

Reliability and Sensitivity Analysis of Composite Power Systems Under Emission Constraints

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Abstract—In view of the increasing role of environmental considerations in power generation, there is an evolving body of reliability methods that are cognizant of these emerging factors. This paper presents a method for analyzing the reliability of composite power systems under the constraints of CO_2 , SO_2 and NO_x emissions. It also utilizes the concept of the shadow price to perform sensitivity analysis with respect to these constraints. The method presented here uses a linearized power flow representation in the form of a linear programming (LP) model similar to those frequently used in power system reliability studies. In addition to power balance and flow constraints normally used, this work introduces a suitably formulated set of emission constraints. The dual solution of the LP provides the shadow prices that are used for determining the sensitivities. This paper also develops relationships for the sensitivities of several reliability indices with respect to the emission allowances. The method is demonstrated on the IEEE-RTS and the results are presented.

Index Terms—emission limits, allowances, reliability, sensitivity, linearized power flow, shadow price, dual solution.

I. INTRODUCTION

FOLLOWING the introduction of the Clean Air Act Amendments of 1990, utilities have been required to reduce and monitor emissions from their power generating units starting from 1995. This act enforced allowances on SO_2 emissions that could be used or traded within the specified banking period [1]. The most significant reductions have occurred very recently. On November 14, 2012, California had its first auction and trading on greenhouse emission allowances [2]. Starting January 2013, California's emission cap-and-trade program came into effect, and greenhouse gas emissions from large electric power plants are required to be reduced by 16% between 2013 and 2020 [2]. The Regional Greenhouse Gas Initiative (RGGI), which consists of nine North-Eastern and Mid-Atlantic states, agreed on reduction of CO_2 budget by 45% by 2014 and the reduction will decrease by 2.5% yearly from 2015 to 2020 [3]. The American Clean Energy and Security Act of 2009 provides for the reduction of carbon emissions by 17% by 2020. The European Union Emissions Trading System reduced greenhouse gas emissions more than 8% between 2008 and 2012 and is expected to reduce greenhouse gas emissions below 20% of the 1990 levels by 2020, starting from 2013 [4].

Several methods have been proposed to reduce emissions from power generating units; these include installing post-combustion cleaning equipment, switching to fuel with less

pollutants, increasing the penetration of the renewable energy, and modifying existing dispatch strategies to include emissions. With these reduction methods, utilities are to operate and expand their generation within the emission allowances.

Some of the earliest work on minimum emission generation dispatch was performed by Gent and Lamont [5]. Nada, Hari and Kothari [6] solved economic emission load dispatch with line capacity constraints. Subsequently, considerable research was reported on variations and extensions of these methods [7]–[13]. These variations use different means of accommodating emissions within the optimization problem. Some of the proposed formulations also take into account the different emission reduction equipment and methods described above, and thereby provide for strategies that may allow utilities to forgo, defer, or minimize additional capital costs. For example, [14] shows how a utility may avoid installation of new emission equipment by changing its commitment and dispatch schedules, or by switching to fuels with low emission (and high cost), or both.

Reducing emissions not only affects power dispatch and cost-effective planning, but also influences system reliability requirements. Imposing emission limits on some specific areas may change reliability-based expansion planning procedures. Planning decisions may comprise a trade-off between the cost of buying extra emission allowances and the cost of improving system reliability. Evaluating system reliability under the condition of reducing the emission became necessary and reliability indices need to be adapted to this new constraint. Therefore, there is an emerging need to include emission considerations in reliability, security and economy studies.

Sensitivity analyses of conventional reliability indices incorporate remedial action through load curtailment minimization under the constraints of power balance, generation capacity limits and transmission capabilities to rank system components according to their risk. However, in accommodating regulatory measures, emission allowances have been considered as additional constraints. While planning studies are concerned with reliability and cost, it is often useful to quantify the sensitivity of the reliability indices of interest to component parameters and operating constraints. In the emerging context, it is reasonable to include emission constraints in the sensitivity analysis of the reliability indices. In other words, sensitivity of reliability indices with respect to the emission limits needs to be added to expansion planning studies. Therefore, generation expansion planning is no longer based only on reliability enhancement or cost minimization.

A considerable amount of research has been conducted on economic/emission dispatching strategies. However, incorpo-

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rating emission allowances in power system reliability studies is rarely found in the literature. In [15], SO_2 , NO_x and CO_2 emissions were considered as constraints in their generation expansion model which is based on multi-area reliability exponential analytic model. Minimization of emissions while maintaining reliability requirements in microgrids is presented in [16]. Most of the prior research has focused on evolutionary techniques and multiobjective minimization function to minimize emissions and generation costs in generation expansion planning [7]–[11], [17].

The work reported in this paper responds to the emerging need for reliability methods that are cognizant of environmental factors. This work calculates the reliability indices of composite power systems and utilizes the concept of the shadow price in ranking the generating units from the reliability point of view while considering the emission allowances as additional constraints. Therefore, this work adds another dimension to system planning processes through the inclusion of emission constraints. Sensitivity analyses of the well known reliability indices, Loss of Load Probability (LOLP), Loss of Load Frequency (LOLF) and Expected Demand Not Supplied (EDNS), are presented. The shadow prices of the generating units are used in evaluating the sensitivity of the EDNS reliability index. Also, a new expression for calculating the sensitivity of the EDNS with respect to the emission limits is developed. The generating units are ranked based on their sensitivities to the emission allowances. Based on this ranking, the emission allowances of generating stations are evaluated to determine a need for expansion. For instance, if a generating plant is ranked high in terms of risk and is viewed as a candidate for reconditioning in order to increase the overall system reliability, the emission allowance should be applied. If the emission limit is reached, the planner should choose between buying emission allowances and investing in another power station that has available emission allowances. The method is applicable to any source that produces undesirable byproducts that are capped by regulatory and other policies, as long as these byproducts are a function of the output power.

