

A Homotopy-based Method for Robust Computation of Controlling Unstable Equilibrium Points

Joydeep Mitra, *Fellow, IEEE*, and Mohammed Benidris, *Member, IEEE*

Abstract—This paper proposes a homotopy-based method for efficiently computing and precisely determining controlling unstable equilibrium points (CUEPs) in transient stability analysis and screening using direct methods. The proposed method is intended to overcome potential numerical convergence problems associated with computing CUEPs. These convergence problems are mostly due to irregularities in the region of convergence of CUEPs which make it difficult to determine suitable initial points. The most commonly used initial points in computing CUEPs are minimum gradient points (MGPs) on the stability boundary which in turn rely on the determination of exit points (EPs—points at which sustained-fault trajectories exit system stability boundaries). However, determination of MGPs and EPs are computationally involved and sophisticated numerical methods have been introduced such as stability-boundary-following procedure and shadowing method to correct the solution trajectory. The proposed method does not depend on MGPs and also does not require accurate EPs. The proposed method maps the solution from EPs to the CUEPs. The proposed method is applied on several known systems including the NE 39 bus system and the reduced WECC system and the results are provided. The results are compared with the traditional methods such as Boundary of stability region based CUEP method (BCU method).

Index Terms—Controlling UEPs, homotopy, region of convergence, transient stability.

I. INTRODUCTION

TRANSIENT stability analysis is a major part of power system operation and planning studies. In particular, modern power systems have grown increasingly vulnerable to transient instabilities, due to reduced regulation capability resulting from increased intermittent renewable energy sources and market operations. Therefore, there is a critical need to develop accurate and computationally efficient transient stability analysis methods. This paper proposes an improvement, based on homotopy, that enhances both the computational efficiency and robustness of direct methods of transient stability analysis.

Direct methods have been amply used in on-line transient stability analysis [1]–[11]. Also, some of the direct methods have been practically tested in some control centers (e.g., PJM [9], [12]) for transient stability screening. However, the major concern in applying these methods is the precise determination of the controlling unstable equilibrium points (CUEPs). The numerical problems associated with calculating CUEPs can be understood thus. Stability boundaries under

given system conditions consist of the stable manifolds of a set of unstable equilibrium points including the CUEP. Therefore, determining CUEPs is not a straightforward task and requires the use of initial points that lie in their regions of convergence. However, the convergence regions of CUEPs are characterized by having fractal shapes which cause local convergence problems [6], [13]–[15]. In other words, it is difficult to ensure that a solution algorithm converges to the CUEP. Several techniques have been introduced to overcome these difficulties but these techniques are also computationally involved.

Several methods have been developed to compute the CUEPs. For example, a fault-dependent CUEP method is proposed in [1]. Also, the Boundary of stability region based CUEP method (BCU method) has been used in [16]. However, correction tools such as stability-boundary-following procedure [6] and shadowing method [17] must usually be applied along with the solution algorithms to correct the trajectory. Although these tools have been successfully applied in computing CUEPs, they are numerically involved and do not guarantee convergence to the correct CUEPs [15].

Homotopy-based approaches hold the potential to accurately calculate CUEPs. They are numerical methods that are used to determine equilibrium points (EqPs) of differential-algebraic system of equations and have been proposed to overcome convergence problems of iterative methods [18]–[20]. Homotopy methods have been used in solving several power system problems including power flow calculations [21]–[24], voltage collapse [25], [26], short circuit analysis [27], optimal power flow [28], and economic dispatch [29].

In transient stability analyses, homotopy-based methods have been applied to calculate *closest* unstable equilibrium points (closest UEPs) [30], [31]. The closest UEP based method approximates the stability boundary using a constant energy surface passing through the closest UEP, which is independent of the fault-on trajectory [5]. A closest UEP is determined from a population of type-1 UEPs (type-1 equilibrium points are defined by having 1-dimensional unstable subspace). In this method, type-1 UEPs are determined by utilizing a randomly selected set of starting points and solving using the concept of bifurcation and singular fixed-point technique. The closest UEPs are characterized by having the smallest value for Lyapunov function among the set of type-1 UEPs. However, not only is the computation of closest UEPs numerically involved (i.e., all UEPs on system stability boundary have to be calculated), but also transient stability analyses based on closest UEPs have been proved to produce considerably conservative results [5], [16]. A homotopy-based

Joydeep Mitra is with the Department of Electrical and Computer Engineering Michigan State University, East Lansing, MI 48823, e-mail: mitraj@msu.edu.

Mohammed Benidris is with the Department of Electrical and Biomedical Engineering; University of Nevada, Reno, Reno, NV 89557, e-mail: mbenidris@unr.edu.

method has been proposed in [32] to compute UEPs. The method presented in [32] is based on locally approximating stability boundaries, which also produces inaccurate results [15]. In previous work by the authors [33]–[36], a homotopy-based method was used to calculate the CUEPs. However, the work presented in [33]–[36] does not provide a comprehensive framework to compute the CUEPs nor does it provide a robust device to guarantee the convergence to the correct CUEPs.

This paper introduces a homotopy-based method to efficiently and robustly compute CUEPs. The proposed method finds CUEPs by tracing solution trajectories from *exit points* (EPs) to CUEPs. The proposed method is not sensitive to initial points as is the case for other iterative methods and guarantees the accuracy of the calculated CUEP. The advantages of the proposed method include: (i) it avoids the need to calculate the minimum gradient points (MGPs) as is the case for other iterative methods including the BCU method; (ii) it does not require detecting accurate EPs—it is robust against inaccuracies in initial points; (iii) it uses an adaptable step size to increase the computation speed; and (iv) it has the ability to distinguish the CUEP from a large set of UEPs on the stability boundary. The following two devices are developed to ensure the accuracy and efficiency of the proposed method: (i) the solution trajectory is corrected based on its sensitivity to the change in the potential energy of the system, and (ii) an adaptable homotopy mapping step procedure is used to increase the computation speed. The proposed method is applied on several systems and the results are compared with those calculated using the BCU method. Also, the robustness of the proposed method against initial points is tested by perturbing the calculated EPs.

