

Power System Reliability Evaluation using a State Space Classification Technique and Particle Swarm Optimisation Search Method

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Abstract—It is well-known that the reliability evaluation of composite power systems is computationally demanding. This work introduces a state space classification (SSC) technique that classifies a systems state space into failure, success, and unclassified subspaces without performing power flow analysis. The SSC technique was developed based on calculating the maximum capacity flow of the transmission lines and the available generation. An algorithm, which is developed based on a directed binary particle swarm optimisation, was developed to search for failure states in the unclassified subspaces. The key element in controlling the particle swarm optimisation (PSO) search method to search for failure states in the unclassified subspaces is the selection of the weighting factors of the velocity update rule. The work presented in this study proposes an intelligent PSO based search method to adjust these weighting factors in a dynamic fashion. The effectiveness of the proposed method was demonstrated on three test systems, the Institute of Electrical and Electronics Engineers reliability test system (IEEE RTS), the modified IEEE RTS and the Saskatchewan Power Corporation in Canada. The results have shown that the reliability indices obtained using the proposed method correspond closely with those obtained using Monte Carlo simulation with less computation burden.

Index Terms—Computation time, particle swarm optimisation, power system reliability.

NOMENCLATURE

Symbol	Description
N_b	number of buses,
N_t	number of transmission lines,
$\hat{B}$	augmented node susceptance matrix ($N_b \times N_b$),
b	transmission line susceptance matrix ($N_t \times N_t$),
$\hat{A}$	element-node incidence matrix ($N_t \times N_b$),
δ	vector of node voltage angles ($N_b \times 1$),
C	vector of bus load curtailments ($N_b \times 1$),
D	vector of bus demand ($N_b \times 1$),
G^{max}	vector of maximum available generation ($N_b \times 1$),
F_f^{max}	vector of forward flow capacities of lines ($N_t \times 1$),
F_r^{max}	vector of reverse flow capacities of lines ($N_t \times 1$),
G	solution vector of the generation at buses ($N_b \times 1$),
K	diagonal line capacity matrix ($N_t \times N_t$),
A	modified element-node incidence matrix ($N_t \times N_b$),
Y_{bus}	bus admittance matrix,

T	matrix transpose,
$LC - Matrix$	line capacity capability flow matrix,
v_i	particle velocity,
x_i	particle position,
$Ppbest_i$	best position, from the probability of a state perspective, particle i has ever encountered,
$Pgbest$	best position, from the probability of a state perspective, the entire group of particles has ever encountered,
$Cpbest_i$	best position, from the load curtailment of a state perspective, particle i has ever encountered,
$Cgbest$	best position, from the load curtailment of a state perspective, the entire group of particles has ever encountered,
c_1, c_2, c_3, c_4	acceleration factors of the velocity update rule,
x_{id}	the d^{th} component of particle i ,
$S(v_{id})$	sigmoid function of d^{th} 's component of particle i ,
V_{max}	ultimate probability limit of a bit in particle i to be zero or one,
F	set of failure states (states that have load curtailment),
S	set of success states (states that do not have load curtailment),
$P_i(x)$	probability of state i ,
$Lc_i(x)$	amount of load curtailment of state i (the sum of the load curtailments at the buses),
$L_i(x)$	amount of the supplied load of state i ,
σ	specified tolerance,
$E[.]$	expectation operator,
$Var(.)$	variance function,
ε	particle probability threshold.

I. INTRODUCTION

Composite system reliability evaluation aims at determining the reliability of the given power system taking into consideration both transmission and generation systems. Numerous techniques have been proposed in the literature to evaluate composite system reliability. In this context, analytical methods [1], [2] and Monte Carlo simulation [3] have been used for composite system reliability evaluation. In evaluating the reliability indices of composite power systems, a power flow or optimal power flow with an objective of minimum load curtailment is usually required to test whether the state under consideration is a failure or success state. The DC

power flow model in conjunction with the linear programming optimisation problem is the most commonly model used in power system reliability evaluation [1], [4]–[9]. However, performing optimal power flow for a huge number of scenarios can be computationally demanding. Consequently, the need for better techniques/methods to reduce the computational burden and the time required in evaluating the reliability indices of composite systems have become evident, and triggered several ongoing efforts to develop improved methods of evaluating power system reliability.