The remainder of this paper is organized as follows: section II reviews the emissions from electric generation, section III presents the modeling of the power grid and inclusion of emission constraints, section IV considers the sensitivity analysis of the reliability indices with respect to the emission allowances, section V illustrates the solution algorithm, section VI presents case studies, section VII discusses the results and section VIII provides some concluding remarks.

II. EMISSIONS FROM ELECTRIC GENERATION

Due to global warming and climate change, several acts and protocols have been introduced in the electric utility industry. These acts and protocols limit the amount of the Carbon Oxides (CO_x), Sulfur Oxides (SO_x) and Nitrogen Oxides (NO_x), enforces emission minimization laws and allows emission allowances trading. For instance, one of the main outcomes of the Kyoto protocol is carbon emission trading in the free market. This trading makes the exchange of emission allowances more flexible and minimizes the overall

operational costs [1]. Also, according to the Clean Air Act of 1990 (CAA), industrialized countries were required to reduce their emission by 5% during the time period of 2008-2012 compared to the emissions in 1990 [18].

A significant amount of energy supply is used in producing electricity world wide, and in most industrialized countries the bulk of electric generation comes from fossil fueled plants. According to the Environmental Protection Agency (EPA), about 16 million tons of SO_2 and 7 million tons of NO_x were emitted by electric utilities in the United States in 1985 [19]. Further, EPA reports that in 1995 about 23 million tons of SO_2 and 19 million tons of NO_x were emitted from all resources. In the UE-27, 40% of energy resources is used in generating electricity [20]. Also, in the UE-27 [20], 55% of the generated electricity in 2005 was supplied by fossil fuel resources. In 2008, 601.32 GW was generated from coal in China which is about 75.87% of the total electric energy generated [21]. Most of these emissions were produced by fossil fuel plants. In 2006, 12 million tons of Sulphur-dioxide emissions were produced in China [15].

Some regulations were introduced and categorized according to some definitions such as, “new source performance”, “modified source”, “reconstruction”, “degree of emission limitation”, etc. For instance, “new source” is defined as any source that is commissioned after the regulation is proposed and is subjected to the NSPS (New Source Performance Standards) requirements even if the regulation is not final [19]. Also, a “modified source” is any source subjected to a physical or operation change that may increase air pollutant or produce a new air pollutant and is subject to the NSPS (CAA).

The goal of reducing the amount of emissions of SO_2 and NO_x is implemented in two stages: Phase I and Phase II. Phase I began in January 1, 1995 and Phase II began in January 1, 2000. For example, in phase I, all units with 100 MW or greater and had SO_2 emission greater than 2.5 lb/mmBtu in 1985 are subjected to emission reduction according to table A of section 404, title IV of the CAA.

III. NETWORK MODELING AND EMISSION CONSTRAINTS

This section describes the formulation and incorporation of the emission in the load curtailment minimization problem. In this paper, the emission allowances are considered as additional constraints to the power system constraints.

A. Modeling of Power System Network

Since this work utilizes the concept of shadow price in ranking system components, linearized or DC load flow is used in modeling the load flow in the network. Linear programming is used with the objective of minimizing load curtailments in the network. The dual solution of the linear programming problem is used to calculate shadow prices of components. The equality constraints are the power balance at each bus and the inequality constraints are the capacity limits of generating units and power carrying capabilities of transmission lines. The linear programming model of the network with DC load flow is given in equation (1) which is adapted from [22].

$$\text{Loss of Load} = \min \left(\sum_{i=1}^{N_b} C_i \right) \quad (1)$$

Subject to

$$\begin{aligned} \hat{B}\delta + G + C &= D \\ G &\leq G^{max} \\ C &\leq D \\ b\hat{A}\delta &\leq F_f^{max} \\ -b\hat{A}\delta &\leq F_r^{max} \\ G, C &\geq 0 \\ \delta &\text{ unrestricted} \end{aligned} \quad (2)$$

and the emission constraints

$$E_j \leq E_j^{max} \quad \forall j \quad (3)$$

In (2), N_b is number of buses, N_t is number of transmission lines, $\hat{B}$ is the augmented node susceptance matrix ($N_b \times N_b$), b is the transmission line susceptance matrix ($N_t \times N_t$), $\hat{A}$ is the element-node incidence matrix ($N_t \times N_b$), δ is the vector of node voltage angles ($N_b \times 1$), C is the vector of bus load curtailments ($N_b \times 1$), D is the vector of bus demand ($N_b \times 1$), G^{max} is the vector of maximum available generation ($N_b \times 1$), F_f^{max} is the vector of forward flow capacities of lines ($N_t \times 1$), F_r^{max} is the vector of reverse flow capacities of lines ($N_t \times 1$), and G is the solution vector of the generation at buses ($N_b \times 1$).

In (3), E_j is the emission of type j , ($j = 1$ for CO_2 , $j = 2$ for SO_2 and $j = 3$ for NO_x), and E_j^{max} is the cap on emission type j . The derivation of the expression for E_j is provided in the next section.

In (2) all generation availability and network constraints have been taken into consideration. Also, in order to get a feasible solution for the standard problem, it has been assumed that one of the bus angles is zero in the constraints (2).

