

Fast economic power dispatch method for power system planning studies

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Abstract: This study describes a method for solving the optimal economic power dispatch problem. The method proposed here uses linear programming, because linear programming-based formulations tend to be flexible, reliable and faster than their non-linear counterparts. The proposed linear programming-based method is developed based on a linearised network model, in which voltage magnitudes and reactive power flows have both been accounted for, unlike traditional linearised power flow methods. A piecewise linear model is developed to handle the thermal capacities of transmission lines. Piecewise linear models are also developed to deal with the exponential loads, cost functions of generators and total power losses. The effectiveness of the proposed method is demonstrated on numerous standard test systems, including the IEEE 300-bus system. Further, a comparison with some other methods, which are available in the literature, is held to validate the consistency of the proposed method in finding the optimal or near-optimal global solution. The results show that the proposed method could lead to optimal or near-optimal global solution, and is appropriate to use for power system planning studies.

Nomenclature

N_b	number of buses
N_t	number of transmission lines
N_g	number of generators
Ψ_k	set of buses connected to bus k
P_k, Q_k	real and reactive power injections at bus k
V_k, V_m	voltages at buses k and m
δ_{km}	voltage angle difference between bus k and bus m
$G_{km} + jB_{km}$	element (k, m) of the bus admittance matrix
b_{kk}	total susceptance of shunt elements connected at bus k
g_{kk}	total conductance of shunt elements connected at bus k
$\Delta V_k, \Delta V_m$	deviations in bus voltages at buses k and m
P_{km}, Q_{km}	real and reactive power owing from bus k to bus m
P_{mk}, Q_{mk}	real and reactive power owing from bus m to bus k
G', B'	modified conductance and susceptance matrices defined in Section 3 ($N_b \times N_b$)
δ	vector of node voltage angles ($N_b \times 1$)
V	vector of bus voltage magnitudes ($N_b \times 1$)
P_G, Q_G	vectors of real and reactive power generation ($N_b \times 1$)
P_D, Q_D	vectors of real and reactive loads ($N_b \times 1$)
$P_G^{\min}, Q_G^{\min}$	vectors of minimum available real power generation ($N_b \times 1$)
$P_G^{\max}, Q_G^{\max}$	vectors of maximum available real power generation ($N_b \times 1$)

$I_f^{\max}, I_r^{\max}$	vectors of forward and reverse flow thermal capacities of transmission lines ($N_t \times 1$)
$V^{\min}, V^{\max}$	vectors of minimum and maximum allowable voltages ($N_b \times 1$)
b	diagonal matrix of transmission lines admittances ($N_t \times N_t$)
A	node-branch incidence matrix ($N_t \times N_b$)
Z_{km}	impedance of branch that connects bus k and bus m
I_{km}^p, I_{km}^q	real and imaginary components of the line current flowing from bus k to bus m
$I_{km}^{\max}$	maximum line current owing from bus k to bus m
θ_l	angle of linear segment l
$P_k^{\text{sp}}, Q_k^{\text{sp}}$	specified real and reactive power at bus k
b_{pk}, b_{qk}	exponent order of the load model at bus k
a_{pk}, c_{pk}	constants associated with the real power load model
a_{qk}, c_{qk}	constants associated with the reactive power load model
$F_i(P_{Gi})$	cost function of generator i
P_{Gi}	output power of generator i
$\alpha_i, \beta_i, \gamma_i$	coefficients of the cost function of generator i
$P_{Gi}^{\min}$	minimum real power limit of generator i
P_L, Q_L	exact real and reactive power losses
$P_L^{\Delta}, Q_L^{\Delta}$	approximate real and reactive power losses

1 Introduction

For about half a century, the optimal power flow problem has been the subject of intensive research all around the world. It

is a crucial tool in the operation and planning stages of any power system. The optimal power flow problem was first formulated and introduced as a network constrained economic dispatch (ED) problem in 1962 by Carpentier [1], and was defined later as an optimal power flow problem by Dommel and Tinney [2]. The task of performing optimal power flow aims essentially at determining the optimal settings of the given power network by optimising certain functions, for example, minimisation of total generation cost, loss or emission, while satisfying a set of operating and technical constraints. Numerous control variables that include generator's real power, transformer's tap changers and phase shifters, static and dynamic VAR compensators are involved in optimising the objective function.

A great variety of solution techniques have been proposed in the literature to solve the optimal ED problem since its inception. Momoh *et al.* presented a review of optimal power flow methods selected in [3, 4]. The challenges to optimal power flows are reported by Momoh *et al.* [5]. Examples of some methods, which are used to solve the ED problem, include non-linear programming [6, 7], sequential quadratic programming (SQP) [8, 9], Newton's method [10], swarm intelligence such as particle swarm optimisation [11, 12], genetic algorithms [13, 14] and linear programming (LP) [15–21]. Among these methods, LP-based methods are recognised as viable and promising tools in solving the constrained ED problem. The inherent features of the LP-based economic dispatch methods include reliability of optimisation, flexibility of the solution, rapid convergence characteristics and fast execution time [22].

