropriate number (between 10 and 20) of clusters. In the *Extended Decomposition* model [10], decomposition is performed for every load cluster, then the computed indices are weighted by the corresponding cluster probabilities and aggregated. This can be made considerably more efficient, with no loss of accuracy, by using the *Simultaneous Decomposition* model [11]. The generation and load models are so constructed that the states are integral multiples of the same increment. Then a reference load state is defined, using which the generation model is modified for every load cluster, and these modified models are interleaved to construct the composite generation model. Then, using the reference load state, the composite state space is decomposed to yield the system and area indices.

Using an identical concept, maintenance intervals can also be taken into consideration [12]. A ‘subcomposite’ generation model can be constructed for every planned outage period, and these can be interleaved together to form the composite generation model over the entire year.

2.4 Coherency and the DC Flow Model

The method of decomposition outlined above is based on the assumption of coherency — i.e., once the boundary states for an *A*, *L* or *B* set has been determined, all the states between the boundaries have the same acceptability or loss of load properties. Coherency is, in fact, the necessary and sufficient condition to the decomposition method.

Now when a DC flow model is used the system remains coherent with respect to the generation units, but a tie-line failure or, indeed, any change in the values of tie-line susceptances (which necessarily accompany changes in tie capacity states) would alter the flow profile, and this may result in a tie capacity violation, thus creating a loss of load state within what would otherwise have been a coherent acceptable set. (Note that the usual capacity flow model precludes this kind of non-coherency, since the line susceptances do not affect the flow profile.)

Therefore, strictly speaking, the inclusion of the DC flow model in the decomposition framework necessitates the assumption of perfectly reliable tie-lines. Studies on real systems [14], however, indicate that because of the relatively low failure rates of tie-lines, it is the tie-line capacities, rather than their failure rates, which have a significant effect on the reliability indices. Moreover, the failure rates of tie-lines can be included by computing indices for a given failure state and then aggregating the indices using conditional probability. This way selected contingency levels of tie-lines can be incorporated.

3 Model Development and Integration

In this section, references will be made concerning concepts and algorithms which are used to set up the models in the present implementation. Details of algorithms referred to may be found in the publications cited.

3.1 Generation Model

Based on the capacity states and forced outage rates of units available in a given area during a given maintenance interval, a discrete Probability Distribution Function is constructed, using the *Unit Addition Algorithm* [9], for every area and every maintenance interval. To enable integration of these models into the composite model, the *PDFs* are so constructed that the capacity difference between any pair of consecutive states (i.e., the step size) is constant over all the *PDFs*.

3.2 Load Model

Based on the hourly forecasted load data in every area over each maintenance interval, a multiarea cluster load model [10, 11] is constructed for every maintenance interval. This means that for every interval we have a load model consisting of an appropriate number (between 10 and 20) of cluster load vectors, each vector associated with a cluster probability. Each cluster load vector consists of the cluster load levels in all the areas.

In these cluster models, every cluster load level is rounded off to the nearest integral multiple of the step size used in constructing the generation models. This is done to enable integration into the composite model.

3.3 Composite State Space

From the cluster load models over all the maintenance intervals, an appropriate load model is chosen for every area, to constitute the reference load level vector. The same reference load vector must serve as reference over the entire period under consideration, so that all the models can be integrated into a single composite state space [11, 12].

For a given maintenance interval, the corresponding generation model in every area is modified for every cluster load level in that interval. This modification consists of ‘shifting’ a generation model by a capacity level equal to the difference between the cluster load and the reference load for that area, so that the margin between any generation level in the modified model and the reference load remains the same as that between the corresponding capacity level in the original model and the cluster load. These modified models are interleaved together to form a subcomposite state space for the given maintenance interval. This is repeated for every maintenance interval, and the subcomposite models are interleaved to produce the composite state space.

The above operations result in a many-to-one mapping of the states in the original state space onto the composite state space. If there are N_i maintenance intervals and N_c cluster load levels, then, in the limiting case, as many as $N_i \times N_c$ states in the original state space can map onto a single state in the composite state space. This ‘condensing’ of the state space is what makes the *Simultaneous Decomposition* algorithm so efficient.

It is necessary to preserve the original generation models, because of the necessity to refer back to these models when computing the indices.