The rest of the paper is organized as follows. Section II explains the development of the energy function from the system dynamical model. Section III discusses the stability boundary and the characteristics of the region of convergence of CUEPs. Section IV explains the proposed homotopy-based method for computing CUEPs. Section V demonstrates the proposed method on several test systems. Also, section V discusses the robustness of the proposed method against the accuracy of initial points and compares the obtained results with the existing iterative methods. Section VI provides concluding remarks.

II. SYSTEM DYNAMICAL MODEL AND ASSOCIATED ENERGY FUNCTION

This section explains the development of the energy function from the system dynamical model and provides bases and definitions to equations and terms used in calculating CUEPs and system energy margins. In this analysis, system dynamical equations and the energy function are referred to the system center of inertia (COI) frame. Using two different grid configurations (i.e., during fault and after clearing the fault), the following dynamical model is used to determine starting points (exit points as explained in section III) and to compute CUEPs respectively.

A. The Dynamical Model

The dynamical model of the generators is described in terms of swing equations. Suppose that a power system consists of n generators with uniform damping distribution coefficients. The dynamical model of the generators in terms of angle and frequency deviations with respect to the COI reference frame can be expressed as follows [4].

$$\dot{\delta}_i = \tilde{\omega}_i, \quad (1)$$

$$\dot{\tilde{\omega}}_i = \frac{1}{M_i} (P_{mi} - P_{ei}) - \frac{1}{M_T} P_{COI} - \lambda \tilde{\omega}_i, \quad (2)$$

where $\tilde{\delta}_i$ and $\tilde{\omega}_i$ are the angle and speed deviations of machine i respectively, λ is a uniform damping constant, P_{ei} and P_{mi} are the electrical output and mechanical input power respectively of machine i , and M_i is the inertia constant of machine i . The angle and speed deviations with respect to the COI reference frame are defined as follows.

$$\tilde{\delta}_i = \delta_i - \delta_o, \quad \tilde{\omega}_i = \omega_i - \omega_o,$$

where

$$\delta_o = \frac{1}{M_T} \sum_{i=1}^n M_i \delta_i, \quad \omega_o = \frac{1}{M_T} \sum_{i=1}^n M_i \omega_i,$$

and δ_i is the power angle and ω_i is angular frequency of generator i , and $M_T = \sum_{i=1}^n M_i$.

The electrical output power of generator i is given in (3).

$$P_{ei} = \sum_{j=1}^n E_i E_j \left[G_{ij} \cos(\tilde{\delta}_i - \tilde{\delta}_j) + B_{ij} \sin(\tilde{\delta}_i - \tilde{\delta}_j) \right] \quad (3)$$

The P_{COI} can be computed as follows.

$$P_{COI} = \sum_{i=1}^n P_{mi} - \sum_{i=1}^n \sum_{j=1}^n E_i E_j \left[G_{ij} \cos(\tilde{\delta}_i - \tilde{\delta}_j) + B_{ij} \sin(\tilde{\delta}_i - \tilde{\delta}_j) \right], \quad (4)$$

where G_{ij} is conductance and B_{ij} is the susceptance elements of Y_{ij} of the reduced admittance matrix Y , and E_i is the internal voltage magnitude of machine i . Equations (1) and (2) can be expressed in a compact form as follows.

$$\dot{x} = F(x). \quad (5)$$

B. System Energy Function

The energy function has been widely used to compute system stability margins and EPs [1], [2]. Computed EPs have been used to locate suitable initial points to generate solution trajectories toward CUEPs. To this end, the energy function associated with the system dynamical model can be expressed as follows—the first term defines the kinetic energy and the second and third terms define the potential energy, where the superscript s denotes the post-fault stable equilibrium point.

$$V = \frac{1}{2} \sum_{i=1}^n M_i \tilde{\omega}_i^2 - \sum_{i=1}^n P_i \left(\tilde{\delta}_i - \tilde{\delta}_i^s \right) - \sum_{i=1}^{n-1} \sum_{j=i+1}^n \left[C_{ij} \left(\cos \tilde{\delta}_{ij} - \cos \tilde{\delta}_{ij}^s \right) - I_{ij} \right], \quad (6)$$

where

$$P_i = P_{mi} - E_i^2 G_{ii}, \quad (7)$$

and I_{ij} represents the energy dissipated in transfer conductances between nodes i and j which is given in (8).

$$I_{ij} = \int_{\tilde{\delta}_i^s + \tilde{\delta}_j^s}^{\tilde{\delta}_i + \tilde{\delta}_j} D_{ij} \cos \tilde{\delta}_{ij} d(\tilde{\delta}_i + \tilde{\delta}_j), \quad (8)$$

where $D_{ij} = E_i E_j G_{ij}$. From (8), it can be seen that the boundaries of the integral depend upon system angle trajectories which are unknown in advance. Several approaches have been used in the literature to avoid the path-dependent integration. In this paper, the approximation proposed in [1] is used which can be expressed as follows.

$$I_{ij} = D_{ij} \frac{\tilde{\delta}_i - \tilde{\delta}_i^s + \tilde{\delta}_j - \tilde{\delta}_j^s}{\tilde{\delta}_i - \tilde{\delta}_i^s - \tilde{\delta}_j + \tilde{\delta}_j^s} [\sin \tilde{\delta}_{ij} - \sin \tilde{\delta}_{ij}^s]. \quad (9)$$

This approximation may introduce errors in the calculated EPs. However, the proposed method is not sensitive to inaccurate EPs in computing CUEPs in contrast to iterative methods since it is equally applicable to network preserving models and is not affected by the transmission model. Thus, the proposed method will accommodate this approximation while being able to accurately compute CUEPs.

The energy function of (6) is a deterministic function, which is derived for conventional generators. Due to uncertainties introduced by renewable energy sources, a stochastic energy function will be needed. Several stochastic energy functions have been developed in [37], [38]. The proposed method can be used with stochastic energy functions for a quantified uncertainty level. However, uncertainty quantification is out of the scope of this paper.