Owing to the liberalisation of energy markets and the introduction of non-utility generation, the power system structure has been recently changed and several real-time engineering applications have emerged. Examples of these applications include reactive power planning using FACTS devices, open access transmission, placement of distributed energy resources, and so forth. In fact, these applications need a power flow study at the first step of the solution. Nevertheless, a vast majority of these applications require repetitive and prompt power flow solutions. Performing full AC power flow (FACPF) gives high calculation precision, however, it is computationally demanding. More prominently, the largest part of the aforesaid problems are essentially non-linear complex combinatorial constrained optimisation problems. The formulation of the non-linear problem, however, tends to be a tedious task and computationally awkward in terms of execution time, storage requirements and programming. In addition, the non-linear nature that results from the interdependency between various variables may increase the complexity of the problem. Given such challenges, it has become necessary to develop linear power flow techniques not only to handle the aforementioned tasks more expeditiously but also to accommodate other developments that are coming in the immediate future. A review of the common power flow methods is presented in Section 2.

2 Review of power flow methods

Power flow analysis has long been performed using FACPF. The solution of the classical power flow problem aims at determining the steady-state voltage phase and magnitude at all buses as well as real and reactive power flows in each line, for specific loading conditions. The fundamental methods that are widely used in solving power flows at transmission level are the Gauss–Seidel, Newton–Raphson

and fast-decoupled method [23–27]. The Gauss–Seidel method is known as a slow-iterative problem-solving technique, because it constantly requires the solution of a set of non-linear equations whose cardinality approximately equals the number of system buses. The Newton–Raphson method is a fast and robust tangential approximation technique, in which the line parameters and the other variables are stored in the Jacobian matrix. The fast-decoupled method [25–27] uses an approximate and constant Jacobian that ignores the dependencies between (a) real power and voltage magnitude and (b) reactive power and voltage angle. It is, therefore, a fast and effective approach in solving power flows at transmission level. However, the convergence of the fast-decoupled method can be an issue if line resistance becomes much higher than its inductive reactance.

A considerable amount of research has been presented in the literature to improve the performance of the fast-decoupled method. In this context, the compensation-based method proposed by Rajicic and Bose [28] is probably the most popular technique, which is used to enhance the performance of the fast-decoupled method; in particular, for systems with high R/X branch ratios. The parallel compensation of the decoupled power flow method has been introduced in [29]. Ejebe *et al.* [30] proposed a fast contingency screening method for voltage security analysis based on fast-decoupled method. The contingency analysis is performed in two stages, which are the screening stage and the solution stage. The screening stage uses a single iteration of the fast-decoupled method. It has been demonstrated in [30] that most of the computational time is utilised for the Q–V half iteration. Consequently, a screening method is developed in [30] in order to reduce the computational time. This screening method was accomplished using sparse vector techniques and was able to solve the Q–V half iteration only.

The DC power flow (DCPF) model [31–35] was devised more than 35 years ago and is still often being used in numerous engineering applications. As a matter of fact, the DCPF model is a linearised version of the FACPF model, but it ignores some aspects of the FACPF model. The DCPF model assumes lossless lines and flat voltage profile at all buses. It is, therefore, non-iterative, linear and absolutely convergent, but with less accuracy than the FACPF model. The DCPF model is widely used in power system operational and planning studies, however, it has some drawbacks. For instance, bus voltages are mainly dependent on line parameters as well as operating conditions; and therefore flat voltage profiles are not always guaranteed. More significantly, voltage limits and reactive power flows are considered as vital constraints in many operational and planning studies and thus cannot be neglected. It is worth mentioning here that the assumptions of the DCPF model are primarily devoted to high-voltage transmission networks, in which the influence of line resistance is insignificant. However, the DCPF model is constantly incorporated with the LP technique and used to solve several engineering applications, in which fast and multiple solutions are required.

This paper proposes an optimisation framework based on LP, to solve the optimal economic power dispatch problem. The main features of the proposed optimisation framework include the following:

- (1) The proposed framework is developed based on a linearised AC power flow model (LACPF), in which the

coupling between active power and voltage magnitude as well as the coupling between reactive power and voltage angle is maintained.

(2) Voltage magnitudes, reactive power flows and all shunt elements have been taken into consideration in the proposed framework.

(3) The current limits constraints are approximated using piecewise linear model. Therefore, the degree of approximation can be improved to the desired level by increasing the number of line segments used, without substantial effect on the main routine and the computational speed.

(4) The optimisation framework presented in this paper is generic in the sense that it can be used for solving several power system operational and planning studies in which repetitive solutions are required.

The rest of this work is organised as follows: In Section 3, the linearised network model is introduced. In Section 4, the validation of the proposed linearised network model is demonstrated. In Section 5, the optimal ED problem is formulated. Case studies and discussion are provided in Section 6. A performance comparison of the proposed method is provided in Section 7. Conclusions are drawn in Section 8.

3 Network model formulation

This section presents the proposed LACPF model. This LACPF model is developed by the authors in [36] and is briefly described here for the sake of completeness. Let us start by writing down the well-known real and reactive power injections at bus k [23]. That is

$$P_k = V_k \sum_{m \in \Psi_k} V_m (G_{km} \cos \delta_{km} + B_{km} \sin \delta_{km}) \quad (1)$$

$$Q_k = V_k \sum_{m \in \Psi_k} V_m (G_{km} \sin \delta_{km} - B_{km} \cos \delta_{km}) \quad (2)$$

In order to approximate the power injection at bus k , let us approximate the magnitude of the voltage at bus k as 1.0 p.u. It is important to note that this is only an approximation that enables linearisation; it is not an assumption that the voltage magnitude equals 1.0 p.u. Therefore, (1) and (2) can now be expressed as follows

$$V_k = 1.0 \pm \Delta V_k \quad (3)$$

$$V_m = 1.0 \pm \Delta V_m \quad (4)$$

$$P_k \simeq \sum_{m \in \Psi_k} V_m (G_{km} \cos \delta_{km} + B_{km} \sin \delta_{km}) \quad (5)$$

$$Q_k \simeq \sum_{m \in \Psi_k} V_m (G_{km} \sin \delta_{km} - B_{km} \cos \delta_{km}) \quad (6)$$

