

Capacity Assistance Distributions for Arbitrarily Configured Multi-Area Networks

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Abstract

This paper presents a method of constructing a probability distribution for the capacity assistance available to a system from its neighboring interconnected areas. The proposed method is capable of dealing with arbitrarily connected networks. Additionally the method considers the correlation between area loads. A discrete joint distribution is used for the multi-area load model, and the capacity assistance distribution is constructed in the form of a joint probability distribution between the assistance levels and the load vectors. The uses of such a joint distribution matrix as a reliability analysis tool as well as an operations planning tool are described. The method is tested using the IEEE-RTS, with 3-area, 4-area, 5-area, and 10-area interconnections.

Keywords: multi-area systems, capacity assistance, reliability analysis, operations planning

1 Introduction

Power system interconnections provide a host of benefits, such as increased stability, economy and security. Several attempts have been made to quantize these benefits, both in terms of overall system operation and in the context of the benefits available to a given area in the interconnected network. The latter context has been gaining more relevance in the light of the current trend towards deregulating the power industry.

Quantization of the benefits available to a given area (*study area*) from its neighbors (*assisting areas*) enables the study area to perform such operations planning activities as unit commitment, scheduling of equipment maintenance, and deciding the amount of spinning reserve to be maintained. Techniques for

assessing interconnection benefits and utilizing this information for reliability and operations planning applications are well established [1–7]. Perhaps the most common form of quantization of interconnection benefits is the *capacity assistance distribution*, which is the probability distribution of the capacity of assistance available to the study area. The derivation of this distribution is easily understood for the two-area case, where the generation reserve distribution of the assisting area is filtered through the capacity distribution of the interconnecting tie-line to yield the assistance distribution [1, 2, 6]. The extension of this method to the radially connected multi-area case is obvious [2, 6]. However, this method did not apply to looped configurations, and the need remained for a method of constructing capacity assistance distributions for arbitrarily configured multi-area networks. The work reported in this paper fulfills this need.

2 Problem Formulation

The structure of a multi-area interconnected power system can be depicted as shown in FIGURE 1. The areas can be divided into the study area and the assisting areas. The problem here is to find the probability of being able to transfer capacity equal to or greater than a given value.

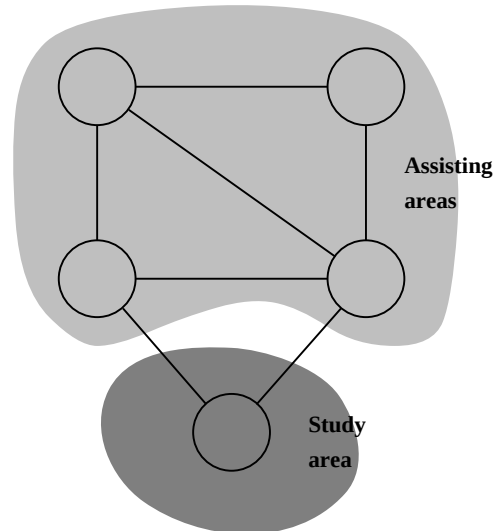


FIGURE 1: PARTITIONING THE MULTI-AREA SYSTEM

The variables involved in this problem are:

1. generators and loads in each area;
2. transmission lines between the areas.

The flow model used will also determine the capacity assistance. The modeling of the above variables is described in the next section.

3 System Modeling

This section briefly describes the models used in the development of the work reported in this paper.

3.1 Generation Model

Based on the capacity states and forced outage rates of units available in a given area, a discrete probability distribution function is constructed, for every assisting area. To enable construction of an integrated state space (described in the next section), the *PDFs* are so constructed that the capacity difference between any pair of consecutive states (i.e., the step size) is constant over all the *PDFs*.

3.2 Transmission Line Model

Based on the capacity states of a tie-line and the probabilities of these states, a transmission model can be constructed in the form of a discrete probability distribution function, over all the capacity levels, including zero. Such models are constructed for all the tie-lines.

3.3 Load Model

Based on the hourly forecasted load data in every area, a discrete joint distribution of area loads is constructed. One method of constructing such a discrete distribution is described in [9]. The distribution is constructed in the form of an appropriate number of multi-area load vectors, each vector associated with a probability. The advantage of using a joint distribution in vector form is that it helps preserve the correlation between area loads.

In constructing such a discrete model, every load value is rounded off to the nearest integral multiple of the step size used in constructing the generation models. This is done to enable construction of the integrated state space.