B. CO_2 , SO_2 , NO_x Emission Constraints

Several methods have been proposed to model the emission function of the thermal generating units. Some of these models are based on the heat rate functions of the generating units with modifying the heat rate coefficients. Some other model the emissions based on the heat rates with multiplying the heat rates with appropriate emission factors. In addition, some models use more detailed models such as including the valve loading effect. The heat rates themselves can be modeled in a quadratic, cubic or exponential functions. Emission rates were modeled using a straight linear form as in [12], [15], [23], combination of a straight line and two exponentials as in [5], a quadratic form as in [6]–[10], [13], [14], [17], [24], [25], a quadratic and an exponential form as in [11], [26], [27], and multiplying the heat rate function with an emission factor as in [28]. In this work, for simplicity, the emissions are modeled based on the heat rates, while ignoring the valve loading effects, and linearizing the quadratic form. The quadratic heat rate for a generating unit i can be expressed as:

$$HR = a_i + b_i P_i(t) + c_i P_i^2(t) \quad (\text{MMBtu/MWh}) \quad (4)$$

where HR is the heat rate, a_i , b_i and c_i are the heat rate coefficients and $P_i(t)$ is the real power generation of unit i at time t . Another widely used unit for heat rates is Btu/kWh. MMBtu/MWh has been used so as to be consistent with units of the emission rates. To convert from MMBtu/MWh to Btu/kWh, the given heat rate can be multiplied by 1,000.

Since the heat rates are functions of the output power and the emissions are functions of the heat rates, the emissions of a generating unit i can be expressed as a function of the output power as follows,

$$E_{ji} = \alpha_{ji} + \beta_{ji} P_i(t) + \gamma_{ji} P_i^2(t) \quad (\text{lbs/MMBtu}) \quad (5)$$

where α , β and γ are the coefficients of the emission rates, j denotes emission type (1 for ECO_2 , 2 for ESO_2 and 3 for ENO_x), ECO_{2i} is the CO_{2i} emission of unit i , ESO_{2i} is the SO_{2i} emission of unit i and ENO_{xi} is the NO_{xi} emission of unit i .

Assuming a power generating plant with n thermal units at time t , the total CO_2 emission is expressed as

$$ECO_2 = \sum_{i=1}^n ECO_{2i}(P_i(t)) \quad (\text{ton/h}) \quad (6)$$

Similarly, the total SO_2 emission is expressed as

$$ESO_2 = \sum_{i=1}^n ESO_{2i}(P_i(t)) \quad (\text{ton/h}) \quad (7)$$

Also, the total NO_x emission is expressed as

$$ENO_x = \sum_{i=1}^n ENO_{xi}(P_i(t)) \quad (\text{ton/h}) \quad (8)$$

These emissions are nonlinear functions in the unit output power. In order to include these emissions as constraints in the linear programming problem, these equations are linearized around the operating points.

IV. SENSITIVITY ANALYSIS

Sensitivity analyses of reliability indices reported in the literature have been based on calculating the amount of change of these indices with respect to component parameters such as availability/unavailability, capacity, failure rate and repair rate. These analyses were conducted either by examining every single state of the system or by Monte Carlo simulation. The analyses were performed by casting the dispatch operation as an optimization problem, through minimizing load curtailment or maximizing load supplying capability. The optimization was constrained by generator and transmission line capacities. Therefore, these sensitivity analyses did not account for other constraints such as emission limits. However, after the introduction of the Clean Air Act, it is necessary for such analyses to accommodate emission limits. This work includes the effect of emission limits on reliability indices frequently used in composite system studies.

Some definitions:

A Loss of Load (LOL) event is one in which a system is unable to meet its total demand.

Loss of Load Probability (LOLP) is the probability of encountering one or more LOL events during a given time period.

Loss of Load Frequency (LOLF) is the expected (average) number of LOL events encountered per unit time during a given period. It is usually expressed in terms of failures/year. *Expected Demand Not Served* (EDNS) is the expected (average) demand that the system is unable to serve as a result of LOL events during a given period. It is calculated as the weighted sum of the demands curtailed during the LOL events, the weights being the probabilities of the corresponding LOL events. It is expressed in MW/year or GW/year.

LOLP and LOLF from their definitions are based on reliability parameters and cannot be directly related to the emission limits. In this paper, sensitivity analysis of the LOLP and LOLF with respect to component reliability parameters is presented to study the effect of imposing emission limits on power system operation. On the other hand, EDNS is based on failure rates, repair rates and units capacities. Since emissions can be related to the output power, the sensitivity of the EDNS with respect to component capacities can be evaluated directly using this dependence.

A. Sensitivity Analysis of LOLP and LOLF with Respect to Component Parameters

Sensitivity studies of LOLP and LOLF to component reliability have been amply described in the literature. Sensitivity analysis can be conducted analytically by enumerating all system states or by simulation. Refs. [29]–[31] provide relationships that are suitable for use in state space enumeration. In this paper, these analytical expressions have been suitably adapted for use in Monte Carlo simulation. Since all the expressions used here are not explicitly developed in [29]–[31], more derivations have been provided in the appendix. As mentioned before, sensitivity analysis of LOLP and LOLF indices can not identify which component was affected by the emission allowances; rather, it shows the influence of reducing the emission limit on system reliability. These indices are used to study the effect of emission reduction on reliability indices.