Let us assume that the phase angle difference between bus k

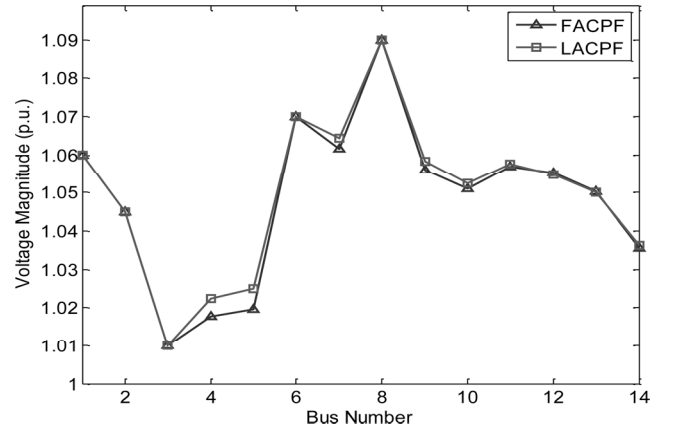


Fig. 1 Voltage profile obtained by the proposed LACPF model and the FACPF model for the IEEE 14-bus system

and bus m is small and apply the cosine and sine rules. That is

$$\cos \delta_{km} = 1 - \frac{\delta_{km}^2}{2} + \dots \quad (7)$$

$$\sin \delta_{km} = \delta_{km} - \frac{\delta_{km}^3}{6} + \dots \quad (8)$$

Therefore, (5) and (6) can be further approximated as

$$P_k \simeq \sum_{m \in \Psi_k} (V_m G_{km} + V_m B_{km} \delta_{km}) \quad (9)$$

$$Q_k \simeq \sum_{m \in \Psi_k} (V_m G_{km} \delta_{km} - V_m B_{km}) \quad (10)$$

With further manipulation, the LACPF model in the compact form is expressed by (11). The reader is advised to refer to [36] for the full development of the LACPF model.

$$\begin{bmatrix} P_K \\ Q_K \end{bmatrix} = \begin{bmatrix} -B' & G \\ -G' & -B \end{bmatrix} \begin{bmatrix} \delta_K \\ V_K \end{bmatrix} \quad (11)$$

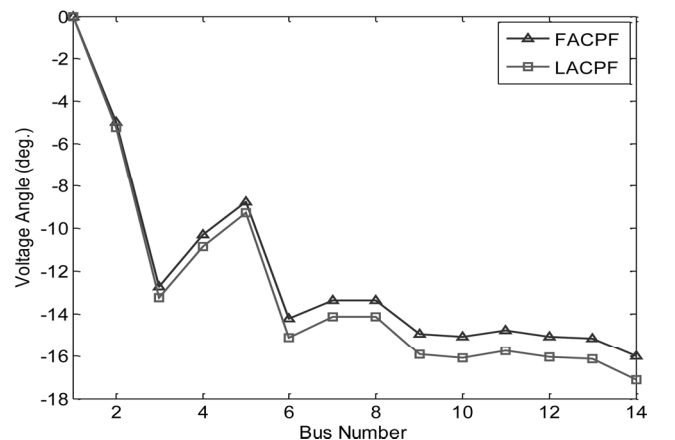


Fig. 2 Angle profile obtained by the proposed LACPF model and the FACPF model for the IEEE 14-bus system

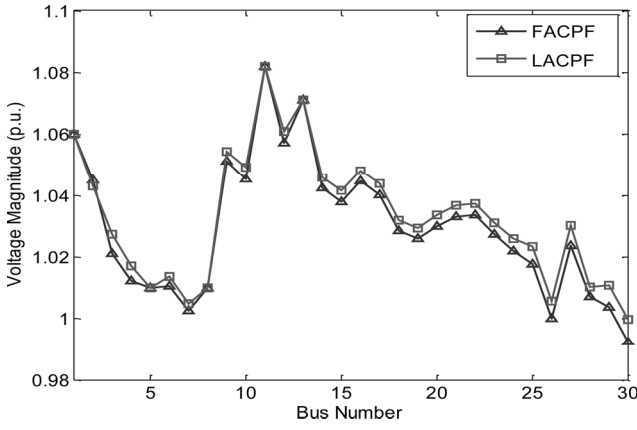


Fig. 3 Voltage profile obtained by the proposed LACPf model and the FACPf model for the IEEE 30-bus system

The diagonal elements of B' and G' can be expressed as

$$B'_{kk} = B_{kk} - b_{kk} \quad (12)$$

$$G'_{kk} = G_{kk} - g_{kk} \quad (13)$$

4 Validation of the proposed network model

To demonstrate the effectiveness of the proposed LACPf model, it has been tested on several IEEE systems. We first apply the proposed LACPf model on the IEEE 14-bus system [37], which represents a portion of the American Electric Power System. The voltage profile and the angle profile of the IEEE 14-bus system, which are obtained by the proposed LACPf model, are shown in Figs. 1 and 2, respectively. For comparison purposes, the voltage profile and the angle profile of the IEEE 14-bus system, which are obtained by the FACPf model, are also shown in Figs. 1 and 2, respectively. We also test the proposed LACPf model on the IEEE 30-bus system and the IEEE 118-bus system [37]. The power flow results in terms of voltage profile and angle profile of the IEEE 30-bus system and the IEEE 118-bus system are shown in Figs. 3–6, respectively. As can be seen from Figs. 1–6, for all test cases, the power flow results obtained by the proposed LACPf model correspond closely with those obtained by the FACPf model.