3.4 DC Flow Model in Decomposition Phase

For the decomposition of any U -set, two partition vectors are required:

1. the u -vector, which defines the lower boundary of the A -set obtained from the current decomposition. (The upper boundary coincides with that of the current U -set.)
2. the v -vector, each element of which defines the minimum responsibility of the corresponding area. The minimum responsibility of an area is the minimum level of generation that area must have, in order that no area suffers loss of load, while the generation at every other area coincides with the corresponding upper boundary in the current U -set.

It is the manner of selection of the partition state vectors that is determined by the flow model used and the loss sharing policy considered. When the DC Flow model is used, the u -vector is determined from the solution of the following Linear Programming problem [15]:

$$\text{Loss of Load} = \text{Min} \sum_{i=1}^{N_a} C_i$$

subject to:

$$\begin{aligned} \hat{B}\theta + G + C &= D \\ G &\leq G^{max} \\ C &\leq D \\ \hat{A}^T b\theta &\leq F_f^{max} \\ -\hat{A}^T b\theta &\leq F_r^{max} \\ G, C &\geq 0 \\ \theta &\text{ unrestricted} \end{aligned} \tag{1}$$

where

- N_a = number of areas
- N_t = number of tie-lines
- C = N_a -vector of area load curtailments
- C_i = i -th element of C , i.e., unsatisfied demand in area i
- D = N_a -vector of net negative injections for No Load Loss Sharing (NLLS) and actual area loads for Load Loss Sharing (LLS)
- G^{max} = N_a -vector of maximum available net positive injections for NLLS and available area generation for LLS; available generation levels coincide with upper bounds of current U -set
- F_f^{max} = N_t -vector of forward flow capacities of tie-lines
- F_r^{max} = N_t -vector of reverse flow capacities of tie-lines
- G = N_a -vector of net positive injections for NLLS and actual area generation for LLS
- θ = N_a -vector of node voltage angles
- b = $N_t \times N_t$ primitive (diagonal) matrix of tie-line susceptances

$$\begin{aligned}
\hat{A} &= N_t \times N_a \text{ element-node incidence matrix} \\
\hat{B} &= N_a \times N_a \text{ augmented node susceptance matrix} \\
&= \hat{A}^T b \hat{A}
\end{aligned}$$

In the NLLS case, the solution for the G -vector needs to be modified to provide the actual area generation; in the LLS case this modification is unnecessary. The solution for the G -vector obtained from this model is basically the u -vector, unless one or more elements of G lie below the corresponding lower bounds of the current U -set, in which case they are set equal to the lower bounds.

For the determination of the v -vector, the i -th element is obtained from the solution of the following LP problem:

$$v_i = \text{Min } G_i$$

subject to:

$$\begin{aligned}
\hat{B}\theta + G &= D \\
G &\leq G^{max} \\
\hat{A}^T b \theta &\leq F_f^{max} \\
-\hat{A}^T b \theta &\leq F_r^{max} \\
G &\geq 0 \\
\theta &\text{ unrestricted}
\end{aligned} \tag{2}$$

The determination of the v -vector, therefore, requires the solution of the LP problem (2) N_a times, once for every area. In the event that (2) fails to find a feasible solution for a certain area j , v_j is set equal to the corresponding upper bound of the current U -set. If (2) yields a solution which lies below the corresponding lower bound of the U -set, it is set equal to the lower bound.

3.5 DC Flow Model in Simulation Phase

In the simulation phase, states are randomly sampled from the undecomposed state space and tested for acceptability using model (1). The only difference is that the G^{max} vector is now determined from the sampled generation states. If the solution to (1) yields a zero loss of load value, the indices remain unaltered; otherwise the probability of this state is computed and the contribution of this state is added to the appropriate indices. This is continued till the standard errors of the computed indices drop below prespecified tolerances.

3.6 Computation of Indices

It has been stated in § 3.3 that each state in the composite state space can be mapped back to as many as $N_i \times N_c$ pre-images in the original state space. The same argument applies to sets of states. So if the probability of a set in the composite state space is to be computed, then the set must first be mapped back to all its pre-images in the original state space, and the sum of the probabilities of all these pre-image sets must be computed to yield the probability of the composite set.

The sum of the probabilities of the L -sets gives us the *system LOLP*, and the sum of the probabilities of the B -sets correspond-

ing to each area loss of load gives us the *area LOLP* for that area.

Determination of the *area EUEs* proceeds as follows. For every composite B -set, all the pre-image sets are determined. For each pre-image set, the mean generation vector is determined. Using the mean generation vector and the actual load vector for the pre-image set, the LP model (1) is executed to provide the mean curtailment vector. The product of this curtailment vector and the probability of the pre-image set gives us the contribution of this pre-image set towards the *area EUEs*. The sum of the contributions of all the pre-image sets is the total contribution of the composite B -set. The total *area EUEs* are computed from all the B -sets. The *system EUE* is the sum of the *area EUEs*.