The sensitivity of LOLP with respect to unavailability u_i of component i can be calculated as follows.

$$\partial LOLP / \partial u_i = \sum_{x \in X} I_f(x) P(x) [(1/u_i) - S_i / (a_i u_i)] \quad (9)$$

where X is the set of all states, x is the state of the system, $P(x)$ is the probability of occurrence of state x , u_i is the probability of failure of component i and a_i is the probability of success of component i . S_i is the state indicator of component i , i.e., $S_i = 0$ if component i is in the down state (failure state) and $S_i = 1$ if component i is in the up state (success state). $I_f(x)$ is the system state indicator function which can be expressed as follows.

$$I_f(x) = \begin{cases} 1 & \text{if } x \text{ is failure state} \\ 0 & \text{if } x \text{ is success state} \end{cases}$$

The sensitivity of LOLP with respect to component failure rate λ_i is given by

$$\partial LOLP / \partial \lambda_i = \sum_{x \in X} I_f(x) P(x) [a_i / \lambda_i - S_i / \lambda_i] \quad (10)$$

The sensitivity of LOLP with respect to component repair rate μ_i is given by

$$\partial LOLP / \partial \mu_i = \sum_{x \in X} I_f(x) P(x) [-a_i / \mu_i + S_i / \mu_i] \quad (11)$$

The sensitivity of LOLF with respect to unavailability u_i of component i can be calculated as follows.

$$\partial LOLF / \partial u_i = \sum_{x \in X} [-I_f(x) P(x) S_i \mu_i / a_i^2 + F(x) P(x) ((1/u_i) - S_i / (a_i u_i))] \quad (12)$$

where $F(x)$ is the sum of the repair rates of a failure state x that crosses the boundary and can be expressed as:

$$F(x) = I_f(x) \sum_{i=1}^m \lambda_i^{in}(x)$$

where m is the number of components, and

$$\lambda_i^{in}(x) = (1 - S_i) \mu_i - S_i \mu_i u_i / a_i$$

The sensitivity of LOLF with respect to component failure rate λ_i is given by

$$\partial LOLF / \partial \lambda_i = \sum_{x \in X} [-I_f(x) P(x) S_i + F(x) P(x) (a_i / \lambda_i - S_i / \lambda_i)] \quad (13)$$

The sensitivity of LOLF with respect to component repair rate μ_i is given by

$$\partial LOLF / \partial \mu_i = \sum_{x \in X} [I_f(x) (1 - S_k) P(x) + F(x) P(x) (-a_i / \mu_i + S_i / \mu_i)] \quad (14)$$

B. Sensitivity Analysis of EDNS

The Expected Demand Not Supplied (EDNS) is an important index because it inherently reflects the severity of reliability events. The sensitivities of this index with respect to u_i , λ_i and μ_i are the same as for the LOLP index except that they are multiplied by the amount of load curtailments. In this subsection, an analytical expression is developed to calculate the sensitivity of the EDNS with respect to emission limits. The development of this expression is based on the linearized relationship between the amount of emission and the output power. The derivation starts with utilizing the partial derivative of the EDNS to the component capacity.

The sensitivity analysis of the EDNS with respect component capacities was derived as follows [30], [31].

$$\partial EDNS / \partial C_i = \sum_{x \in X} I_f(x) P(x) \partial L_c(x) / \partial C_i \quad (15)$$

where C_i is the capacity of component i and $L_c(x)$ is the total load curtailment when the system is at state x .

The derivative of the total load curtailment with respect to component capacity can be expressed as

$$\partial L_c(x) / \partial C_i = \begin{cases} \pi_{g,i} & \text{if component } i \text{ is generator} \\ \pi_{t,ij} & \text{if component } i \text{ is circuit} \end{cases}$$

where $\pi_{g,i}$ and $\pi_{t,ij}$ are the Lagrange multipliers or shadow prices of generation capacity constraints and transmission

lines carrying capability constraints respectively. $\pi_{g,i}$ can be calculated directly from the optimization problem of (1). However, $\pi_{t,ij}$ depends on circuit parameters which are circuit capacity and susceptance. These two parameters are dependent variables and can not be treated separately. Pereira and Pinto [32], combined the effect of circuit capacity and susceptance on circuit sensitivity and developed the following expression.

$$\pi_{t,ij} = (\pi_{d,i} - \pi_{d,j})(\theta_j - \theta_i) \quad (16)$$

where $\pi_{d,i}$ and $\pi_{d,j}$ are Lagrange multipliers or shadow prices of load constraints of buses i and j respectively and θ_j and θ_i are voltage angles of buses j and i respectively.

Sensitivity of EDNS with respect to circuit capacity or susceptance is inherently not affected by the emission limits; however, with generation redispatch under the condition of emission limits reduction they may become active constraints.

In section II it was stated that the emission functions are linearized about the operating points. Therefore, the effect of the emission limits on EDNS sensitivity can be evaluated by developing a relationship between shadow price of a generating unit with its emission. From (1), the shadow price of a generating unit can be expressed as

$$\pi_{g,i} = \partial L_c(x) / \partial g_i \quad (17)$$

where g_i is the capacity of generator i .

Now a new term is introduced which is the partial derivative of total load curtailment with respect to emission limit. This term can be thought of it as a new constraint to (1). This new constraint will produce a new shadow price which relates the emission limit to total load curtailment. The partial derivative of total load curtailment with respect to emission limits can be expressed as

$$\pi_{E,ij} = \partial L_c(x) / \partial E_{ij} \quad (18)$$

where $\pi_{E,ij}$ and E_{ij} are the shadow price of the emission limit and the emission limit of unit i for emission type j . $\partial L_c(x) / \partial E_{ij}$ can be expressed as

$$\partial L_c(x) / \partial E_{ij} = \partial L_c(x) / \partial g_i \cdot \partial g_i / \partial E_{ij} \quad (19)$$

The first term of the right hand side of (19) is the shadow price of the generating capacity. The second term can be calculated using the relationship of generation output power and emission. This relationship can be given as,

$$E_{ij} = E_{c,ij} \cdot g_i \quad (20)$$

where $E_{c,ij}$ is the linearized emission coefficient of unit i for emission type j .