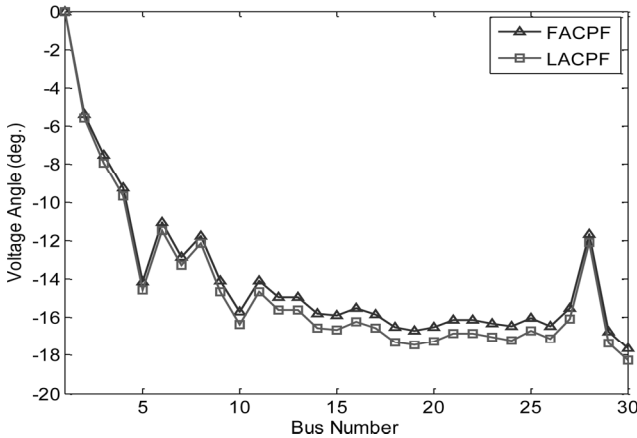


Fig. 4 Angle profile obtained by the proposed LACPf model and the FACPf model for the IEEE 30-bus system

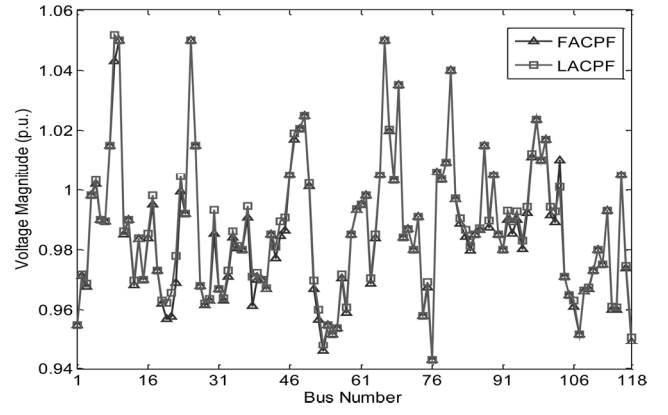


Fig. 5 Voltage profile obtained by the proposed LACPf model and the FACPf model for the IEEE 118-bus system

A comparison has been held between the results obtained by the proposed LACPf model and those obtained by the traditional DCPf model. Owing to space constraints, we provided the largest error obtained by each model. Table 1 shows the results obtained by both models. As can be seen from Table 1, compared to the DCPf model, the proposed LACPf model provides less voltage error for all test systems.

5 Formulation of the ED problem

5.1 Power balance equations

Power balance equations are equality constraints that represent the sum of power at a bus. The power balance equations are posed as

$$B'\delta - GV + P_G = P_D \quad (14)$$

$$G'\delta + BV + Q_G = Q_D \quad (15)$$

5.2 Real and reactive power constraints

Real power limits of generators can be bounded by the maximum and the minimum operating values. The reactive power limits, on the other hand, can be constrained by two ways, which are constant maximum and minimum reactive power or using the P - Q power capability curve. If the P - Q power capability curve is used, the maximum reactive

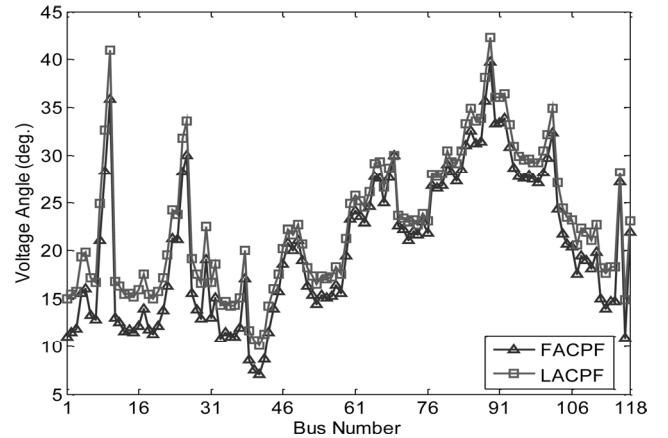


Fig. 6 Angle profile obtained by the proposed LACPf model and the FACPf model for the IEEE 118-bus system

Table 1 Comparison of the largest error obtained by the DCPF model and the proposed LACPF model

System/model	DCPF model		LACPF model	
	Voltage error, %	Corresponding bus number	Voltage error, %	Corresponding bus number
14 bus system	8.26	8	0.53	5
30 bus system	7.58	11	0.76	30
118 bus system	6.04	76	1.01	38

power produced or absorbed by a generator can be determined from the real power output according to the P - Q power capability curve. For the sake of simplicity, this paper bounds the real and reactive power of each generator by its upper and lower power limits. That is

$$P_G^{\min} \leq P_G \leq P_G^{\max} \quad (16)$$

$$Q_G^{\min} \leq Q_G \leq Q_G^{\max} \quad (17)$$

5.3 Voltage constraints

This paper bounds the upper and lower voltage limits to $\pm 5\%$, which means that the upper voltage bound is 1.05 p.u. and the lower voltage bound is 0.95 p.u. Consequently, the voltage limits are expressed as

$$V^{\min} \leq V \leq V^{\max} \quad (18)$$

5.4 Thermal constraints of transmission lines

The formulation of the current constraints is derived in this section. For a transmission line connected between bus k and bus m , and with an impedance of Z_{km} , the current flowing out from bus k towards bus m can be calculated as

$$\hat{I}_{km} = \frac{\hat{V}_k - \hat{V}_m}{\hat{Z}_{km}} \quad (19)$$

Let us make the approximation that voltage angle difference is small and line resistance is less than its reactance. This assumption will not prevent line resistance from being included in line constraints, rather it would help in making linearisation. Thus, (19) can be written as

$$|I_{km}| \simeq \sqrt{(\delta_k - \delta_m)^2 + (V_k - V_m)^2} / |Z_{km}| \quad (20)$$