3.4 Power Flow Model

The manner in which power transfers occur between the areas is represented by an appropriate power flow model. In the studies reported here, the capacity flow model [8, 9] was used, but the technique can be extended to include the DC flow model using the approach described in [10]. It is relevant to note that the future trend will be to use flow technologies such as FACTS to achieve increased control of flow and higher capacity utilization of transmission lines. This will be an important factor in high performance power flow networks in the deregulated

environment. In this scenario the capacity flow model will be appropriate representation of power transfers.

4 Derivation of the Assistance Distribution

The proposed method uses the state space decomposition technique. The technique, as well as the manner in which it is adapted to construct the capacity assistance distribution, are described in this section.

4.1 Characterization of State Space

For any given load scenario, the available area generations and the tie-line capacities will determine whether or not the area loads will be satisfied. We can thus define a state space as the set of all possible combinations of generation levels and tie-line capacities. In general, therefore, a system with N_A areas and N_T tie-lines will have a discrete state space of dimension $(N_A + N_T)$, each axis consisting of the area generation or tie-line capacity levels, zero levels included.

This definition of state space applies to a single load scenario. The treatment of temporal load variation is accommodated by using a discrete joint distribution of area load vectors and constructing an integrated state space, as described here.

Consider a load model with N_L multi-area load vectors. From this load model, an appropriate load level is chosen for every area, to constitute the reference load level vector. For every load vector in the load model, the generation model in each area is modified. This modification consists of ‘shifting’ a generation model by a capacity level equal to the difference between the load level and the reference level for that area, so that the margin between any generation level in the modified model and the reference load remains the same as that between the corresponding capacity level in the original model and the vector load. These modified models are interleaved together to form what will be referred to as an integrated generation model. When the integrated generation model is used to represent the area generation in the $(N_A + N_T)$ -dimensional state space described above, the resulting state space is referred to as the integrated state space.

It is necessary to preserve the original generation models, because of the necessity to refer back to these models when computing the probabilities of states.

The above operations result in a many-to-one mapping of the states in the original state space onto the integrated state space. For a load model with N_L vectors, as many as N_L states in the original state space can map onto a single state in the integrated state space. This ‘condensing’ of the state space is highly effective in the treatment of temporal load variations in the method of *State Space Decomposition*, described below.

4.2 Simultaneous Decomposition

The method of state space decomposition [8] recursively decomposes the state space into disjoint sets of acceptable, unclas-

sified, and failure states, for a given load scenario. This method has traditionally been used for reliability analysis, where the reliability indices are computed from the probabilities of the decomposed sets. The method, when applied to the integrated state space described above, enables inclusion of a more complete load model, and is known as simultaneous decomposition [9, 10].

4.3 Adapting the Method of Decomposition

The method of simultaneous decomposition generates a large number of sets, and for large systems this number often grows unmanageably large, especially if failure sets are also generated. Besides, the determination of a partition state for generation of failure sets is considerably more cumbersome than the determination of a partition state for generation of an acceptable set. Now the proposed method uses only acceptable sets, so the generation of failure sets is conveniently dispensed with.

Yet another modification is required. The criterion for acceptability of a set of states is redefined. Whereas for purposes of reliability an ‘acceptable set’ is defined as a set of states everyone of which results in the total system load being satisfied, for the present purpose this definition is modified to refer to a set of states which satisfy the load in the study area, under the agreed prioritization of assistance.

The adapted decomposition technique is now explained as follows. The original state space is first treated as an unclassified set (U -set); based on the maximum capacity levels available in this U -set, the system load curtailment is minimized; then the combination of the lowest capacity states which yield zero curtailment in the study area constitutes a partition vector which will be called the u -vector. The u -vector has the property that all capacity levels between and including the u -vector and the upper boundary of the U -set will be acceptable states and will constitute an acceptable set (A -set). Using the u -vector, the original U -set is now decomposed into an A -set, and $N_A + N_T - n_1$ disjoint U -sets, where n_1 is the number of single level components in the original U -set (i.e., the maximum generation or transmission level in the U -set coincides with the corresponding minimum level). Each of the resulting U -sets is similarly decomposed. This is continued until all the A -sets in the state space have been identified. If the number of sets generated in the process grows too large, U -sets of extremely small probability (e.g., 10^{-16} or lower) may simply be omitted.