III. COMPUTATION OF CUEPs

Accurate determination of CUEPs is crucial in transient stability assessment since the values of the energy function at CUEPs are used in computing system stability margins. An equilibrium point x^* (a post-fault stable equilibrium point (post-fault SEP) or a UEP) represents the solution of the dynamic system given in (5) if $F(x^*) = 0$. A CUEP is embedded in a large set of unstable equilibrium points on the stability boundary (type-1 UEPs) which complicates the process of detecting and distinguishing it from other UEPs. Mathematically, a CUEP is a UEP whose stable manifold, $W^s(X_{co})$, intersects with the sustained-fault (fault-on) trajectory $(\delta(t), \omega(t))$ [1], [6], [15], [16]. The point of intersection is called the exit point or EP as shown in Fig. 1.

The symbols given in Fig. 1 are defined as follows. X_{co} is the CUEP; X_s is the post-fault SEP; X_s^{pre} is the pre-fault SEP; $A(X_s)$ is the region of attraction (ROA) or region of stability; $\partial A(X_s)$ is the boundary of the ROA; $R(X_{co})$ is the convergence region of the CUEP; $W^s(X_{co})$ is stable manifold of the CUEP; δ_1 and δ_2 are the power angles; EP is the exit point; and MGP is the minimum gradient point. It should be noted here that $A(X_s)$ represents the stability region

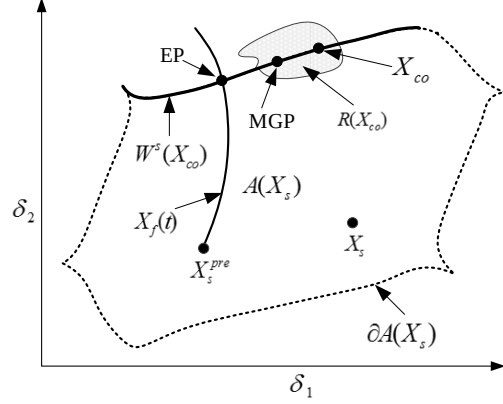


Fig. 1. A typical system stability boundary and region of convergence of CUEPs using Newton methods.

of the system with respect to X_s and $R(X_{co})$ represents the numerical convergence region in computing the CUEP.

The region of convergence of an EqP is numerically defined as a set of system state vectors wherein starting from any point inside this region, a numerical method iteratively converges to the desired solution. Regions of convergence of CUEPs are characterized by having fractal and irregular shapes where their boundaries and sizes vary with the numerical method being used [6]. These irregularities make it difficult to find initial points that are guaranteed to lie inside the regions of convergence of CUEPs for the numerical method being used; they are the main reasons of the lack of robustness of iterative methods to accurately determine CUEPs. In other words, if the initial point is not within the regions of convergence of the CUEP for the numerical method being used, the solution process either diverges or converges to an undesirable equilibrium point.

MGPs are commonly used as initial points to compute CUEPs [15]–[17], [32], [39]–[41]. An MGP is a point representing a system state vector that lies on the system stability boundary (see Fig. 1). An MGP is numerically determined by tracing the post-fault trajectory of the system dynamic model to find the first local minimum value of the norm of its vector field [6]. The search process to locate an MGP starts from a point on the stability boundary known as exit point or EP (see Fig. 1). An EP is a point representing a system state vector at which the sustained-fault trajectory crosses (i.e., exits) the stability boundary of the post-fault SEP. Numerically, it is determined by searching for the first maximum value of the potential energy of the post-fault network configuration on the path of the sustained-fault trajectory [6].

EPs are commonly determined by simulating system fault-on trajectories using time-domain simulations. Large integration step sizes are usually utilized until an EP is located within a certain range. Then, some numerical devices, such as quadratic interpolation and golden section, are used to accurately determine the EP.

From the above discussion it is clear that: (a) the successful determination of CUEPs depends upon the accuracy of the

calculated MGPs [15], [17], [32], [41], [42]; (b) the accuracy of the MGPs depends on the precise determination of EPs [6]; and (c) precise determination of EPs is computationally involved. Therefore, small errors in the values of EPs have the potential to undermine the process of computing CUEPs. Indeed, precise determination of EPs does not guarantee accurate computation of MGPs and accurate computation of MGPs does not guarantee successful determination of CUEPs.

IV. THE PROPOSED HOMOTOPY-BASED METHOD

We propose a homotopy-based approach to compute CUEPs. Homotopy methods are continuation methods that are recognized by being globally convergent and insensitive to starting points [18]–[20]; thus, they overcome the difficulties related to precise determination of initial points within regions of convergence of CUEPs. The proposed method uses EPs as initial points thus avoiding the need to compute MGPs as is required in applying iterative methods. Also, since homotopy-based methods are insensitive to starting points, precise determination of EPs is not necessary. However, applying homotopy methods in computing CUEPs faces two challenges as follows: (i) their convergence to CUEPs depends on the direction of the mapping process which in turn depends on the position of the initial point as explained in section IV-B (Fig. 4 and Fig. 5), and (ii) they are slow in performance. Changes in potential energy values along angle trajectories between exit points and angle values after one homotopy iteration are used to overcome challenge (i) and an adaptable step size is used to overcome challenge (ii). This section explains the proposed homotopy-based method and briefly presents the well-known BCU approach for the sake of comparison.

A. The BCU Method

The BCU method is a systematic approach that uses artificial reduced-state models to compute CUEPs of the original system [3], [6]. It has been found that computing CUEPs from the reduced-state model requires less computational burden than computing them from the original model [6]. Each power system dynamical model has an equivalent reduced-state model that captures the dynamic as well as static properties of the original model. For instance, consider the following system [6]:

$$T\dot{x} = -\frac{\partial U}{\partial x}(x, y) + g_1(x, y), \quad (10)$$

$$\dot{y} = z, \quad (11)$$

$$M\dot{z} = -D_m z - \frac{\partial U}{\partial y}(x, y) + g_2(x, y), \quad (12)$$

where T is a positive definite matrix, x , y and z represent state variables of the system, M and D_m are diagonal positive definite matrices (they can represent moments of inertia and damping coefficients, respectively), and $g_1(x, y)$ and $g_2(x, y)$ are the network transfer conductances.