Alternatively, (20) can be represented as

$$|I_{km}| \simeq \sqrt{(I_{km}^p)^2 + (I_{km}^q)^2} \quad (21)$$

where

$$I_{km}^p = \frac{(\delta_k - \delta_m)}{|Z_{km}|} \quad (22)$$

$$I_{km}^q = \frac{(V_k - V_m)}{|Z_{km}|} \quad (23)$$

As can be seen from (22) and (23), the line current can be expressed by two linear components, however, its magnitude is still non-linear.

As can be seen from Fig. 7, the line current constraints form a circle with a radius of $I_{km}^{\max}$, which is formed by the locus of the real and imaginary components expressed by (21). With further analysis, a linear equation for each segment can be obtained as

$$I_{kml}^p \cos \theta_l + I_{kml}^q \sin \theta_l - I_{km}^{\max} = 0 \quad (24)$$

where

$$\theta_l \in \{0, 2\pi\} \quad (25)$$

It is interesting to highlight here that when θ_l is set equal to zero in (24), we obtain the same constraints as those of the traditional DCPF model. The current limit constraints in the proposed optimisation framework can thus be expressed as

$$bA'\delta + bA''V \leq I_f^{\max} \quad (26)$$

$$-bA'\delta - bA''V \leq I_r^{\max} \quad (27)$$

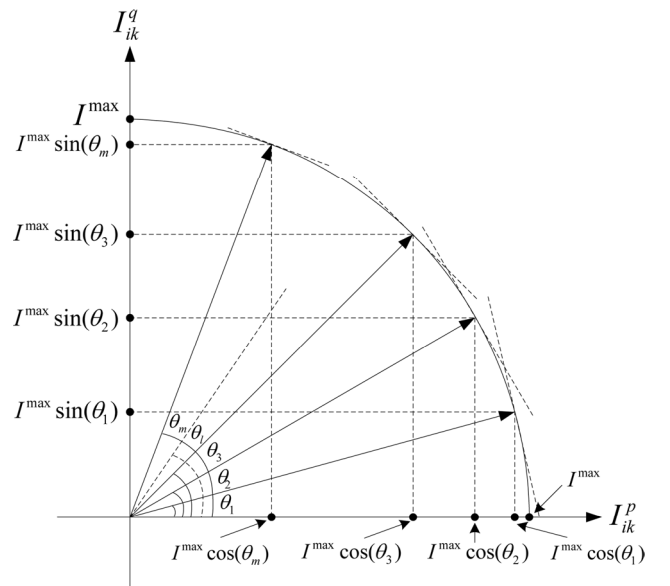
where

$$A' = A \cos \theta_l \quad (28)$$

$$A'' = A \sin \theta_l \quad (29)$$

5.5 Modelling of exponential loads

For an exponential load model, the real and reactive power at bus k can be calculated as

**Fig. 7** Linearisation of line capacity constraints

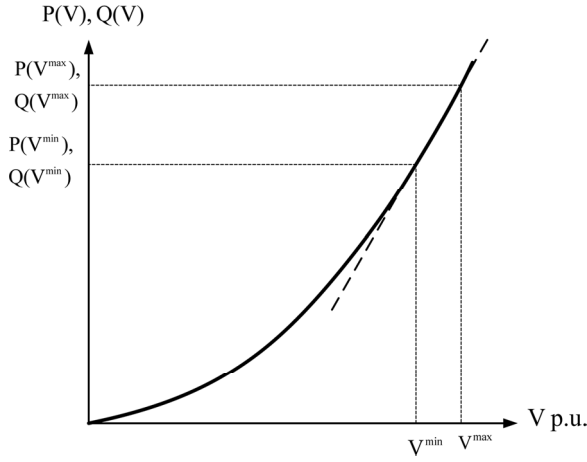


Fig. 8 Exponential load model

$$P_k = P_k^{\text{sp}} (V_k)^{b_{pk}} \quad (30)$$

$$Q_k = Q_k^{\text{sp}} (V_k)^{b_{qk}} \quad (31)$$

As the upper and lower voltage limits are, respectively, set equal to 1.05 and 0.95 p.u., the exponential load model can be linearised at very high accuracy, as can be seen from Fig. 8. Therefore, the linearised form of the exponential load model can be expressed as

$$P_k = a_{pk} V_k + c_{pk} \quad (32)$$

$$Q_k = a_{qk} V_k + c_{qk} \quad (33)$$

where

$$a_{pk} = \frac{(V^{\text{max}})^{b_{pk}} - (V^{\text{min}})^{b_{pk}}}{V^{\text{max}} - V^{\text{min}}} \quad (34)$$

$$c_{pk} = P_k^{\text{sp}} (V^{\text{max}})^{b_{pk}} - a_{pk} V^{\text{max}} \quad (35)$$

$$a_{qk} = \frac{(V^{\text{max}})^{b_{qk}} - (V^{\text{min}})^{b_{qk}}}{V^{\text{max}} - V^{\text{min}}} \quad (36)$$

$$c_{qk} = Q_k^{\text{sp}} (V^{\text{max}})^{b_{qk}} - a_{qk} V^{\text{max}} \quad (37)$$