In the simulation phase, of course, only individual states are sampled; the contribution of each sampled state can be computed directly from the probability of the set, and the curtailment vector obtained from a single execution of LP model (1).

This indicates a further motivation for switching over to simulation after a certain level of decomposition. Beyond this level, not only does the contribution of each cycle of decomposition become very small, the number of B -sets produced at each level also increases exponentially, thereby increasing the number of times (1) has to be executed. This drastically increases the computational burden, since (1) is itself an iterative algorithm.

The above consideration motivated the development of a more efficient embodiment of the DC flow model. This embodiment, as developed by the authors, is outlined in the next section.

4 A Computationally Efficient Embodiment of the DC Flow Model

Consider the model given below.

$$\text{Loss of Load} = \text{Min} \sum_{i=1}^{N_a} C_i$$

subject to:

$$\begin{aligned}
\hat{B}\theta + G + C &= D \\
G + X_1 &= G^{max} \\
\hat{A}^T b \theta + X_2 &= F_f^{max} \\
-\hat{A}^T b \theta + X_3 &= F_r^{max}
\end{aligned} \tag{3}$$

$$G, C, X_1, X_2, X_3 \geq 0 \text{ and } \theta \text{ unrestricted}$$

where X_1, X_2, X_3 are slack variables.

This is not only a restatement of model (1) in standard LP form, but also has two major differences:

1. The constraint $C \leq D$ has been discarded. Since the sum of the area load curtailments is being minimized, the structure of the problem will inherently maintain C below D , and hence the constraint is redundant. Discarding this constraint has the desirable effect of reducing the dimension of the feasibility space by N_a .

2. The first constraint equation, which is the power balance equation, is not augmented by artificial variables, even though it is an equality constraint. Rather, it is left as an equality, since one can intuitively see that in the power balance equation, the area load curtailments effectively serve as slack variables. This formulation therefore obviates the need for artificial variables, and enables the problem (3) to be solved directly, without using the two-phase method.

There are other aspects to this formulation which enable an efficient implementation. Before these are discussed, it is appropriate to restate model (2) in standard LP form:

$$v_i = \text{Min} G_i$$

subject to:

$$\begin{aligned} \hat{B}\theta + G &= D \\ G + X_1 &= G^{max} \\ \hat{A}^T b\theta + X_2 &= F_f^{max} \\ -\hat{A}^T b\theta + X_3 &= F_r^{max} \end{aligned} \quad (4)$$

$$G, X_1, X_2, X_3 \geq 0 \text{ and } \theta \text{ unrestricted}$$

The need for artificial variables in model (4) may be avoided by using a duplex algorithm consisting of alternate executions of dual and primal simplex procedures to ensure both optimality and feasibility.

In models (3) and (4), the θ -vectors need to be split into

$$\theta = \theta' - \theta'', \quad \theta', \theta'' \geq 0$$

to conform to the standard LP model. This means that models (3) and (4) will have constraint coefficient matrices of sizes $(2N_a + 2N_t) \times (5N_a + 2N_t)$ and $(2N_a + 2N_t) \times (4N_a + 2N_t)$, respectively. The feasibility spaces for both models will be of dimension $(2N_a + 2N_t)$.

Now in both models if we select the $(2N_a + 2N_t)$ rightmost columns to constitute the starting bases, then these starting bases will be of the form:

$$[I_{(2N_a + 2N_t)}] \quad \text{for model (3)}$$

and

$$\begin{bmatrix} I_{N_a} & 0 & 0 \\ I_{N_a} & I_{N_a} & 0 \\ 0 & 0 & I_{2N_t} \end{bmatrix} \quad \text{for model (4)}$$

Note that not only are both these starting bases guaranteed to be invertible, their inverses can also be formed directly, without actual inversion:

$$[I_{(2N_a + 2N_t)}] \quad \text{for model (3)}$$

and

$$\begin{bmatrix} I_{N_a} & 0 & 0 \\ -I_{N_a} & I_{N_a} & 0 \\ 0 & 0 & I_{2N_t} \end{bmatrix} \quad \text{for model (4)}$$

If in addition to directly constructing the inverses of the starting bases, the method of product form inversion is used for the determination of every modified basis inverse in subsequent LP iterations, both accuracy and speed of the LP solutions can be substantially increased. Indeed, the entire Decomposition-Simulation can be performed without ever completely inverting a matrix.