Therefore, from (20), partial derivative of the output power with respect to the amount of emission can be calculated as follows.

$$\partial g_i / \partial E_{ij} = 1 / E_{c,ij} \quad (21)$$

Now, the partial derivative of the total load curtailment with respect to the emission allowances can be given by

$$\partial L_c(x) / \partial E_{ij} = \partial L_c(x) / \partial g_i \cdot 1 / E_{c,ij} \quad (22)$$

This can be expressed in terms of shadow price or Lagrange multiplier as

$$\pi_{E,ij} = \pi_{g,i} / E_{c,ij} \quad (23)$$

Therefore, the sensitivity of the EDNS with respect to emission allowances now can be defined as,

$$\partial EDNS / \partial E_{ij} = \sum_{x \in X} I_f(x) P(x) \partial L_c(x) / \partial E_{ij} \quad (24)$$

Expressed in terms of shadow price or Lagrange multiplier,

$$\partial EDNS / \partial E_{ij} = \sum_{x \in X} I_f(x) P(x) (\pi_{g,i} / E_{c,ij}) \quad (25)$$

Equations (24) and (25) calculate the sensitivity of the EDNS with respect to emission allowances. Therefore, by solving the optimization problem of (1) and getting the shadow prices of the generating units, the shadow prices of the emission limits are calculated by dividing the shadow prices of the generating units by the linearized emission coefficients.

V. SOLUTION ALGORITHM

This section describes the approach used for calculating the sensitivity analysis of LOLP, LOLE and EDNS with respect to component reliability and emission allowance limits of various system components. As mentioned above, emission allowances are considered as additional constraints. In this paper, sensitivity analysis was carried out using Monte Carlo simulation and therefore the above expressions need to be modified to be updated for every sampled state. In other words, in using Monte Carlo simulation, instead of using $P(x)$ in the above expressions, for every failure state these indices are updated by using “ $1/N$ ” instead of $P(x)$ where N is the number of samples.

The stopping criterion used in this method consists of comparing the coefficient of variation of a selected index (e.g., the EDNS) to a pre-specified tolerance, as is usual in Monte Carlo methods. The simulation steps are summarized in Fig. 1. It is also possible to accelerate the convergence by using the method of state space pruning [33].

VI. CASE STUDIES

This section illustrates the method on the IEEE RTS test system. The IEEE RTS system consists of 24 buses, 33 transmission lines, 5 transformers and 32 generating units. Total installed generating capacity is 3405 MW and system peak load is 2850 MW. The data of this system is given in [34].

The IEEE RTS-79 has two nuclear steam generating units each of which 400 MW and six hydro driven generating units each of which 50 MW. These units do not have SO_2 , NO_x or CO_2 emissions.

The emission rates were calculated from the heat rates. The emission coefficients were calculated from the emission rates based on 2nd order polynomials.

System states were sampled and the shadow prices for the generating units were calculated for every state. Also, for every state the partial derivative of the total load curtailment with respect to generation capacity is calculated from the optimization problem of (1). The emission curves were linearized

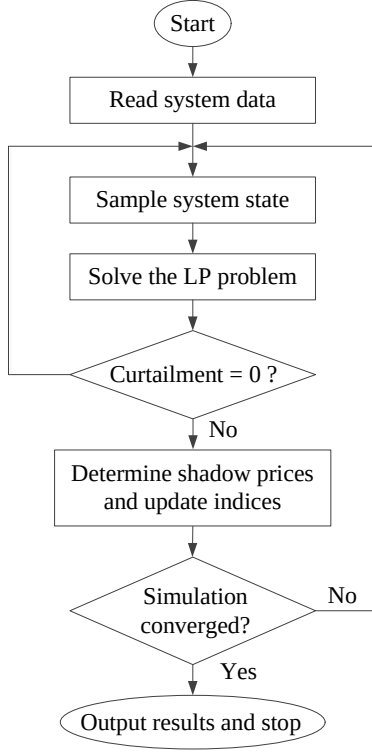


Fig. 1. Flowchart of the proposed method

about the operating points for two linear segments. After the simulation converges to the specified accuracy, the sensitivity of the EDNS to the emission limits is summed up as shown in (25).

Two case studies have been performed: a base case, and one in which the emission allowances are reduced by 5%. The annualized reliability indices of the IEEE RTS are shown in table I. The results for two more reliability indices, Loss of Load Expectation (LOLE) and Loss of Energy Expectation (LOEE, also called unexpected unserved energy or EUE), are also presented. LOLE and LOEE can be calculated directly from LOLP and EDNS respectively by multiplying by the study period in hours.

The sensitivity results of the LOLP with respect to reliability parameters for the base case and reduced emission case are shown in tables II and III respectively. The sensitivity results of the LOLP with respect to reliability parameters for the base case and reduced emission case are shown in tables IV and V respectively. The sensitivity results of the EDNS with respect to reliability parameters for the base case and reduced emission case are shown in tables VI and VII respectively. Table VIII shows the sensitivity of the EDNS with respect to ESO_2 , ENO_x and ECO_2 for the base case. Table IX shows the sensitivity of the EDNS with respect to ESO_2 , ENO_x and ECO_2 for the reduced emission case.