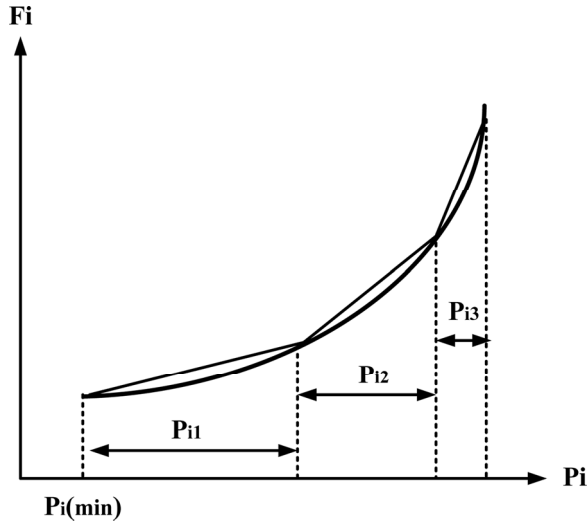


Fig. 9 Linearisation of cost function

5.6 Cost functions of generators

The cost function of a conventional generator i is usually expressed as a second-order quadratic polynomial as

$$F_i(P_{Gi}) = \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2 \quad (38)$$

This paper adapted the linearisation model developed in [31]. Thus, the non-linear cost function can be approximated by a series of straight line segments, as depicted in Fig. 9. The three linear segments shown in Fig. 9 are represented as P_{i1} , P_{i2} and P_{i3} . Further, the slope of each linear segment is given as μ_1 , μ_2 and μ_3 , respectively.

The cost function is then represented as [31]

$$F_i(P_{Gi}) = F_i(P_{Gi}^{\text{min}}) + \mu_1 P_{i1} + \mu_2 P_{i2} + \mu_3 P_{i3} \quad (39)$$

with

$$0 \leq P_{ih} \leq P_{ih}^+, \quad h = 1, 2, 3 \quad (40)$$

Therefore, the cost function is obtained by including all the linear segments in the P_{ih} . That is

$$P_{Gi} = P_{Gi}^{\text{min}} + P_{i1} + P_{i2} + P_{i3} \quad (41)$$

5.7 Inclusion of losses

The LACPF model is a lossless model, as can be seen from (11). This section presents a method to include the losses in the proposed framework. Let us consider the simple transmission circuit shown in Fig. 10. The real and reactive power flowing out from bus k to bus m can be calculated as [23]

$$P_{km} = V_k^2 g_{km} - V_k V_m g_{km} \cos \delta_{km} - V_k V_m b_{km} \cos \delta_{km} \quad (42)$$

$$Q_{km} = -V_k^2 b_{km} + V_k V_m b_{km} \cos \delta_{km} - V_k V_m g_{km} \sin \delta_{km} \quad (43)$$

The real and reactive power flowing out from bus m towards bus k can be obtained in a similar manner. That is

$$P_{mk} = V_m^2 g_{km} - V_k V_m g_{km} \cos \delta_{km} - V_k V_m b_{km} \cos \delta_{km} \quad (44)$$

$$Q_{mk} = -V_m^2 b_{km} + V_k V_m b_{km} \cos \delta_{km} - V_k V_m g_{km} \sin \delta_{km} \quad (45)$$

Therefore, the real and reactive power losses can be obtained as

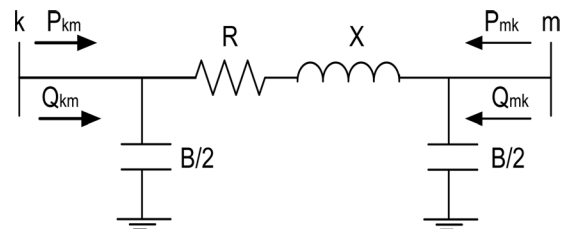


Fig. 10 Transmission line π model

$$P_L = g_{km}(V_k^2 + V_m^2 - 2V_k V_m \cos \delta_{km}) \quad (46)$$

$$Q_L = -b_{km}(V_k^2 + V_m^2 - 2V_k V_m \cos \delta_{km}) \quad (47)$$

Let us use the approximations $V_k \simeq 1.0$ p.u., $V_m \simeq 1.0$ p.u. and $\cos \delta_{km} \simeq 1 - \frac{\delta_{km}^2}{2}$, then the approximate real and reactive losses formulas can be written as

$$P_L^\Delta = g_{km} \delta_{km}^2 \quad (48)$$

$$Q_L^\Delta = -b_{km} \delta_{km}^2 \quad (49)$$

Equations (48) and (49) explicitly indicate that the active and reactive power losses are quadratic forms of δ_{km} . It should be noted here that (48) and (49) include the losses produced by the series impedances of transmission lines only. Losses produced by shunt elements of transmission lines have already been accounted for in the original formulation expressed by (11). References [38–40] describe piecewise methods to linearise transmission line losses.

5.8 Entire optimisation framework

This section presents the entire optimisation framework for solving the optimal ED problem. The objective function used here is to minimise the total generation cost.