4.4 Constructing the Assistance Distribution

Suppose that in the study area the generation is set to zero, and the load is assigned a constant value of a_0 ; then, if the resulting state space is subjected to decomposition, the total probability of the A -sets equals the probability of the assisting areas being able to supply power to the study area, to the extent of a_0 or more. This idea is used to construct the assistance distribution, as explained here.

It has been stated in section 4.1 that each state in the integrated state space can be mapped back to as many as N_L pre-images in the original state space. The same argument applies to sets of states. Now if an A -set in the integrated state space is mapped back to each of its pre-images in the original state space, and the probability of each such pre-image is computed, then the probabilities of the N_L pre-image sets form a probability vector of size N_L . If the probability vectors corresponding to all the A -sets are added, the sum will be a vector

$$\mathbf{p}_0 = [p_{01} \ p_{02} \ \cdots \ p_{0N_L}] \quad (1)$$

of which the j -th entry has the property that

$$p_{0j} = P\{A \geq a_0, \mathbf{L} = \mathbf{l}_j\} \quad (2)$$

where

A is the random variable representing the assistance available to the study area

a_0 is the level of assistance

$\mathbf{L}$ is the vector random variable representing the load vector

$\mathbf{l}_j$ is the j -th load vector

Now the method can be described as follows.

1. In setting up the state space, generation in the study area is artificially set to zero.
2. An assistance level a_i is selected from the set

$$\{0, a_1, a_2, \dots, a_K\}$$

where a_K is the maximum level of assistance available to the study area (this will usually be limited by the total capacity of the tie-lines connecting the study area to the assisting areas) and the a_i s increase by the step size of the generation model.

3. In setting up the state space, every load vector is modified so that the element corresponding to the study area equals the value of a_i selected above.
4. For every such value of a_i , a decomposition is performed, and a vector $\mathbf{p}_i$ of the form (1) is obtained. This is done for all values of a_i .
5. The $\mathbf{p}_i$ vectors thus obtained are now stacked to produce the matrix $\mathcal{P}$, which has the following form.

$$\mathcal{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1N_L} \\ p_{21} & p_{22} & \cdots & p_{2N_L} \\ \vdots & \vdots & & \vdots \\ p_{K1} & p_{K2} & \cdots & p_{KN_L} \end{bmatrix} \quad (3)$$

where

$$p_{ij} = P\{A \geq a_i, \mathbf{L} = \mathbf{l}_j\} \quad (4)$$

Notice that (4) implies that

$$p_{ij} = P\{A \geq a_i, L_s = l_{js}\} \quad (5)$$

where the subscript s refers to the element corresponding to the study area. The values $\{l_{js}; j = 1, 2, \dots, N_L\}$ are the actual load values (s -th elements of the load vectors) for the study area.

The purpose of building such a joint distribution is to preserve the effect of the correlation between area loads. This will become clearer in the next section.

5 Using the Assistance Distribution

This section describes some of the uses of the capacity assistance distribution. The use of the distribution in determining the reliability of the study area is described in some detail, as this will result in a clearer understanding of the distribution described by (3), (4) and (5). The use of the distribution for operations planning applications are discussed qualitatively.

5.1 Reliability Analysis

Reliability analysis of a study area connected to an assisting area is typically done using capacity assistance distribution method [1]. This method can be readily extended to several assisting areas when the assisting areas are not interconnected [2, 6]. In the past it has not been possible to extend this approach when the assisting areas are interconnected. However using the approach described in this paper it is now possible to achieve this. Assuming the assistance is denoted by A , a loss of load event occurs when:

$$G_s + A - L_s < 0 \quad (6)$$

where G_s and L_s are, respectively, the random variables representing the generation and the load at the study area. G_s also has a discrete probability distribution.

Therefore the loss of load probability of the study area, $LOLP_s$, is given by

$$\begin{aligned} LOLP_s &= P\{G_s + A - L_s < 0\} \\ &= \sum_n P\{G_s < g_n\} P\{L_s - A = g_n\} \end{aligned} \quad (7)$$

where

$$\begin{aligned} P\{L_s - A = g_n\} &= \sum_j P\{L_s = l_{js}, A = l_{js} - g_n\} \\ &= \sum_j [(p_{ij} - p_{(i+1)j}) \mid a_i = l_{js} - g_n] \end{aligned} \quad (8)$$

5.2 Generation Reserve Planning

Using the reliability evaluation technique outlined above, a series of risk assessments are performed, for several levels of generation reserve. This is not too time consuming, since once the assistance distribution $\mathcal{P}$ has been determined, the discrete

convolution described above is extremely fast. (The determination of $\mathcal{P}$ may be time consuming for large systems.) The level of reserve which provides an acceptably low level of risk may be determined from the results thus obtained. This method is discussed in [2].