Several methods have been introduced in the literature of power system reliability evaluation to reduce the search space and the computation effort. In [4], Singh and Mitra have introduced the concept of the state space pruning to reduce the search space. In their work, an arbitrary set of coherent acceptable subspaces is pruned out from the state space using the concept of partitioning vectors; then, Monte Carlo simulation is performed for the remaining unclassified subspaces. The number of pruned subspaces is system dependent and it is a trade-off between the time required to prune the acceptable subspaces and the time required to perform Monte Carlo simulation on the rest of the subspaces. In a subsequent publication [5], Mitra and Singh have applied the state space pruning to calculate the frequency and duration indices of composite power systems. A state space partitioning technique has been introduced in [10] which is also intended to classify and reduce the search space. Consequently, several other researchers have developed the concept of state space pruning based on the meta-heuristic population-based intelligent search methods [11]–[16]. The genetic algorithm and the modified genetic algorithm have been used in [11]–[13] to prune the state space and Monte Carlo simulation has been then used for the remaining part of the state space. Binary particle swarm optimisation (BPSO) technique [14], [15] and the artificial immune systems optimisation technique [16] have been utilised to prune the state space. A comparative study of using population-based intelligent search methods in power system reliability, in particular, genetic algorithms, repulsive binary particle swarm optimisation, and binary ant colony optimisation has been presented in [17]. It is worth pointing out here that in the work presented in [11]–[17], population-based intelligent search methods have been utilised as heuristic techniques to reduce the search space where the selection of the acceleration factors has been a key element to achieve the best performance of these methods.

The use of Binary Particle Optimisation (BPSO) search method in power system reliability is found to be an efficient tool in calculating the reliability indices. However, the task of choosing the weighting factors associated with the velocity update rule, to prevent the particles from searching in one direction, tends to be tedious and time consuming as trial and error approaches are often times being used. Consequently, the development of an automated approach to adjust these weighting factors could save some effort in solving the task of reliability evaluation efficiently.

In this work, a state space classification (SSC) technique is used to classify the state space (all possible outcomes of the combination of generation, transmission and load statuses)

into success, failure and unclassified subspaces to reduce the computation effort. Most of the existing state space classification techniques have been based on pruning success subspaces and performing Monte Carlo simulation on the undiscovered subspaces. The proposed technique is different from the existing state space classification techniques in the sense that it classifies the state space into success, failure, and unclassified subspaces; then, BPSO search method is applied to search for failure states in the unclassified subspace. BPSO search method has been used in power system reliability evaluation by summing up the probabilities of the visited states which, in some cases, takes long time to converge. In this work, a new method, which is developed based on the “duality” concept and tracing the trajectories of the reliability indices, is introduced to compensate for the unclassified subspaces. This method is developed based on an intelligent, bounded and directed BPSO. The BPSO based search method dynamically adjusts the weighting factors associated with the velocity update rule of the particles so that the swarm has always forced to fly on the unclassified subspace. An intelligence factor is added to the BPSO that reverses the direction of the particles to search in the unclassified subspace (to prevent the particles from searching in the already classified success and failure subspaces). The proposed algorithm tracks the behaviour of the particles and then adjusts the coefficients of the velocity update rule correspondingly. The behaviour of the swarm can be examined by tracing the trajectories of the reliability indices of the visited states.

In this work, we have evaluated four reliability indices, namely: LOLP, EDNS, LOEE, LOLE. The definitions of these indices are:

A Loss of Load (LOL) event is one in which a system is unable to meet its total demand.

Loss of Load Probability (LOLP) is the probability of encountering one or more LOL events during a given time period.

Expected Demand Not Served (EDNS) is the expected (average) demand that the system is unable to serve as a result of LOL events during a given period. It is expressed in MW/year or GWh/year.

Loss of Energy Expectation (LOEE) is the expected (average) energy that the system is unable to serve as a result of LOL events during a given period. It is expressed in MWh/year or GWh/year.

Loss of Load Expectation (LOLE) is the expected (average) of encountering one or more LOL events during a given time period. It is expressed in hr/year.

The remainder of this paper is organised as follows: Section II presents the modelling of power system networks and power systems reliability evaluation. Section III discusses the development of the proposed state space classification technique. Section IV introduces the proposed dynamically directed particle swarm optimisation search method. Section V shows the solution algorithm. Section VI shows the applications of the proposed methods. Section VII presents and discusses the results of the proposed methods. Section VIII provides concluding remarks.

II. NETWORK MODELLING AND RELIABILITY EVALUATION

In evaluating the reliability indices of composite systems, an optimal power flow with an objective of minimum load curtailment is usually performed. The DC power flow model has been widely utilised in reliability assessment of power systems due to its simplicity of formulation and implementation [1], [4]–[9].