The reduced-state model of the system given in (10), (11), and (12) can be expressed as follows [6].

$$T\dot{x} = -\frac{\partial U}{\partial x}(x, y) + g_1(x, y), \quad (13)$$

$$\dot{y} = -\frac{\partial U}{\partial y}(x, y) + g_2(x, y). \quad (14)$$

The step-by-step procedure of computing CUEPs using the BCU method can be described as follows [16].

- 1) Simulate the sustained-fault trajectory (fault-on trajectory) using time-domain methods;
- 2) Determine the EP using the procedure given in section III;
- 3) Simulate the trajectory of post-fault configuration of the reduced-state model and use the EP as an initial point and to find the MGP;
- 4) Compute the CUEP using the MGP as an initial point;
- 5) Determine the CUEP of the original system.

In applying this procedure, many numerical problems may arise that adversely affect the computation processes [6]. For instance, inaccurate EPs will make the solution procedure converge to a MGP that may not lie inside the region of convergence of the CUEP. If that is the case, the solution procedure may diverge or converge to a UEP that is not the CUEP of interest. Fig. 2 shows two cases wherein the EPs are not on the stability boundary—EP1 is outside the ROA and EP2 is inside the ROA. Each EP converges to a MGP (EP1 converges to MGP1 and EP2 converges to MGP2) where both do not converge to the CUEP.

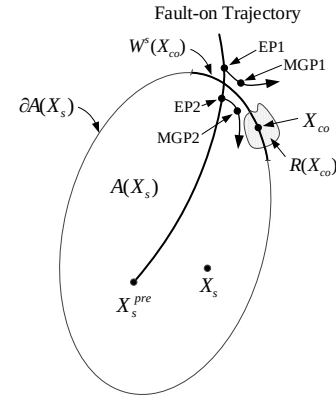


Fig. 2. Effects of inaccurate EPs on the solution procedure.

From the foregoing discussions, we can conclude that accurately determining EPs is crucial for successful computation of CUEPs. A stability-boundary-following procedure has been proposed in [6] to detect inaccuracies in determining EPs and to correct their rays.

The stability-boundary-following approach can be outlined as follows: simulate the trajectory of the post-fault configuration model starting from the determined EP for a few iterations and examine whether an MGP has been encountered. If it has been encountered, examine whether the CUEP can be determined. If the MGP has not been encountered or the CUEP cannot be determined, draw a ray connecting the post-fault SEP and the most recent point on the sustained-fault trajectory. Then, search for a new MGP on this ray. The process continues

until the CUEP is determined. For the case wherein the EP lies inside the ROA, the stability-boundary-following procedure is shown in Fig. 3.

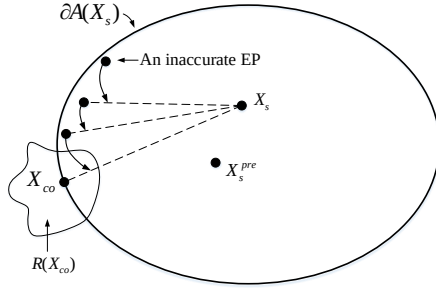


Fig. 3. Adjustments of the EP rays using the stability-boundary-following approach.

B. Homotopy-based Approach

There are several traditional iterative methods to solve (5) including Newton methods. However, Newton methods converge to the desired solution only if the initial point lies within its region of convergence. Homotopy-based approaches have been proposed to solve local convergence problems of the iterative methods because they are globally convergent [18]–[20]. In other words, homotopy methods converge to a solution disregarding starting points being used provided that there is no turning point, bifurcation point, or singularities in the solution trajectory. If there are multiple EqPs, homotopy has the ability to find all of them. However, starting from points in the vicinity of the solution of interest can help to find it first. Therefore, with the EP as the initial starting point, homotopy-based methods can determine CUEPs first because they are close to the EPs in terms of the energy value.

Homotopy methods are numerical methods to compute the equilibrium points of non-linear system of equations (e.g., $F(x^*) = 0$). The homotopy-based methods trace solution trajectories through an incremental mapping factor to arrive to the desired solution. This method uses path continuation to find the solutions as given in (15)—the solution trajectory starts at a known solution x^0 that satisfies $\mathcal{G}(x^0) = 0$. Equation (15) shows the most widely used homotopy function.

$$\mathcal{H}(x, t) = tF(x) + (1 - t)\mathcal{G}(x) = 0, \quad (15)$$

where t is the mapping factor and it is gradually increased using an adaptable step size to find the solution (i.e., $\mathcal{H}(x, 0) = \mathcal{G}(x)$ and $\mathcal{H}(x, 1) = F(x)$).

Conceptually, the function $\mathcal{G}(x)$ can be arbitrarily chosen as long as it has a known solution. However, its choice affects the convergence speed of homotopy-based methods [18]. Newton homotopy function is the most commonly used homotopy function. It uses the original system for the known solution as given in (16). The advantages of the Newton homotopy function are as follows: (i) it has large convergence region [39], [43]; and (ii) same system model can be used to start

the continuation processes instead of determining an arbitrary function with a known solution.

$$\mathcal{G}(x) = F(x) - F(x^0), \quad (16)$$

where x^0 is the starting point. Substituting for $\mathcal{G}(x)$ in (15), the homotopy function will become function of $F(x)$ as follows.

$$\mathcal{H}(x, t) = F(x) - (1 - t)F(x^0) = 0. \quad (17)$$

We used the Newton homotopy to compute the CUEPs. In our case, the system dynamical model of (1) and (2) represents the $F(x)$ function; and x^0 is the EP. Mathematically, the homotopy function for the system of dynamic equations of (1) and (2) is given in (18).

$$\begin{aligned} \mathcal{H}(x, t) = & \frac{1}{M_i} (P_{mi} - P_{ei}(x)) - \frac{1}{M_T} P_{COI}(x) \\ & - (1 - t) \left[\frac{1}{M_i} (P_{mi} - P_{ei}(x^0)) \right. \\ & \left. - \frac{1}{M_T} P_{COI}(x^0) \right] = 0. \end{aligned} \quad (18)$$