This approach may be used both for short term spinning reserve planning, as well as for long term static reserve planning.

5.3 Maintenance Scheduling

The assistance distribution matrix $\mathcal{P}$ is not constant, but varies with planned outages of generators in the assisting areas. The distribution $\mathcal{P}$ would therefore be constant over every maintenance interval over the entire system. In each such interval, risk assessments are performed to determine the minimum level of reserve which must be maintained in the study area, and based on this information a decision is made as to which generators (in the study area) can be taken out for maintenance.

5.4 Firm Contracts

Firm contracts between areas will affect the routing of power through the tie-lines. Risk assessments may be performed for various levels of firm contracts to determine the most acceptable level. Likewise, for given levels of firm contracts, the assistance distribution may be used to determine the levels of generation reserve which must be maintained.

6 Test Cases and Results

The proposed method was applied to reliability analysis of a study area connected to several areas. Four cases with different numbers of assisting areas were studied. These configurations are shown in FIGURE 2.

In all the cases, each area is identical to the IEEE-RTS with peak generation 3405 MW and peak load 3000 MW. The generation data for the system can be found in [11]. The tie-lines were assumed perfectly reliable, each with a capacity of 300 MW. The load model used consisted of ten multi-area load vectors, as shown in TABLE A-I in the Appendix. For each load vector, the assistance distribution was convolved with the generation distribution and load in the study area. These results were then combined for all load vectors to arrive at the $LOLP$ for the study area. The $LOLP$ was also calculated using the simultaneous decomposition-simulation method [9]. These results, compared in TABLE I, show very good agreement. The

TABLE I: COMPARISON OF RESULTS

CASE	AREA 1 LOLP	
	proposed method	decomposition simulation
CASE 1	6.456×10^{-5}	6.456×10^{-5}
CASE 2	3.496×10^{-5}	3.496×10^{-5}
CASE 3	2.826×10^{-5}	2.827×10^{-5}
CASE 4	2.640×10^{-5}	2.629×10^{-5}

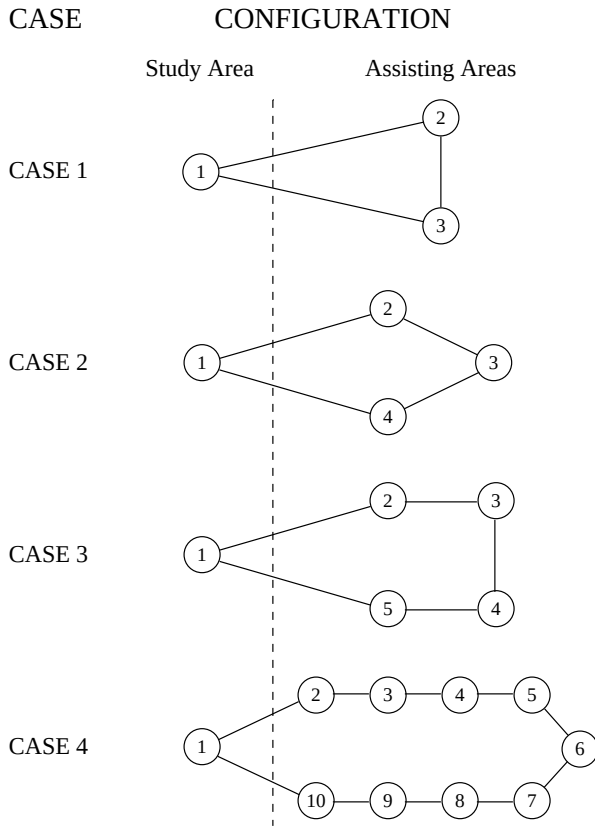


FIGURE 2: CONFIGURATION OF TEST CASES

advantage with the proposed method is that one has to deal only with the acceptable sets in the assisting set of areas.

7 Conclusion

A methodology for constructing capacity assistance distributions when the assistance areas are interconnected is described in this paper. Using the concept of simultaneous decomposition, a set of distributions for load vectors representing joint discrete load distributions can be simultaneously obtained. This method has been applied to the reliability analysis of an area, and the results compared with those obtained from the full decomposition-simulation method.

This type of distribution can be useful for many applications while taking into account the assistance available from other areas. For example when a generator wants to contract power sale to a major customer, it must assess its risk and in so doing needs to know the assistance available in case of a shortfall.