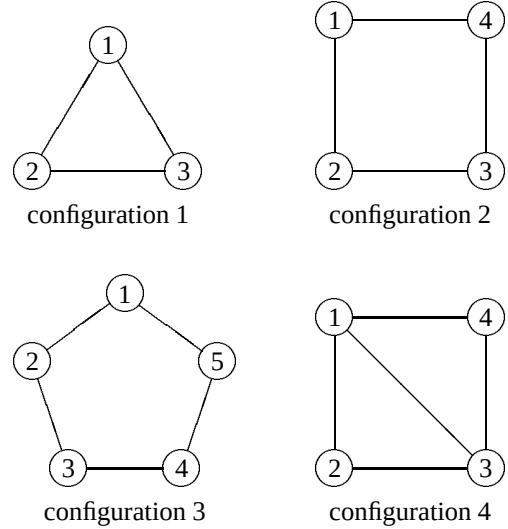
Sections 3 and 4 comprehensively outline the method actually implemented by the authors. The next section presents the test cases and results.

5 Test Cases and Results

In this section test cases and results are reported which support the validity of the implementation and highlight some of the salient aspects of the method.

5.1 Case Bank

It is convenient to first define the test cases used in this study. Four configurations are defined:



Every area in the above configurations is identical to the IEEE-RTS [16], with a maximum area generation of 3405 MW; the hourly load model is used with a peak of 3000 MW, i.e., with 13.5% reserve. Every inter-area tie has a capacity of 300 MW, in both forward and reverse directions. The tie-line susceptances are different for the different cases, tabulated below:

CASE	CONFIGURATION	TIE-LINE SUSCEPTANCES
CASE 1	configuration 1	all ties: $-40 pu$
CASE 2	configuration 2	all ties: $-40 pu$
CASE 3	configuration 3	all ties: $-40 pu$
CASE 4	configuration 1	ties 1-2, 2-3: $-40 pu$ tie 1-3: $-10 pu$
CASE 5	configuration 4	all ties: $-40 pu$
CASE 6	configuration 4	ties 1-2, 2-3, 3-4, 4-1: $-40 pu$ tie 1-3: $-10 pu$

The assistance policy assumed is No Load Loss Sharing (NLLS). When there is a loss of load, the curtailments are allocated among the areas suffering loss of load in such a manner

as to minimize the total curtailment, while ensuring that the assisting areas first meet their entire area demands. The model, however, allows implementation of other policies. The Load Loss Sharing policy, for instance, can be implemented as explained in the legend describing model (1). Area assistances can be prioritized by using a weighted sum of area curtailments in model (3). Other policies may be incorporated in the constraint equations or as additional constraints.

For these test cases no planned outage is considered, i.e., the entire year is treated as a single interval with full generation availability, except for forced outages.

5.2 Model Validation

The validity of the integrated model described in sections 3 and 4 was established by comparing its performance with that of a model based entirely on Monte Carlo simulation. Briefly, the two models are defined as:

Simultaneous Decomposition-Simulation using DC flow constraints; generation model constructed with step size Z_S ; N_c -cluster load model rounded off to multiples of Z_S . LP models (3) and (4) implemented with direct construction of inverses of starting bases and product form inversion. ... (DSIDS)

and

Explicit Simulation over complete state space spanned by load, generation, and tie-line states. Loads are considered on an hourly basis; hourly loads are traversed sequentially and generation states are drawn for the given hourly loads. LP model (1) implemented with two-phase simplex algorithm. ... (EXSIM)

For the studies reported here, the following parameters were used in the above models: In DSIDS, step size was $Z_S = 50$ MW; number of load clusters was $N_c = 10$. The simulation part of DSIDS was required to converge to the same tolerance level as EXSIM was, i.e., in both models the standard errors were required to converge to within $\pm 2.5\%$ of the corresponding mean values.

The models were tested on **CASE 1**, **CASE 2** and **CASE 3**; their performances are compared in TABLE 1.

It should be noted that the first difference between the two models lies in load modeling. Whereas EXSIM uses exact hourly load data over the entire year, DSIDS models the load by clustering the entire hourly load model into a relatively small number (10 in these studies) of cluster states. The second difference is that DSIDS is a hybrid model, using part analytical approach and part sampling, while EXSIM is purely a simulation model. These two differences can lead to some variation in the results obtained by the two models. The third difference is in the LP formulation and solution techniques. This difference affects only the computation time and not the accuracy.