The direction of the homotopy search for the solution (or mapping) depends upon the numerical sign of t —its sign is positive for forward mapping and negative for backward mapping. In case of backward mapping, the homotopy function changes to:

$$\mathcal{H}(x, t) = tF(x) + (1 + t)\mathcal{G}(x) = 0. \quad (19)$$

1) *Mapping Direction*: It should be noted here that the direction of the mapping is a key factor in finding the CUEPs. The direction of mapping processes depends upon the location of the EP with respect to the CUEP on the solution path (i.e., either the EP is located to the right or to the left of the CUEP on the stability boundary as shown in Fig. 1—in Fig. 1, the CUEP is located to the right of the EP but it can be to the left of it). Since the direction is not known in advance, the solution path will converge to a solution (equilibrium point) but it may not be the CUEP. Fig. 4 shows a scenario in which the CUEP is in the path of the forward mapping from the EP. For this scenario, the backward mapping would converge to an EqP but not the CUEP. Fig. 5 shows a scenario in which the CUEP is in the path of backward direction. For this scenario, the forward mapping would converge to an EqP but not the CUEP.

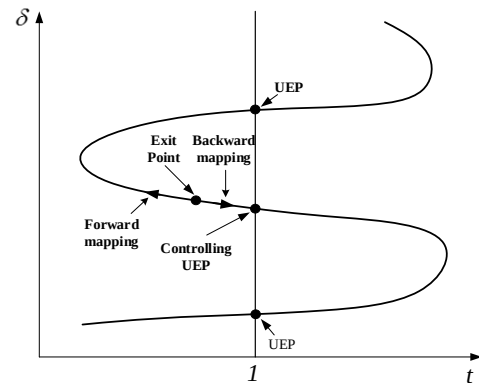


Fig. 5. The forward mapping process will not converge to the CUEP.

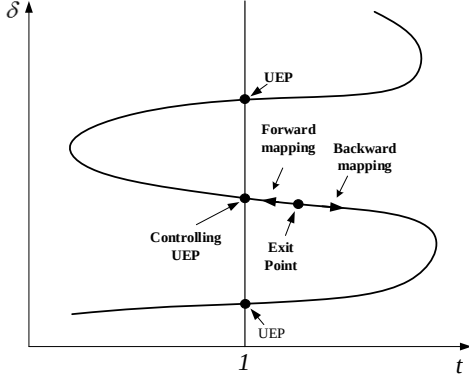


Fig. 4. The backward mapping process will not converge to the CUEP.

During the computation processes, the algorithm starts by assuming forward mapping and proceeds to perform one homotopy iteration (i.e., complete solution of Newton method) starting from an EP. The algorithm traces the change in potential energy values along the angle trajectory between the EP and angle values after one homotopy iteration—the second and third terms of (6) are used to determine the potential energy function values. If the potential energy value is increasing, the point is deemed far from the EP and the algorithm switches to backward mapping.

2) *Adaptable Step Size*: An adaptable step size is utilized to increase the computation speed of the homotopy method. The step size is adjusted based on the homotopy function norm at each iteration. Several adaptable step size strategies can be used. In this paper, we use the following device to adjust the step size. If the function norm is small enough (e.g., smaller than a threshold), the step size is increased; and the step size is decreased if the function norm is larger than the threshold. Several threshold values can be used to decide whether to increase or decrease the step size. At each iteration, the step size is calculated as follows,

$$t_{k+1} = t_k \frac{\epsilon}{\mathcal{H}_n}. \quad (20)$$

where ϵ is the specified threshold value and $\mathcal{H}_n$ is the homotopy function norm. A threshold value of 0.002 is used for the tested systems. For example, if the function norm is 0.01, the step size is multiplied by 0.2 (i.e., the step size is decreased by a factor of 5). Also, if the function norm is 0.0004, the step size is multiplied by 5 (i.e., the step size is increased by a factor of 5).

The number of homotopy iterations depends strongly upon the proximity of CUEPs to EPs. For the studied systems, starting from the EPs, the number of homotopy iterations to find the CUEPs has been found between one to three iterations. This can be explained as follows. If the solution algorithm starts from the MGP, it requires only one homotopy iteration (i.e., same as the BCU method). On the other hand, if the solution algorithm starts from the EP, it may take more than one homotopy iteration. However, since determining MGPs and accurate EPs are computationally complicated and expensive, homotopy methods offer advantages over the

conventional iterative methods; not to mention being globally convergent.

3) *Procedure*: The computation procedure of CUEPs using the proposed method is shown in Fig. 6.

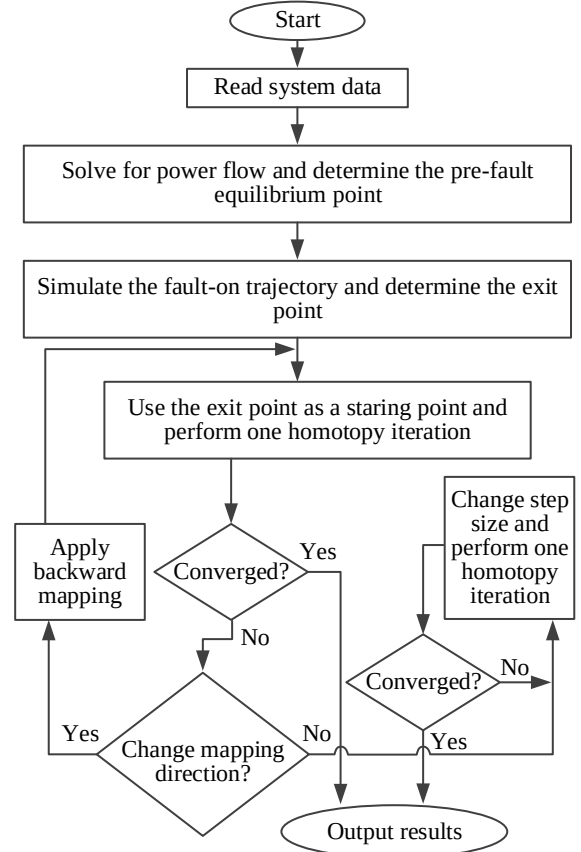


Fig. 6. Procedure Flowchart for Computing CUEPs.