The minimisation problem is formulated as

$$\text{Total cost} = \min \left(\sum_{i=1}^{N_g} F_i(P_{Gi}) \right) \quad (50)$$

subject to

(1) Real and reactive power injections

$$\begin{aligned} B' \delta - GV + P_G &= P_D \\ G' \delta + BV + Q_G &= Q_D \end{aligned} \quad (51)$$

(2) Real and reactive power constraints

$$\begin{aligned} P_G^{\min} &\leq P_G \leq P_G^{\max} \\ Q_G^{\min} &\leq Q_G \leq Q_G^{\max} \end{aligned} \quad (52)$$

(3) Line capacity constraints

$$\begin{aligned} bA' \delta + bA'' V &\leq I_f^{\max} \\ -bA' \delta - bA'' V &\leq I_r^{\max} \end{aligned} \quad (53)$$

(4) Transformer tap setting limits

$$T_S^{\min} \leq T_S \leq T_S^{\max} \quad (54)$$

(5) Voltage bound constraints

$$|V^{\min}| \leq |V| \leq |V^{\max}| \quad (55)$$

(6) Angle constraints

$$\delta \text{ is unrestricted} \quad (56)$$

6 Demonstration and discussion

The effectiveness of the proposed ED method is demonstrated on the IEEE reliability test system (IEEE RTS), the IEEE 30-bus system and the IEEE 300-bus system. The results are provided and discussed in the subsequent section.

6.1 IEEE RTS test case

The proposed method for optimal economic power dispatch is demonstrated on the IEEE RTS [41]. The IEEE RTS system consists of 24 buses, 35 transmission lines, 5 transformers and 32 generators, which are placed on 10 buses. The IEEE RTS has one reactor and one synchronous condenser. The total generation of this system is 3405 MW. The total real and reactive peak loads, respectively, are 2850 MW and 580 MVar. The IEEE RTS system is used here to verify the effectiveness of the proposed optimal ED method. In this context, four methods have been utilised to solve the ED problem. These methods are the FACPFBased ED, LACPFBased ED lossy model, LACPFBased ED lossless model and the DCPFBased ED. The results of the FACPFBased ED are obtained using Matpower simulation package [42].

The real power and voltage magnitudes of the generators of the IEEE RTS are presented in Table 2. As can be seen from Table 2, the results of the optimal economic power dispatch of the 10 units, which are obtained by the LACPFBased ED (lossy model), correspond closely with those obtained by the FACPFBased ED. A slight difference in the power dispatch of units 7 and 13 can be attributed, in part, to the number of linear segments involved in the linearisation of the cost curves of these units. Further, most of voltage magnitudes obtained by the proposed model are in close agreement with those obtained by the FACPFBased ED. The deviation in bus voltages obtained by the LACPFBased ED (lossy model) is always less than 5%, which is considered to be an acceptable figure from practical perspective, particularly for many off-line planning studies.

The total generation cost, total generation and total losses of the IEEE RTS are presented in Table 2. As can be seen from Table 2, the results obtained by the proposed LACPFBased ED (lossy model) correspond closely with those obtained by the FACPFBased ED. The results of the total generation cost, total generation and total losses, which are obtained by both methods were almost similar. While the results of the total generation cost, total generation and total losses obtained by the proposed LACPFBased ED (lossy model) and the FACPFBased ED were almost similar, the proposed method is considered to be more flexible for power system planning and the studies that require repetitive and multiple solutions.

6.2 IEEE 30-bus system test case

The proposed ED method has been implemented on the IEEE 30-bus system [37]. The IEEE 30-bus system consists of 30 buses, 41 transmission lines and 2 shunt capacitors, which are placed at bus 5 and bus 24, respectively. This system has six generators, which are placed at bus 1, bus 2, bus 13, bus 22, bus 23 and bus 27, respectively. The IEEE 30-bus system has four tap-changing transformers. The total real and reactive power peak loads on the IEEE 30-bus system are 189.2 MW and 107.2 MVar, respectively.

Table 2 ED results of the IEEE RTS obtained with four different methods

Power, MW; Voltage, p.u.	FACPF-based ED	LACPF-based ED		DCPF-based ED
		Lossy	Lossless	
P_{G1}	184	184	184	184
P_{G2}	184	184	184	184
P_{G7}	211.03	240.924	193	193
P_{G13}	235.74	207	207	207
P_{G15}	167	167	167	167
P_{G16}	155	155	155	155
P_{G18}	400	400	400	400
P_{G21}	400	400	400	400
P_{G22}	300	300	300	300
P_{G23}	660	660	660	660
V_1	1.047	1.047	1.047	1.0
V_2	1.047	1.046	1.047	1.0
V_7	1.036	1.012	1.036	1.0
V_{13}	1.033	1.017	1.016	1.0
V_{15}	1.041	0.990	0.988	1.0
V_{16}	1.044	0.991	0.991	1.0
V_{18}	1.050	0.991	0.994	1.0
V_{21}	1.050	0.998	0.999	1.0
V_{22}	1.050	1.015	1.020	1.0
V_{23}	1.050	1.010	1.010	1.0
t. cost (\$/hr)	63352.21	63226.58	60530.83	60530.83
t. gen. (MW)	2896.77	2897.92	2850	2850
t. losses (MW)	46.77	47.92	0	0

The real power and voltage magnitudes of the generators of the IEEE 30-bus system are, respectively, presented in Table 3. As can be seen from Table 3, the results of the optimal economic power dispatch of the six generators, which are obtained by the LACPF-based ED (lossy model), correspond closely with those obtained by the FACPF-based ED. Furthermore, voltage magnitudes obtained by the LACPF-based ED (lossy model) also correspond closely with those obtained by the FACPF-based ED. In this context, the percentage of voltage errors obtained by the proposed method at buses 1, 2, 13, 22, 23 and 27, respectively, are 0.00%, 0.88%, 4.39%, 0.88%, 0.58% and 3.91%, which are less than 5%.