The studies reported in TABLE 1 indicate that the results obtained by the two methods are quite close, but that DSIDS is several orders of magnitude faster than EXSIM. The agreement between the results validates the models and assumptions used

in the development of the DSIDS model.

When EXSIM was used to simulate from the 10-cluster load model, the indices and the CPU times were close to those obtained using the exact hourly load model. The cluster model basically discretizes the hourly load model into N_c cluster states. The probability of cluster states is an approximation of the distribution of exact hourly loads. Therefore the drawing of a given cluster state is an approximation of the event of drawing an hourly load state belonging to the set represented by that cluster state. This explains why both the load models required approximately equal numbers of samplings to meet the same convergence criterion.

5.3 Sensitivity to Threshold Probability

The *Threshold Probability* p_0 is the minimum probability a set (U , L or W) must have in order to be subjected to decomposition. If the probability of a set is less than p_0 , it is set aside for the simulation phase. The level of p_0 therefore determines the stage at which decomposition stops and simulation begins. Understandably, the computation time is sensitive to the value of p_0 . This sensitivity is illustrated in TABLE 2.

These studies indicate that for every topology there is an optimum threshold probability p_0^* , which takes the lowest computation time. Though it is not possible to determine the exact value of p_0^* for every case, values of p_0^* in the order of 10^{-7} to 10^{-6} seemed to perform satisfactorily for all cases studied.

5.4 Sensitivity to Tie-Line Susceptances

The DC flow model distinguishes itself from the capacity flow model by its sensitivity to tie-line susceptances. This is illustrated through the following examples:

1. **CASE 4** differs from **CASE 1** in that the susceptance of tie-line 1–3 has been lowered, while its capacity remains the same. This actually results in a ‘weakening’ of tie 1–3, so that flows in the other ties are now preferred. Since both the other ties connect to area 2, the latter naturally has a higher reliability than areas 1 and 3, as reflected in TABLE 3.
2. In **CASE 5**, the cross-tie is electrically ‘stronger’ than in **CASE 6**; consequently, areas 1 and 3 have higher levels of reliability in **CASE 5** than they do in **CASE 6**. This, too, is reflected in TABLE 3.

Since **CASE 4**, **CASE 5**, and **CASE 6** bring out the most interesting aspects of this study, it is relevant to compare the results obtained for the two models, for these cases. TABLE 4 shows the results obtained from EXSIM; the results in TABLE 3 were, of course, obtained from DSIDS.

5.5 Effect of Interconnection

It can be seen from TABLE 1 that as the number of interconnected systems increases, the area reliability levels improve, as expected. However, since in each case every area is connected to only two other areas, there is a transmission bottleneck (since

TABLE 1: PERFORMANCE COMPARISON BETWEEN MODELS DSIDS AND EXSIM

CASE	DSIDS					EXSIM				
	LOLE (h/year)		EUE (MWh/year)		CPU time ¹ (hh:mm:ss)	LOLE (h/year)		EUE (MWh/year)		CPU time ² (hh:mm:ss)
	SYSTEM	AREA	SYSTEM	AREA		SYSTEM	AREA	SYSTEM	AREA	
CASE 1	1.52	0.63	282.0	94.0	0 : 00 : 30	1.60	0.69	278.0	92.7	3 : 55 : 00
CASE 2	1.23	0.35	191.8	48.0	0 : 02 : 30	1.26	0.38	191.0	47.8	11 : 12 : 00
CASE 3	1.16	0.26	173.9	34.8	0 : 09 : 00	1.26	0.30	179.0	35.8	19 : 52 : 00

TABLE 2: SENSITIVITY TO THRESHOLD PROBABILITY

CASE	CPU time ³ for different threshold probabilities (p_0)					Optimal p_0^*
	$p_0 = 10^{-5}$	$p_0 = 10^{-6}$	$p_0 = 10^{-7}$	$p_0 = 10^{-8}$	$p_0 = 0.0$	
CASE 1	0 : 01 : 10	0 : 01 : 10	0 : 00 : 30	0 : 00 : 30	0 : 00 : 30	0.0
CASE 2	0 : 04 : 00	0 : 02 : 30	0 : 04 : 30	0 : 08 : 20	****	10^{-6}
CASE 3	0 : 21 : 00	0 : 11 : 40	0 : 09 : 00	****	****	10^{-7}