VII. RESULTS AND DISCUSSION

From tables I and II it is clear that the generating unit (350 MW) at bus 23 has the highest effect on the LOLP followed by the generating units (400 MW) of buses 18 and 21 for the two study cases. Consequently, these generating units have

the largest impact on system reliability. In other words, these buses are at the highest risk and the most beneficial way to make the system more reliable is to increase the availability of these stations by adding new units, or by modernizing existing units to improve reliability. When the emission allowances were reduced (case study 2), these sensitivities increased accordingly.

In comparison to the case study 1, the sensitivities of LOLP of the generating units of the case study 2 have increased due to the decrease in the emission limits. Such decrease in the emission limits has forced the generating units to have larger shadow prices values than it was in case study 1. By decreasing the emission limits, the emission limits constraints became binding constraints and generating capacities became redundant constraints.

From tables III and IV it is clear that the generating unit (350 MW) at bus 23 has the highest effect on the EDNS for both study cases. Also, after reducing the emissions, the sensitivities of the EDNS with respect to the emission limits have increased dramatically. For instance, for the SO_2 air pollutant, sensitivity of the 350 MW unit with respect to the emission has increased by about 2.6 times and for the 197 MW unit has increased by about 4.9 times. The factor of increase in the sensitivity of the EDNS with respect to emission limit due reducing the emissions are shown in table X.

From the above discussion it is obvious that reducing the emission allowance limits leads to a huge change in system reliability indices. From these studies, planners can decide where to invest by trading off between reliability requirements and the cost of reducing the emissions or buying emission allowances for the affected areas.

TABLE I
ANNUALIZED INDICES OF THE IEEE RELIABILITY TEST SYSTEM, RTS

	LOLP	EDNS MW	LOEE MWh/yr	LOLE hr/yr
Base Case	0.08496	14.76607	128996	742.21056
Reduced Emission	0.14093	26.49218	231435	1231.16448

TABLE II
SENSITIVITY ANALYSIS OF LOLP WITH RESPECT TO RELIABILITY PARAMETERS FOR THE BASE CASE

Bus No. & Gen. Cap.	$\partial LOLP / \partial u_i$ $\times 10^{-4}$	$\partial LOLP / \partial \lambda_i$ $\times 10^{-2}$	$\partial LOLP / \partial \mu_i$ $\times 10^{-4}$
#15G12	0.23468	0.13523	-0.27598
#22G50	1.57580	0.30889	-0.31201
#16G155	14.87768	5.48451	-22.85212
#23G155	13.65057	5.03215	-20.96728
#15G155	14.49772	5.34444	-22.26849
#13G197	15.36175	6.93199	-36.48416
#23G350	42.33958	35.83622	-311.61931
#18G400	30.31500	35.21391	-480.18962
#21G400	30.42285	35.33918	-481.89795

VIII. CONCLUSION

Most of the emission studies in the literature focused on minimizing emission with economic dispatch and introducing new concepts to solve multi-objective functions. The goal was

TABLE III
SENSITIVITY ANALYSIS OF LOLP WITH RESPECT TO RELIABILITY
PARAMETERS FOR THE REDUCED EMISSION CASE

Bus No. & Gen. Cap.	$\partial LOLP / \partial u_i$ $\times 10^{-4}$	$\partial LOLP / \partial \lambda_i$ $\times 10^{-2}$	$\partial LOLP / \partial \mu_i$ $\times 10^{-4}$
#15G12	1.39303	0.80272	-1.63820
#22G50	4.97198	0.97461	-0.98445
#16G155	17.80651	6.56419	-27.35080
#23G155	16.73514	6.16924	-25.70517
#15G155	18.46648	6.80748	-28.36451
#13G197	19.05290	8.59762	-45.25063
#23G350	46.66679	39.49877	-343.46757
#18G400	33.29836	38.67937	-527.44598
#21G400	32.84112	38.14824	-520.20328

TABLE IV
SENSITIVITY ANALYSIS OF LOLF WITH RESPECT TO RELIABILITY
PARAMETERS FOR THE BASE CASE

Bus No. & Gen. Cap.	$\partial LOLF / \partial u_i$ $\times 10^{-4}$	$\partial LOLF / \partial \lambda_i$ $\times 10^{-2}$	$\partial LOLF / \partial \mu_i$ $\times 10^{-4}$
#15G12	0.23468	0.13523	-0.27598
#22G50	1.57580	0.30889	-0.31201
#16G155	14.87768	5.48451	-22.85212
#23G155	13.65057	5.03215	-20.96728
#15G155	14.49772	5.34444	-22.26849
#13G197	15.36175	6.93199	-36.48416
#23G350	42.33958	35.83622	-311.61931
#18G400	30.31500	35.21391	-480.18962
#21G400	30.42285	35.33918	-481.89795

TABLE V
SENSITIVITY ANALYSIS OF LOLF WITH RESPECT TO RELIABILITY
PARAMETERS FOR THE REDUCED EMISSION CASE

Bus No. & Gen. Cap.	$\partial LOLF / \partial u_i$ $\times 10^{-4}$	$\partial LOLF / \partial \lambda_i$ $\times 10^{-2}$	$\partial LOLF / \partial \mu_i$ $\times 10^{-4}$
#15G12	1.39303	0.80272	-1.63820
#22G50	4.97198	0.97461	-0.98445
#16G155	17.80651	6.56419	-27.35080
#23G155	16.73514	6.16924	-25.70517
#15G155	18.46648	6.80748	-28.36451
#13G197	19.05290	8.59762	-45.25063
#23G350	46.66679	39.49877	-343.46757
#18G400	33.29836	38.67937	-527.44598
#21G400	32.84112	38.14824	-520.20328