V. NUMERICAL EXAMPLES

The proposed homotopy-based method was demonstrated on two systems: reduced WECC (Western Electricity Coordinating Council) system and NE (New England) 39 bus system. EPs were used as initial points to compute CUEPs. The proposed method succeeded in computing all CUEPs using the EPs as initial points. This section discusses the results for the test systems and examines the sensitivity of the proposed method to initial points.

A. Computation of CUEPs

This section discusses the computation of EPs and CUEPs for the reduced WECC and the NE 39 bus test systems.

1) *The Reduced WECC System*: Dynamic data of the reduced WECC system are given in [6]. We have chosen this system because (i) it has been widely used for transient stability analyses; thus, the results of the proposed method can be compared with existing results; and (ii) it is a small system so that it allows comprehensive analysis of the performance of the proposed method against initial points.

The analysis starts with calculating the EPs. An EP is determined by searching for the first maximum value of the potential energy of the post-fault network configuration on the path of the sustained-fault trajectory. The golden section approach is used to accurately determine an EP after locating it within a certain range using a large time step.

Since this system consists of six transmission lines, twelve contingencies are considered; assuming faults occur near each end of each line. The EPs that correspond to the list of contingencies are given in Table I.

TABLE I
A LIST OF POSSIBLE CONTINGENCIES FOR THE REDUCED WECC AND THEIR CORRESPONDING EPs

Faulted bus	Tripped line	EPs, (rad)		
		$\tilde{\delta}_1$	$\tilde{\delta}_2$	$\tilde{\delta}_3$
5	4–5	−0.85604	2.24468	1.95048
4	4–5	−0.83157	2.05223	2.16748
6	4–6	−0.82663	2.00656	2.22575
4	4–6	−0.82573	2.00656	2.22575
7	5–7	−0.84828	2.68764	0.94769
5	5–7	−0.77351	2.07331	1.66662
9	6–9	−0.50293	0.51129	2.86278
6	6–9	−0.75797	1.83370	2.05405
8	7–8	−0.77159	1.74633	2.34681
7	7–8	−0.73581	2.28572	0.91888
9	8–9	−0.45872	0.48828	2.56452
8	8–9	−0.76930	1.74136	2.33940

The EPs of Table I are used as starting points to compute CUEPs. The solution trajectories of the first three contingencies are shown in Fig. 7, Fig. 8 and Fig. 9 respectively. To trace solution trajectories and to show them on three-dimensional graphs, generator #1 is chosen as a reference for plotting purposes only (the mapping factor t is plotted along the x axis, and the y and z axes represent the angle difference of generator 2 and generator 3 respectively with respect to generator 1). This is done to reduce the number of axes and facilitate visualization. In actual implementation, we preserve all machine angles and use the COI as reference. These trajectories show how the proposed method generates a sequence of steps to reach the CUEPs from the EPs as the homotopy mapping factor, t , changes. Whereas an adaptable step size for the homotopy factor was utilized in the original calculations, in these plots we used a fixed step homotopy factor so that the solution trajectory can be clearly shown. It can be seen from Fig. 7 and Fig. 8 that the CUEPs are far from the EPs in comparison with Fig. 9. Thus, the number of iterations for the contingencies of Fig. 7 and Fig. 8 are more than those for the contingency of Fig. 9.

The values of ω_i along the solution trajectories shown in Fig. 7, Fig. 8 and Fig. 9 are always zeros. Along these trajectories, deviation of angular frequency (ω) is zero because, at each mapping step of the homotopy, we solve the system of equations for δ which represents an equilibrium point for the $\mathcal{H}(x, t) = tF(x) + (1 - t)G(x) = 0$. For example, at a CUEP, ω is zero whereas $\delta = \text{CUEP}$ because the CUEP represents an equilibrium point where $\omega = d\delta/dt = 0$.

The solution algorithm uses the EPs given in Table I as initial points to find the CUEPs. The calculated CUEPs of the

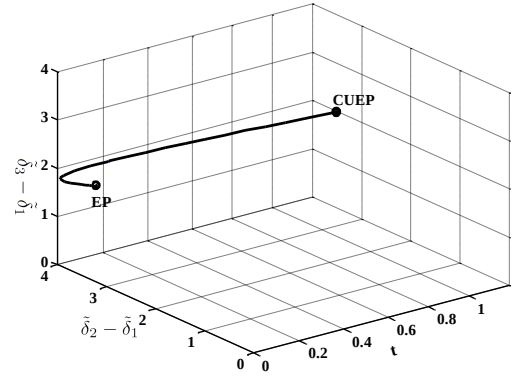


Fig. 7. Solution mapping trajectory for the first contingency in Table I.

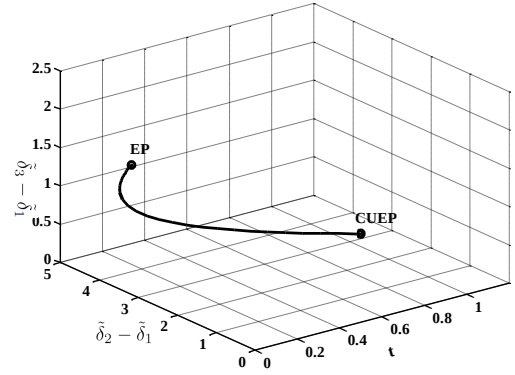


Fig. 8. Solution mapping trajectory for the second contingency in Table I.

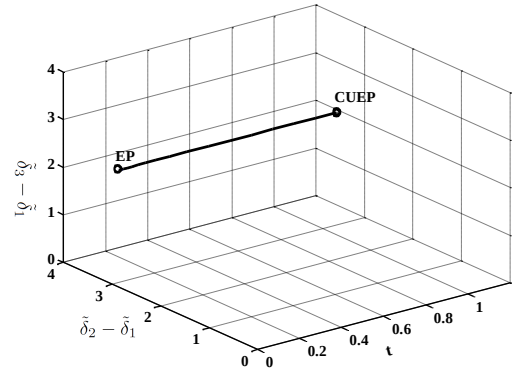


Fig. 9. Solution mapping trajectory for the third contingency in Table I.

list of contingencies are given in Table II. We validate our results by comparing them with previously published results: in Table III, we provide CUEPs that were calculated using the BCU method and reported in [6].