TABLE 3: SENSITIVITY TO TIE-LINE SUSCEPTANCES

CASE	LOLE (h/year)			EUE (MWh/year)		
	SYSTEM	AREAS 1 & 3	AREA 2	SYSTEM	AREAS 1 & 3	AREA 2
CASE 1	1.52	0.63	0.63	282.0	94.0	94.0
CASE 4	2.64	1.10	0.84	418.2	151.8	114.6

CASE	LOLE (h/year)			EUE (MWh/year)		
	SYSTEM	AREAS 1 & 3	AREAS 2 & 4	SYSTEM	AREAS 1 & 3	AREAS 2 & 4
CASE 5	0.79	0.16	0.33	137.4	23.6	45.1
CASE 6	0.92	0.23	0.34	156.4	32.0	46.2

TABLE 4: RESULTS FROM MODEL EXSIM FOR CASE 4, CASE 5, AND CASE 6

CASE	LOLE (h/year)			EUE (MWh/year)		
	SYSTEM	AREAS 1 & 3	AREA 2	SYSTEM	AREAS 1 & 3	AREA 2
CASE 4	2.74	1.15	0.92	402.0	139.5	123.0

CASE	LOLE (h/year)			EUE (MWh/year)		
	SYSTEM	AREAS 1 & 3	AREAS 2 & 4	SYSTEM	AREAS 1 & 3	AREAS 2 & 4
CASE 5	0.84	0.20	0.35	144.0	29.5	43.0
CASE 6	0.98	0.24	0.37	159.0	33.0	46.5

tie capacities are same in all cases), and the improvement in reliability is not very significant.

Now in the 4-area case, when a cross-tie is connected between areas 1 & 3, the improvement in both area and system reliability levels is quite significant. (Compare **CASE 2** in TABLE 1 with **CASE 5** and **CASE 6** in TABLE 3.)

While these studies have demonstrated the applicability of DC flow constraints to the method of state space decomposition for cases with upto 5 interconnected areas, decomposition models [13] have been used on systems with larger numbers of interconnected areas and tie lines, and it has been observed that in such cases the size of the state space, and consequently the computation time, increases with the number of interconnections.

¹On a VAX 9000, with optimal threshold probability p_0^* (see TABLE 2).

²On a VAX 9000.

³On a VAX 9000; a '****' entry implies that the time was inordinately long.

6 Discussion and Conclusion

The studies reported in section 5 show that the implemented hybrid model is much faster than the explicit simulation model, and at the same time the results from the two models compare favorably. While the improved formulation and implementation of the LP algorithm is partly responsible for the increased efficiency, it is the basic approach that makes most of the difference. In the purely Monte Carlo approach, the complete original state space is subjected to random sampling. On the other hand, the simultaneous decomposition-simulation process stratifies the states so that sampling is performed from sets of states which are closer in relationship. For instance, in an L -set, all states are loss of load states, differing only in the areas that suffer loss of load. It can be seen that even a relatively low level of decomposition ($p_0 = 10^{-5}$) can lead to a large saving in computation time.

The contributions of this paper can now be summarized as follows:

1. Integration of suitable models into an analytical framework which can accommodate the DC load flow constraints in a computationally tractable form.
2. Formulation of a computationally efficient embodiment of the DC flow model, which reduces the dimensions of the feasibility spaces, obviates the need for artificial variables, and enables the LP forms to be solved without ever completely inverting a matrix.
3. Development and implementation of a hybrid methodology by integrating the aforementioned analytical framework with a suitable Monte Carlo model to enable reliability evaluation of interconnected systems of practical dimensions within a reasonable time frame.

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Biographies

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Discussion

Quan Chen, New England Power Planning, Holyoke, MA: The authors are to be congratulated for their contribution in an important and difficult field. Multi-area reliability evaluations have important applications in power system planning. I am particularly interested to know the following:

1. How different is the reliability index calculated by using the transportation model and by using the DC model? For example, for the Configuration 4 in Section 5.1.
2. What are authors' comments on incorporating an HVDC tie line in the model?
3. From my knowledge, the NARP model of ERCOT uses a Monte Carlo simulation technique and an LP model. Is there any difference between the EXSIM model (Section 5.2) and the NARP model of ERCOT?