TABLE VI
SENSITIVITY ANALYSIS OF EDNS WITH RESPECT TO RELIABILITY
PARAMETERS FOR THE BASE CASE

Bus No. & Gen. Cap.	$\partial EDNS / \partial u_i$ $\times 10^{-4}$	$\partial EDNS / \partial \lambda_i$ $\times 10^{-2}$	$\partial EDNS / \partial \mu_i$ $\times 10^{-4}$
#15G12	0.23468	0.13523	-0.27598
#22G50	1.57580	0.30889	-0.31201
#16G155	14.87768	5.48451	-22.85212
#23G155	13.65057	5.03215	-20.96728
#15G155	14.49772	5.34444	-22.26849
#13G197	15.36175	6.93199	-36.48416
#23G350	42.33958	35.83622	-311.61931
#18G400	30.31500	35.21391	-480.18962
#21G400	30.42285	35.33918	-481.89795

to minimize emissions and fuel costs to accommodate for the requirements of the CAA of 1990. Minimizing the emission is one solution among many methods such as utilizing fuels with less air pollutant, exploring new techniques to reduce the amount of emissions, etc. Minimizing the emissions can help in redispatching the existing generators. The need to establish strategies to include the CAA 1990 in future planning is necessary. Inclusion of emission constraints in planning studies as described in this paper helps guide investment strategies with respect to reconstruction or upgrades.

The work presented here responds to the emerging need for reliability methods that are cognizant of environmental factors. This work introduced a framework for evaluating the effect of the emission rates and allowances on system reliability and determining the sensitivity of commonly used reliability indices to equipment and emission constraints.

The Expected Demand Not Supplied (EDNS) is used as a key reliability index. The sensitivity of the EDNS with respect to emission allowances is evaluated and used to study the effect of emission allowances on system reliability. From the sensitivity results, components can be ranked from reliability point of view. Therefore, the component that have the highest effect on system reliability can be identified. Also, if the reliability of the system needs to be improved, the sensitivity analyses can identify which component to increase its emission limit.

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TABLE VII
SENSITIVITY ANALYSIS OF EDNS WITH RESPECT TO RELIABILITY
PARAMETERS FOR THE REDUCED EMISSION CASE

Bus No. & Gen. Cap.	$\partial EDNS/\partial u_i$ $\times 10^{-4}$	$\partial EDNS/\partial \lambda_i$ $\times 10^{-2}$	$\partial EDNS/\partial \mu_i$ $\times 10^{-4}$
#15G12	1.39303	0.80272	-1.63820
#22G50	4.97198	0.97461	-0.98445
#16G155	17.80651	6.56419	-27.35080
#23G155	16.73514	6.16924	-25.70517
#15G155	18.46648	6.80748	-28.36451
#13G197	19.05290	8.59762	-45.25063
#23G350	46.66679	39.49877	-343.46757
#18G400	33.29836	38.67937	-527.44598
#21G400	32.84112	38.14824	-520.20328

TABLE VIII
SENSITIVITY ANALYSIS OF EDNS WITH RESPECT TO EMISSION
ALLOWANCE LIMITS FOR THE BASE CASE

Bus No. & Gen. Cap.	$\partial EDNS/\partial ESO_2$ $\times 10^{-4}$	$\partial EDNS/\partial ENO_x$ $\times 10^{-4}$	$\partial EDNS/\partial ECO_2$ $\times 10^{-4}$
#15G12	-34.95128	-13.98051	-0.04112
#16G155	-94.96082	-37.98433	-0.09044
#23G155	-47.96955	-19.18782	-0.04569
#15G155	-94.63840	-37.85536	-0.09013
#13G197	-47.58625	-19.03450	-0.05598
#23G350	-267.88743	-107.15497	-0.25513

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TABLE IX
SENSITIVITY ANALYSIS OF EDNS WITH RESPECT TO EMISSION
ALLOWANCE LIMITS FOR THE REDUCED EMISSION CASE

Bus No. & Gen. Cap.	$\partial EDNS/\partial ESO_2$ $\times 10^{-4}$	$\partial EDNS/\partial ENO_x$ $\times 10^{-4}$	$\partial EDNS/\partial ECO_2$ $\times 10^{-4}$
#15G12	-39.70206	-15.88082	-0.04671
#16G155	-274.08317	-109.63327	-0.26103
#23G155	-57.60845	-23.04338	-0.05487
#15G155	-229.14951	-91.65980	-0.21824
#13G197	-233.52268	-93.40907	-0.27473
#23G350	-702.68535	-281.07414	-0.66922

TABLE X
PERCENTAGES OF INCREASE IN THE SENSITIVITIES OF EDNS WITH
RESPECT TO EMISSION ALLOWANCE LIMITS DUE TO REDUCED
EMISSIONS

Bus No. & Gen. Cap.	Ratio of Increase
#15G12	1.14
#16G155	2.89
#23G155	1.20
#15G155	2.42
#13G197	4.90
#23G350	2.62

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APPENDIX

DERIVATION OF THE SENSITIVITY RELATIONSHIPS

Here, only the derivations of the sensitivity of the LOLP index with respect to reliability parameters are provided. The

remaining sensitivities for LOLF and EDNS can be easily derived in the same manner. Some of the derivations presented below have been adapted from [30]. However, this reference does not provide the complete derivations.