A. Network Modelling

Linear programming with the objective of minimum load curtailment is used in calculating the reliability indices of composite power systems. If under any scenario, the curtailment is unavoidable, the linear programming minimises the amount of load curtailment with satisfying the network constraints. Generally, using DC power flow model, there are three main constraints which are power balance at the buses (equality constraints) and capacity limits of the generators and power carrying capacities of the transmission lines (inequality constraints). The load curtailment minimisation problem can be presented as follows [1]:

$$\text{Loss of Load} = \min \left(\sum_{i=1}^{N_b} C_i \right) \quad (1)$$

subject to

$$\begin{aligned} \hat{B}\delta + G + C &= D \\ G &\leq G^{max} \\ C &\leq D \\ b\hat{A}\delta &\leq F_f^{max} \\ -b\hat{A}\delta &\leq F_r^{max} \\ G, C &\geq 0 \\ \delta &\text{ unrestricted} \end{aligned} \quad (2)$$

In the standard minimisation problem given by (1) and (2), all generation and network constraints have been taken into consideration. Moreover, it has been assumed that one of the bus angles is zero in the constraints (2) to work as a reference bus.

B. Calculation of the Reliability Indices

In this work, we evaluate the well-known composite power system reliability indices, namely LOLP, EDNS, LOEE and LOLE. A stopping criterion has been applied to stop the algorithm from sampling if it converges to the desired accuracy.

1) *The duality Concept:* The concept of duality has been applied in several disciplines to calculate/estimate a parameter from its complementary value. The duality concept in composite power system reliability evaluation can be defined as: *a power system reliability index that can be evaluated from the failure states has a complementary value that can be evaluated from the success states as long as the boundaries of the index and its dual value are known.* For example, the boundaries (minimum and maximum values) of the probability of system failure and the probability of system success is between 0 and 1. Also, the boundaries of expected load curtailment and the expected load supplied is between 0 and the peak load.

2) *Calculation of probability indices:* The estimated value of the LOLP index can be calculated as follows,

$$\text{LOLP} = \sum_{i \in F} P_i(x) \quad (3)$$

The estimated dual value of the LOLP index (DLOLP index) can be expressed as,

$$\text{DLOLP} = 1 - \sum_{i \in S} P_i(x) \quad (4)$$

The loss of load probability is approximated by the trajectory of the crossings between the tangents of the LOLP and DLOLP. The LOLP index is monotonically increasing function and the DLOLP index is monotonically decreasing function and eventually the LOLP and DLOLP indices will reach each other. The loss of load expectation index (LOLE) is the loss of load probability index multiplied by the study period.

3) *Calculation of energy indices:* The estimated value of the EDNS index can be calculated as follows,

$$\text{EDNS} = \sum_{i \in F} P_i(x) \times Lc_i(x) \quad (5)$$

The dual value of the EDNS index is the expected supplied demand. The estimated dual value of the EDNS index (DEDNS) can be expressed as,

$$\text{DEDNS} = \text{Peak Load} - \sum_{i \in S} P_i(x) \times L_i(x) \quad (6)$$

The expected demand not supplied index is approximated by the trajectory of the crossings between the tangents of the EDNS and DEDNS. The EDNS index is monotonically increasing function and the DEDNS index is monotonically decreasing function and eventually the EDNS and DEDNS indices will reach each other. The loss of energy expectation index (LOEE index) is the expected demand not supplied index multiplied by the study period.

4) *A Stopping Criterion:* A convergence criterion should be applied to stop algorithm if there is not much change in the reliability indices. In power system reliability analysis using Monte Carlo simulation, it has been found that energy indices are the slowest indices from convergence view point [25]. In this work, the stopping criterion was applied on the EDNS index for the SSC method, the directed BPSO search method and Monte Carlo simulation (Monte Carlo simulation was used to validate the results and to compare the calculation speeds).

The stopping criterion considering the EDNS index can be expressed as,

$$\sigma = \frac{\sqrt{\text{Var}(\text{EDNS})}}{E[\text{EDNS}]} \quad (7)$$

where $E[\cdot]$ is the expectation operator and $\text{Var}(\cdot)$ is the variance function.

After few iterations, the amount of the change in σ is calculated, if this amount is less than or equal to the specified tolerance, the algorithm is terminated; otherwise, the search continues.

III. STATE SPACE CLASSIFICATION (SSC) TECHNIQUE

Searching for failure states in the entire state space is time consuming; and hence, a heuristic state space classification technique is required to reduce the computation time. A state space classification technique is presented and described in this section. This technique is developed based on calculating the power carrying capabilities of the available transmission lines. This technique models power system networks based on network configuration, available generation, loading conditions and the availabilities and power carrying capabilities of the transmission lines.