A.T.Holen (The Norwegian Institute of Technology), **A.Johannessen**, **O.Kvennås** (The Norwegian Electric Power Research Institute), Trondheim, Norway: The authors are to be congratulated for demonstrating the feasibility of including power flow constraints in a macro type multi-area reliability model based on analytical and hybrid solution techniques. The susceptance modelling of tie-lines allows a more realistic representation of flow constraints than a flow capacity model, and we believe the sensitivity of LOLE and EUE to line susceptance demonstrated by the test cases in the paper justify that it is worth looking carefully at this approach, particularly for systems where the tie-lines may be bottlenecks for area and system reliability. This is normally the situation for the systems we are looking at. Our specific questions and comments are:

1. Computation time, number of areas and capacity step-size. We believe it is a common experience with multi-area models that computation time seems to increase rather sharply when number of areas are increasing. We think this is also demonstrated by the test cases in the paper. The number of states, and consequently computation time, also depend upon the capacity (and load) step size. Do the authors have any experience and/or advice about the maximum practical number of areas that can be handled with "reasonable" computation time and state of the art work-stations or PCs? Is increase of step size a recommended "medicine" to reduce computational burden or is that hazardous with respect to accuracy?
2. The LP optimum criterion. The criterion applied in the paper is minimization of total curtailed load. Recent customer surveys have demonstrated that cost of interrupted load depends a lot on customer category. Hence, it may be justified to apply different costs for different areas, provided that areas have a different mix of customer categories. A modification of the criterion used in the LP solution to include cost of curtailed power rather than the power itself might contribute to focus on different area characteristics and reveal possible transmission bottlenecks. This is probably becoming more interesting as we are moving towards open transmission access and increased power wheeling. We would appreciate the authors' comment on this.

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Gene Preston (City of Austin Electric Utility, member of the Electric Reliability Council of Texas Reliability Task Force, Austin, Texas): J. Mitra and C. Singh have given us a new tool to evaluate bulk system reliability. As a member of the ERCOT group that helped in the development of the N Area Reliability Program, I will share some ideas that went into the NARP program's development. The authors may want to comment on differences between NARP and the methods presented in the paper.

Early in the NARP program development, the ERCOT committee decided that the transportation model (referred to as a capacity flow model in this paper) would not give the correct distributions of power between load areas in the ERCOT system. This decision eliminated programs based on the transportation model.

The DC model was created to have incremental flows

between areas as close as possible to those observed in a full AC load flow. The original DC model was prepared by a committee as a cut and try process and took some effort to calibrate. The process is now automated with a computer program that creates a DC link model from inter-area tie flows from the output reports of running full AC load flows.

Experience with the NARP program shows transmission link capacity limits are reached when relatively large numbers of generators are forced off line (10 to 30 typically). These generator outages result in large enough power shifts between areas to encounter DC link constraints and load is shed. The load shedding allocation is determined by solving an LP with loss of load minimized. The authors may want to comment on how load shedding is assigned to more than one area when the objective function in the LP has no preference between the areas. The authors may also want to comment on computer run time versus generation reserve margin. ERCOT's experience is that a very high generation reserve margin may require very long run times to find failure states.

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J. Mitra, C. Singh, (Texas A & M University, College Station, TX): The authors would like to thank the discussers for their insightful comments and queries.

Dr. Chen expressed an interest in comparing indices obtained from the transportation model with those obtained from the DC flow model. For CASE 1, CASE 2, CASE 3 and CASE 5, the transportation model results in slightly more optimistic indices, as shown below for CASE 5, corresponding to Configuration 4.

LOCATION	LOLE (h/year)	EUE (MWh/year)
SYSTEM	0.67	129.2
AREAS 1 & 3	0.14	22.7
AREAS 2 & 4	0.29	41.9

The difference results from the fact that in the DC flow model the tie-line flows are dictated by the network topology and tie-line susceptances, while in the transportation model they are only constrained by tie-line capacities; as a result, the transportation model is more effective in exploiting the tie-line capacities, and therefore yields more optimistic indices. CASE 4 and CASE 6 cannot, of course, be handled by the transportation model, because of its inability to accommodate tie-line susceptances.