A. Sensitivity of LOLP to Component Failure Probability

$$\begin{aligned} \partial LOLP / \partial u_i &= \sum_{x \in X} \{ [\partial I_f(x) / \partial u_i] P(x) + I_f(x) [\partial P(x) / \partial u_i] \} \\ &= \sum_{x \in X} I_f(x) \{ [\partial [P(S_1)P(S_1) \cdots P(S_i) \cdots P(S_m)] / \partial u_i] \} \\ &= \sum_{x \in X} I_f(x) P(x) [1/P(S_i)] [\partial P(S_i) / \partial u_i] \end{aligned}$$

If $S_i = 1$,

$$\begin{aligned} [1/P(S_i)] [\partial P(S_i) / \partial u_i] &= (1/P(S_i = 1)) (\partial P(S_i = 1) / \partial u_i) \\ &= (1/a_i) (\partial a_i / \partial u_i) = -1/a_i = -S_i/a_i \end{aligned}$$

If $S_i = 0$,

$$\begin{aligned} [1/P(S_i)] [\partial P(S_i) / \partial u_i] &= (1/P(S_i = 0)) (\partial P(S_i = 0) / \partial u_i) \\ &= (1/u_i) (\partial u_i / \partial u_i) = 1/u_i = (1 - S_i)/u_i \end{aligned}$$

Therefore, in general,

$$\begin{aligned} [1/P(S_i)] [\partial P(S_i) / \partial u_i] &= (1 - S_i)/u_i - S_i/a_i \\ &= (1/u_i) - S_i/(a_i u_i) \end{aligned}$$

Then,

$$\partial LOLP / \partial u_i = \sum_{x \in X} I_f(x) P(x) [(1/u_i) - S_i/(a_i u_i)]$$

B. Sensitivity of LOLP to Component Failure Rate

$$\begin{aligned} \partial LOLP / \partial \lambda_i &= \sum_{x \in X} \{ [\partial I_f(x) / \partial \lambda_i] P(x) + I_f(x) [\partial P(x) / \partial \lambda_i] \} \\ &= \sum_{x \in X} I_f(x) \{ [\partial [P(S_1)P(S_1) \cdots P(S_i) \cdots P(S_m)] / \partial \lambda_i] \} \\ &= \sum_{x \in X} I_f(x) P(x) [1/P(S_i)] [\partial P(S_i) / \partial \lambda_i] \end{aligned}$$

If $S_i = 1$,

$$\begin{aligned} [1/P(S_i)] [\partial P(S_i) / \partial \lambda_i] &= (1/P(S_i = 1)) (\partial P(S_i = 1) / \partial \lambda_i) \\ &= (1/a_i) (\partial a_i / \partial \lambda_i) = -S_i/(\lambda_i + \mu_i) \end{aligned}$$

If $S_i = 0$,

$$\begin{aligned} [1/P(S_i)] [\partial P(S_i) / \partial \lambda_i] &= (1/P(S_i = 0)) (\partial P(S_i = 0) / \partial \lambda_i) \\ &= (1/u_i) (\partial u_i / \partial \lambda_i) \\ &= (a_i/u_i) (1/(\lambda_i + \mu_i)) \\ &= (a_i/u_i) ((1 - S_i)/(\lambda_i + \mu_i)) \end{aligned}$$

Therefore, in general,

$$\begin{aligned} [1/P(S_i)] [\partial P(S_i) / \partial \lambda_i] &= -S_i/(\lambda_i + \mu_i) + (a_i/u_i) ((1 - S_i)/(\lambda_i + \mu_i)) \\ &= a_i/\lambda_i - S_i/\lambda_i \end{aligned}$$

Then,

$$\partial LOLP / \partial \lambda_i = \sum_{x \in X} I_f(x) P(x) [a_i/\lambda_i - S_i/\lambda_i]$$

C. Sensitivity of LOLP to Component Repair Rate

$$\begin{aligned} \partial LOLP / \partial \mu_i &= \sum_{x \in X} \{ [\partial I_f(x) / \partial \mu_i] P(x) + I_f(x) [\partial P(x) / \partial \mu_i] \} \\ &= \sum_{x \in X} I_f(x) \{ [\partial [P(S_1)P(S_1) \cdots P(S_i) \cdots P(S_m)] / \partial \mu_i] \} \\ &= \sum_{x \in X} I_f(x) P(x) [1/P(S_i)] [\partial P(S_i) / \partial \mu_i] \end{aligned}$$

If $S_i = 1$,

$$\begin{aligned} [1/P(S_i)] [\partial P(S_i) / \partial \mu_i] &= (1/P(S_i = 1)) (\partial P(S_i = 1) / \partial \mu_i) \\ &= (1/a_i) (\partial a_i / \partial \mu_i) \\ &= (u_i/a_i) (1/(\lambda_i + \mu_i)) \\ &= (u_i/a_i) (S_i/(\lambda_i + \mu_i)) \end{aligned}$$

If $S_i = 0$,

$$\begin{aligned} [1/P(S_i)] [\partial P(S_i) / \partial \mu_i] &= (1/P(S_i = 0)) (\partial P(S_i = 0) / \partial \mu_i) \\ &= (1/u_i) (\partial u_i / \partial \mu_i) \\ &= -1/(\lambda_i + \mu_i) = -(1 - S_i)/(\lambda_i + \mu_i) \end{aligned}$$

Therefore, in general,

$$\begin{aligned} [1/P(S_i)] [\partial P(S_i) / \partial \mu_i] &= (u_i/a_i) (S_i/(\lambda_i + \mu_i)) - (1 - S_i)/(\lambda_i + \mu_i) \\ &= -a_i/\mu_i + S_i/\mu_i \end{aligned}$$

Then,

$$\partial LOLP / \partial \mu_i = \sum_{x \in X} I_f(x) P(x) [-a_i/\mu_i + S_i/\mu_i]$$

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