2) *New England Test System*: The New England 39 bus system consists of ten generators, thirty-nine buses, eleven transformers and thirty-five transmission lines [44]. The proposed homotopy-based method succeeded in determining the CUEPs of all possible $N - 1$ contingencies. However, owing to the space constraint, we provide only the CUEPs of fifteen contingencies. The computed CUEPs are provided in Table IV where the angles are referred to the COI reference frame.

TABLE II
CUEPs OF THE WECC SYSTEM USING THE PROPOSED METHOD

Faulted bus	Tripped line	CUEPs, (rad)		
		$\tilde{\delta}_1$	$\tilde{\delta}_2$	$\tilde{\delta}_3$
5	4–5	−0.83237	2.07428	2.14475
4	4–5	−0.83237	2.07428	2.14475
6	4–6	−0.82661	2.08214	2.05402
4	4–6	−0.82661	2.08214	2.05402
7	5–7	−0.75983	1.95215	1.80718
5	5–7	−0.75983	1.95215	1.80718
9	6–9	−0.75862	1.85769	1.99796
6	6–9	−0.75862	1.85769	1.99796
8	7–8	−0.35001	0.07388	2.58618
7	7–8	−0.54302	2.17976	−0.37641
9	8–9	−0.29150	−0.10171	2.50047
8	8–9	−0.29150	−0.10171	2.50047

TABLE III
CUEPs OF THE WECC SYSTEM USING THE BCU METHOD [6].

Faulted bus	Tripped line	CUEPs, (rad)		
		$\tilde{\delta}_1$	$\tilde{\delta}_2$	$\tilde{\delta}_3$
5	4–5	−0.83640	2.07970	2.14660
4	4–5	−0.83640	2.07970	2.14660
6	4–6	−0.82560	2.08300	2.05490
4	4–6	−0.82560	2.08300	2.05490
7	5–7	−0.75890	1.95280	1.80790
5	5–7	−0.75890	1.95280	1.80790
9	6–9	−0.75760	1.85830	1.99860
6	6–9	−0.75760	1.85830	1.99860
8	7–8	−0.34950	0.07450	2.58640
7	7–8	−0.54240	2.18020	−0.37550
9	8–9	−0.29100	−0.10110	2.50080
8	8–9	−0.29100	−0.10110	2.50080

B. Impacts of starting points

To assess the robustness of the proposed method against the accuracy of starting points, we performed two case studies as follows: (i) disturbing the EPs, and (ii) determining the number of homotopy steps for the cases of using EPs and MGPs as initial points.

1) *Disturbing the EPs*: It is much easier to determine less accurate EPs than determining exact EPs. Yet less accurate EPs are expected to introduce convergence problems when applying iterative methods. In this case study, we have disturbed the EPs that are given in Table I, by either selecting a point prior to or after detecting the exit point along the fault-on trajectory as shown in Fig 3 (e.g., EP2 and EP1 respectively), to make them inaccurate. A time-step of 0.01 second is used to select the disturbed EPs along the fault-on trajectory. Whereas the proposed method has succeeded in calculating CUEPs providing that the EPs are disturbed, the conventional iterative methods have failed in calculating them. If EPs are significantly disturbed, the algorithm would converge either to the post-fault stable equilibrium point (X_s) if EP2 is close to X_s^{pre} or to a UEP that is not on the stability boundary (i.e., not type-1 UEP) if EP1 is far (several time-steps after detecting the EP on the fault-on trajectory) from the stability boundary.

TABLE IV
CUEPs OF THE NE 39 BUS SYSTEM FOR A LIST OF POTENTIAL CONTINGENCIES.

Tripped Line From To	CUEPs (rad) ($\tilde{\delta}_1, \tilde{\delta}_2, \dots, \tilde{\delta}_{10}$)					
1*	2	−1.3013	2.7573	2.6174	2.2003	2.2820
		2.2171	2.2400	2.2316	2.3452	1.8976
1	2*	−1.3013	2.7573	2.6174	2.2003	2.2821
		2.2171	2.2400	2.2316	2.3452	1.8976
1*	39	−1.3214	2.7956	2.6579	2.2356	2.3163
		2.2522	2.2748	2.2653	2.3758	1.9345
1	39*	−1.3214	2.7956	2.6579	2.2356	2.3163
		2.2522	2.2748	2.2653	2.3758	1.9345
2*	3	−0.1708	−0.1564	−0.1575	−0.1872	−0.1021
		−0.1707	−0.1474	2.5571	1.8152	−0.2649
2	3*	−0.9547	2.2214	2.1303	1.8522	1.9678
		1.8677	1.8953	0.7979	2.7504	0.0082
2*	25	−0.9795	2.5886	2.2507	1.7183	1.8343
		1.7308	1.7590	1.8709	2.0038	0.3010
2	25*	−0.9394	1.3543	1.4139	1.7046	1.8381
		1.7146	1.7453	2.7322	3.0795	0.0744
3*	4	−0.9903	2.0047	1.9228	1.6103	1.7240
		1.6247	1.6525	1.4133	3.1593	0.7425
3	4*	−0.0259	2.3553	0.1709	−0.3027	−0.2265
		−0.2867	−0.2647	−0.2244	−0.1602	−0.5248
3*	18	−0.0092	2.3121	0.0146	−0.3639	−0.2894
		−0.3467	−0.3248	−0.1515	−0.1371	−0.4528
3	18*	−0.0092	2.3121	0.0146	−0.3639	−0.2894
		−0.3467	−0.3248	−0.1515	−0.1371	−0.4528
4*	5	0.0104	2.3336	0.0366	−0.3865	−0.3129
		−0.3705	−0.3490	−0.2470	−0.1928	−0.5536
4	5*	0.0104	2.3336	0.0366	−0.3865	−0.3129
		−0.3705	−0.3490	−0.2470	−0.1928	−0.5536
4*	14	−0.0288	2.3148	0.0147	−0.2831	−0.2086
		−0.2665	−0.2448	−0.1601	−0.0940	−0.4726