The total generation cost, total generation and total losses of the IEEE 30-bus system are given in Table 3. As can be seen from Table 3, the results obtained by the proposed LACPF-based ED (lossy model) correspond closely with those obtained by the FACPF-based ED. The results of the total generation cost, total generation and total losses that

are obtained by both methods were almost similar. The results presented in this case study show that the proposed method can handle the optimal ED problem more efficiently than some other traditional linearised methods, which are developed based on the DCPF model.

6.3 IEEE 300-bus system test case

To show the effectiveness of the proposed LACPF-based ED method, it is applied on the IEEE 300-bus large-scale transmission system. The IEEE 300-bus system consists of 69 generators, 60 load tap changers, 304 transmission lines and 195 loads. The data for this system can be found in [37]. Owing to space constraints, we presented the results of the total generation cost, total generation and total losses in Table 4. As can be noticed from Table 4, the results of the optimal economic power dispatch, which are obtained by the LACPF-based ED (lossy model), correspond closely with those obtained by the FACPF-based ED. The results presented in this case study show the suitability of the proposed method in handling optimal power flows of large-scale systems.

The errors of the total generation and total generation cost of the IEEE RTS, the IEEE 30-bus system and the IEEE 300-bus system, which are obtained by the DCPF-based ED method and the proposed LACPF-based ED method, are summarised in Table 5.

Table 3 ED results of the IEEE 30-bus system obtained with four different methods

Power, MW; Voltage, p.u.	FACPF-based ED	LACPF-based ED		DCPF-based ED
		Lossy	Lossless	
P_{G1}	39.92	45	45	45
P_{G2}	53.03	48.958	47.20	47.884
P_{G13}	14.96	15	15	15
P_{G22}	22.46	21	21	21
P_{G23}	16.06	16	16	16
P_{G27}	45.58	45	45	44.316
V_1	1.050	1.050	1.050	1.0
V_2	1.028	1.037	1.036	1.0
V_{13}	1.048	1.002	1.003	1.0
V_{22}	1.019	1.010	1.011	1.0
V_{23}	1.026	1.020	1.022	1.0
V_{27}	1.050	1.009	1.008	1.0
t. cost (\$/hr)	578.29	579.72	572.95	572.95
t. gen. (MW)	192.01	190.96	189.20	189.20
t. losses (MW)	2.81	2.84	0	0

Table 4 ED results of the IEEE 300-bus system obtained with four different methods

Item/Method	FACPF-based ED	LACPF-based ED		DCPF-based ED
		Lossy	Lossless	
t. cost (10^5 \$/hr)	7.20	7.21	7.06	7.06
t. gen. (MW)	23829.9	23849.52	23525.85	23525.85
t. losses (MW)	304.05	323.67	0	0

Table 5 Comparison of total cost and total generation errors obtained by the proposed LACPF-based ED method and the DCPF-based ED method

System/Model	DCPF-based ED		LACPF-based ED	
	Cost error, %	Gen. error, %	Cost error, %	Gen. error, %
IEEE test system				
RTS	4.45	1.61	0.04	0.19
30-bus system	1.46	0.92	0.55	0.25
300-bus system	0.08	1.28	1.94	0.14

Table 6 Comparison of method performance

Comparison item	Method ref. [11]	Method ref. [42]	Proposed method
t. cost (\$/hr)	575.41	578.29	579.72
t. gen. (MW)	191.85	192.01	190.96
t. losses (MW)	2.86	2.81	2.84

7 Performance comparison of the proposed method

To test the consistency of the proposed LACPF-based ED method in finding the optimal or near-optimal global solution, it has been compared with some other methods, which are presented in the literature. We used a method that is developed based on hybrid particle swarm optimisation [11], and also a method that is developed based on SQP using Matpower simulation package [42]. The IEEE 30-bus system is considered in this comparative study. The comparison results are shown in Table 6. As can be seen from Table 6, the proposed LACPF-based ED method could lead to optimal or near-optimal global solution similar to that solution, which is obtained using non-linear means.

8 Conclusion

This paper has presented a method for solving the optimal economic power dispatch problem. The method presented in this paper is developed based on LP and a linearised network model, in which voltage magnitudes and reactive power flows have both been accounted for, unlike traditional linearised power flow methods. Piecewise linear models are developed to handle the thermal capacities of transmission lines, exponential loads, cost curves of generating units and line losses. The effectiveness of the proposed method is demonstrated on the IEEE reliability test system, the IEEE 30-bus system and the IEEE 300-bus large-scale system. The results of the optimal economic power dispatch of these three systems, which are obtained by the proposed method, were almost similar to those results, which are obtained by non-linear means. There was a slight difference in the power dispatch of some units of the IEEE reliability test system, which can be attributed, in part, to the number of linear segments involved in the linearisation of the cost curves of the generators of this system. A comparison with two other methods, which are developed based on hybrid particle swarm optimisation and sequential quadratic programming, is held to validate the consistency of the proposed method in finding the optimal or near-optimal global solution. The results show that the proposed method could consistently lead to optimal or

near-optimal global solution, and is appropriate to use for power system planning studies.

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