It should be pointed out that it is possible to extend the approach to DC load flow models. It appears, however, that to make use of full capacity flows the networks in future will be reengineered using flow technologies like FACTS to make them respond like capacity flow models. Therefore the proposed method should be a useful decision tool.

Acknowledgment

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The generation and hourly load data used for the test cases reported in section 6 are available in [11]. The area generation was 3405 MW, and the peak load was 3000 MW (i.e., 13.5% reserve). The hourly load data was converted to a discrete multi-area correlated load vector, consisting of ten vectors, using the method outlined in [9]. The model thus obtained is shown in TABLE A-I.

TABLE A-I: THE LOAD MODEL

Load Vector					Probability
j	l_{j1}	l_{j2}	$\dots$	l_{jN_A}	$P\{\mathbf{L} = \mathbf{l}_j\}$
1	1300	1300	$\dots$	1300	2.454212×10^{-1}
2	1600	1600	$\dots$	1600	1.951694×10^{-1}
3	1850	1850	$\dots$	1850	1.571658×10^{-1}
4	2050	2050	$\dots$	2050	1.224817×10^{-1}
5	2200	2200	$\dots$	2200	8.802655×10^{-2}
6	2350	2350	$\dots$	2350	8.642399×10^{-2}
7	2500	2500	$\dots$	2500	6.261447×10^{-2}
8	2650	2650	$\dots$	2650	3.010531×10^{-2}
9	2750	2750	$\dots$	2750	9.844323×10^{-3}
10	2900	2900	$\dots$	2900	2.747253×10^{-3}

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TABLE A-II: CAPACITY ASSISTANCE DISTRIBUTION FOR TEN-AREA CASE

A	Probability distribution of assistance to area 1											
	$p_{ij} = P\{A \geq a_i, L_1 = l_{j1}\}$											
	$l_{11} = 1300$	$l_{21} = 1600$	$l_{31} = 1850$	$l_{41} = 2050$	$l_{51} = 2200$	$l_{61} = 2350$	$l_{71} = 2500$	$l_{81} = 2650$	$l_{91} = 2750$	$l_{101} = 2900$		
$a_1 = 0$	0.2454212	0.1951694	0.1571658	0.1224817	0.0880266	0.0864240	0.0626144	0.0301022	0.0098341	0.0026903		
$a_2 = 50$	0.2454212	0.1951694	0.1571658	0.1224817	0.0880266	0.0864240	0.0626143	0.0301014	0.0098324	0.0026839		
$a_3 = 100$	0.2454212	0.1951694	0.1571658	0.1224817	0.0880266	0.0864240	0.0626142	0.0301006	0.0098307	0.0026780		
$a_4 = 150$	0.2454212	0.1951694	0.1571658	0.1224817	0.0880266	0.0864240	0.0626140	0.0300993	0.0098284	0.0026711		
$a_5 = 200$	0.2454212	0.1951694	0.1571658	0.1224817	0.0880266	0.0864240	0.0626138	0.0300975	0.0098248	0.0026642		
$a_6 = 250$	0.2454212	0.1951694	0.1571658	0.1224817	0.0880266	0.0864239	0.0626133	0.0300950	0.0098202	0.0026551		
$a_7 = 300$	0.2454212	0.1951694	0.1571658	0.1224817	0.0880266	0.0864238	0.0626125	0.0300906	0.0098152	0.0026449		
$a_8 = 350$	0.2454212	0.1951694	0.1571657	0.1224803	0.0880181	0.0863733	0.0624773	0.0298519	0.0096157	0.0024849		
$a_9 = 400$	0.2454212	0.1951694	0.1571656	0.1224789	0.0880122	0.0863440	0.0623554	0.0297120	0.0095379	0.0024184		
$a_{10} = 450$	0.2454212	0.1951694	0.1571654	0.1224757	0.0880020	0.0863070	0.0622698	0.0295031	0.0094420	0.0023675		
$a_{11} = 500$	0.2454212	0.1951694	0.1571649	0.1224699	0.0879755	0.0862417	0.0621416	0.0292857	0.0092333	0.0023019		
$a_{12} = 550$	0.2454212	0.1951694	0.1571640	0.1224617	0.0879458	0.0860744	0.0618636	0.0290070	0.0089975	0.0021796		
$a_{13} = 600$	0.2454212	0.1951693	0.1571622	0.1224477	0.0879086	0.0859596	0.0614390	0.0283667	0.0088578	0.0020481		