*: Fault near this bus

2) Comparison Between EPs and MGPs as Initial Points:

Although the proposed method is intended to avoid the necessity of calculating accurate EPs and MGPs in determining the CUEPs, this section provides a comparison between using EPs and MGPs as initial points. For each type of initial point, we recorded the number of homotopy iterations to arrive to the desired solution. Table V shows the number of steps that are required to calculate the CUEPs for the reduced WECC test system using the EPs and MGPs as initial points. Each homotopy step is equivalent to a full solution of Newton-Raphson method (a conventional iterative method). Although computing the CUEPs using the MGPs as initial points requires less number of iterations than using EPs, the computation burden of determining a MGP inside the region of convergence of a CUEP and the possibility of failure of locating a MGP makes the use of EPs as initial points more attractive.

From the above results it is clear that the proposed method requires less computation efforts than the BCU method since it avoids the necessity of computing MGPs and does not require accurate EPs. As mentioned in section III, not only determin-

ing accurate EPs and MGPs is computationally involved but also inaccurate determination of EPs and MGPs leads to failure of the BCU method in computing CUEPs.

TABLE V
TESTING OF THE PROPOSED METHOD IN CASE OF USING EPs AND MGPs AS INITIAL POINTS.

No.	Fault at Bus	Tripped Line		Number of homotopy steps using:	
		From	To	EPs	MGPs
1	5	4	5	2	1
2	4	4	5	2	1
3	6	4	6	2	1
4	4	4	6	2	1
5	7	5	7	2	1
6	5	5	7	3	1
7	9	6	9	2	1
8	6	6	9	2	1
9	8	7	8	3	1
10	7	7	8	2	1
11	9	8	9	2	1
12	8	8	9	2	1

C. Computation of Critical Clearing Times

In this section we compute fault critical clearing times (CCTs) using both our method and time-domain simulation. The CCTs for the New England 39 bus system are given in Table VI. The CCTs for the reduced WECC systems are given in [16]. From Table VI, it is clear that both methods provide similar results which confirms the robustness of the proposed method. For the contingencies that involve the removal of line 3–18, both methods produce very close results. The highest difference between the two methods occurs when any contingency involves removal of a line connected to bus 1. In all cases, the direct method produces smaller CCTs than the time-domain based method due to its conservativeness. Although this is generally the case, direct methods may overestimate CCTs due to the approximation of the energy function particularly when a large number of generators start deviating from the rest of the system [6].

TABLE VI
CRITICAL CLEARING TIME (CCT) OF THE NE 39 BUS SYSTEM.

Faulted bus	Tripped line	Critical Clearing Time (sec)	
		Time-domain	Direct Method
1	1–2	0.2351	0.2293
2	1–2	0.0841	0.0822
1	1–39	0.2963	0.2908
39	1–39	0.2548	0.2537
2	2–3	0.1984	0.1980
3	2–3	0.1853	0.1847
2	2–25	0.1246	0.1241
25	2–25	0.0989	0.0985
3	3–4	0.1948	0.1946
4	3–4	0.1881	0.1877
3	3–18	0.1865	0.1862
18	3–18	0.1785	0.1785
4	4–5	0.1976	0.1969
5	4–5	0.1871	0.1865
4	4–14	0.1948	0.1942

VI. CONCLUSION

This paper has presented a homotopy-based method to efficiently and precisely calculate the CUEPs in transient stability analysis and screening using direct methods. Convergence devices have been introduced to improve the efficiency of computation and ensure the accuracy of the CUEPs. An adaptable homotopy mapping technique was utilized to increase the computation speed, and a solution trajectory correction technique was used to ensure that the solution algorithm converges to the correct CUEP during the mapping process. Also, the proposed method is not sensitive to starting points as it is the main concern in computing CUEPs (inaccuracies in exit points or minimum gradient points or both result in iterative methods failing to compute CUEPs). In other words, the proposed method does not require determining minimum gradient points nor does it require computation of accurate exit points. The proposed method was applied on several systems including a real system and the results are compared with the BCU method. We applied the method successfully to the Southern California Edison system under the Department of Energy contract that funded this work but are unable to report results because of confidentiality requirements. Further, the robustness of the proposed method against the inaccuracy of the initial points is tested by perturbing the computed exit points. The proposed method was found more robust to accuracy of the initial points in comparison with the other methods.

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Joydeep Mitra (S'94–M'97–SM'02–F'19) received the B.Tech. (Hons.) degree in electrical engineering from the Indian Institute of Technology Kharagpur, Kharagpur, India, in 1989, and the Ph.D. degree in electrical engineering from Texas A&M University, College Station, TX, USA, in 1997.

He is currently an Associate Professor of electrical engineering with Michigan State University, East Lansing, MI, USA, the Director of the Energy Reliability and Security Laboratory, and a Senior Faculty Associate with the Institute of Public Utilities. He is co-author of the book, "Electric Power Grid Reliability Evaluation: Models and Methods," and of IEEE Standard 762. He was a recipient of the IEEE-PES Roy Billinton Power System Reliability Award. His research interests include reliability, planning, stability, and control of power systems.



Mohammed Benidris (S'10–M'14) received a Ph.D. degree in Electrical Engineering from Michigan State University, East Lansing in 2014, and MSc. and BSc. degrees in Electrical Engineering from University of Benghazi, Benghazi, Libya in 2005 and 1998 respectively.

He is an Assistant Professor of Electrical Engineering with the University of Nevada, Reno (UNR), Reno, NV, USA. Prior to joining the UNR, he was Research Associate and Visiting Lecturer at Michigan State University. Prior to this, he was an Assistant Lecturer at the department of Electrical and Electronics Engineering, University of Benghazi, Benghazi, Libya. His main research interests include power system reliability and stability and resilience of cyber-physical energy systems.