Inclusion of HVDC tie-lines involves a simple extension of the LP model (1). Since power flows in HVDC lines are completely controllable and are independent of network topology and reactances, an inflow or outflow through a HVDC tie can be regarded as an increase in generation or load, respectively. In fact, if all the interconnecting ties were HVDC, a transportation model would be adequate. As such, HVDC ties can be included in the DC flow framework by modifying model (1) to:

$$\text{Loss of Load} = \text{Min} \sum_{i=1}^{N_g} C_i$$

subject to:

$$\begin{aligned}
\hat{B}\theta + \hat{A}_{dc}^T F_{dc} + G + C &= D \\
G &\leq G^{max} \\
C &\leq D \\
(\hat{A}^T b)^T \theta &\leq F_f^{max} \\
-(\hat{A}^T b)^T \theta &\leq F_r^{max} \\
F_{dc} &\leq F_{dc,f}^{max} \\
-F_{dc} &\leq F_{dc,r}^{max} \\
G, C &\geq 0 \\
\theta &\text{ unrestricted}
\end{aligned}$$

where

F_{dc} = vector of HVDC tie-line flows

$F_{dc,f}^{max}$ = vector of forward flow capacities of HVDC tie-lines

$F_{dc,r}^{max}$ = vector of reverse flow capacities of HVDC tie-lines

$\hat{A}_{dc}$ = element-node incidence matrix of HVDC ties only

All other variables are the same as described before, pertaining only to AC components.

Dr. Chen's observation regarding the similarity between the NARP model used by ERCOT and the EXSIM model described in the paper is justified; both models are in fact identical.

Mr. Preston has raised some interesting issues based on his experience with the NARP program, which, as we have stated earlier, is identical to the EXSIM model referred to in the paper. It is true that the objective function in the LP models shows no preference between the areas when it comes to distributing the curtailments. However, the manner of this distribution merits comment. The LP starts out with all curtailments equal to zero. Since it seeks an optimal solution by moving from one corner point on the feasible polytope to an adjacent one, and comes to a stop as soon as an optimal point is encountered, the resulting allocation of curtailments is such that the number of areas suffering loss of load is kept at a minimum. In other words, the curtailments at the remaining areas do not change from their starting values of zero. Mr. Preston's observation regarding the increase in run time for large reserve margins is also appropriate. This can be understood as follows. An increase in the reserve margin results in improved reliability, i.e., in a reduced LOLP. This means that in a given number of sampled states, the proportion of loss of load states is lower; hence a larger number of states must be sampled before the indices converge.

Dr. Holen, Dr. Johannesen and Dr. Kvennås have also addressed some pertinent issues. It is true that the computation time increases sharply with the number of areas. This is primarily due to two reasons: one is that the size of the state space increases; for instance, if every area is identical to the IEEE-RTS, and there are n interconnected areas, then the state space

contains 69^n generation states for a 50 MW step-size. The other reason is that the dimensionality of the LP problem increases with the number of areas and number of tie-lines, and this has a significant effect on the run time, since the LP takes up the bulk of the solution time. As such, it is difficult to estimate a limit on the number of areas that can be reasonably handled by the proposed method. This is because this limit depends primarily on the number of generation states in the areas. It is more realistic to relate the computation time to the product of the numbers of generation levels in all the areas. As pointed out by the discussers, the machine used for the computations is also an important factor. For instance, the 3-area case with complete decomposition took about 300 seconds of CPU time on a SPARC-20 workstation with a 50 MHz processor. While increasing the step-size may affect the computation time, this should be exercised with caution. Our experience has shown that while the LOLEs are not severely affected by the step-size, the EUEs begin to diverge if the step-size becomes too large. For instance, when a step size of 100 MW was used, the following results were obtained.

CASE	LOLE (h/year)		EUE (MWh/year)	
	SYSTEM	AREA	SYSTEM	AREA
CASE 1	1.65	0.70	360.9	120.3
CASE 2	1.27	0.37	242.8	60.7
CASE 3	1.22	0.28	226.0	45.2

The observation that assigning different costs to different areas can accommodate variations in cost of interrupted loads in different customer categories is also appropriate. In fact, a wide range of situations and constraints may be accommodated by varying the cost coefficients in the objective function and by altering or adding to the constraints. The inclusion of HVDC ties is a case in point. Another example is that in a deregulated environment, it may be cheaper in some instances to shed loads than to import power from another area. This condition may be incorporated by including in the objective function a weighted sum of the θ variables, in addition to the curtailments.

Incidentally, there is a typographical error in LP model (1). In the expressions for the tie-line flows, the θ vector is premultiplied by the transpose of the matrix $(\hat{A}^T b)$, rather than by the $(\hat{A}^T b)$ matrix itself. This omission of the transpose notation has been repeated in (2), (3) and (4). The model including HVDC ties, appearing in this discussion, shows the constraint equations in the correct form.

The authors once again thank the discussers for their valuable comments and questions.

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