A. Failure Subspace

By examining the linear programming problem of (1) and (2), it can be noted that, for any sampled state, if one of the following two conditions is satisfied, the state under consideration is a failure state: (1) the total system generation is less than the total system load and (2) the sum of the capacities of the transmission lines that are connected to a bus is less than the sum of the loads at the bus. Given these two conditions, we can construct a heuristic technique that classifies the state space without performing power flow analysis. The procedures can be described as follows,

- 1) Sample system state using Monte Carlo state sampling method,
- 2) Subtract the loads connected to the generation buses from the sampled amount of generation at the buses,
- 3) Construct a line capacity capability flow matrix (LC-Matrix) for the given sampled configuration. This matrix can be constructed in the same manner as constructing a " Y_{bus} " from the branch-node incidence matrix ($\bar{A}$) except that instead of using "-1" for the branch that is assumed to enter a node, use "1". In this case, the modified incidence matrix is denoted A . Further, instead of using the diagonal susceptance matrix for the case of finding " Y_{bus} ", use a diagonal capacity matrix (K), that is, the diagonal entries of this matrix are the power carrying capacity limits of the transmission lines. The LC-Matrix can be expressed as follows,

$$\text{LC-Matrix} = A^T K A \quad (8)$$

- 4) At any bus, if the total generation is larger than the corresponding diagonal element of the LC-Matrix, adjust the generation at that bus downward to the corresponding diagonal element. This can be justified by the fact that transmission lines are not allowed to carry more than their limits disregarding the amount of the available generation,
- 5) For a state x , if the value of any of the diagonal elements of the LC-Matrix is less than the absolute value of the power injection at the corresponding bus or the total generation is less than the total demand, this state is guaranteed a failure state,
- 6) Check for convergence. If the stopping criterion (section II-B4) has been met, stop; otherwise go to step 1.

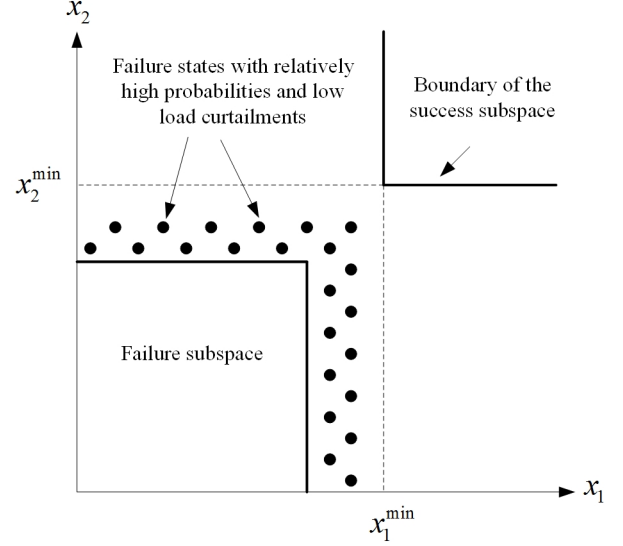


Fig. 1. State space representation of two dimensional system (two components with multiple outcomes)

B. Success Subspace

The state space classification technique described in section III-A can detect the boundary of the failure subspace, but cannot determine the boundary of the success subspace. However, since the reliability indices are evaluated based on the failure states, analysis of the entire success subspace is not important. A large portion of the success subspace can be truncated by considering all the transmission lines in the up state and determining the vector of minimum generation $((x_1^{\min}, x_2^{\min}))$ of Fig. 1 that satisfies the load by solving the problem of (1) and (2) for this scenario. Any state in the state space that has a generation vector larger than or equal to the minimum generation vector (the boundary of the success subspace) is guaranteed to be a success state.

C. State Space Description

The state space classification method described in section III-A determines the failure subspace and the state space classification method described in section III-B determines the boundary of the success subspace. The remaining subspaces are termed unclassified subspaces. Also, most of the unclassified failure states lay around the boundary of the failure subspace. Therefore, an intelligent search method that is bounded between the boundaries of the success and failure subspaces (the unclassified subspaces) can efficiently converge to the failure states. In this paper, a method based on directed BPSO is used to search for these states. The boundaries of the failure and success subspaces are used to control the behaviour of the swarm as shown in section IV. Fig. 1 shows the state space classification of a two dimension hypothetical system.

IV. DYNAMICALLY-DIRECTED BINARY PARTICLE SWARM OPTIMISATION SEARCH METHOD

In performing the state space classification technique, some failure states, which represent subsets of the unclassified

subspaces, may not be captured. In this context, two failure scenarios may exist and cannot be classified by the state space classification technique as failure states. In the first scenario, an area with loads only is either isolated from the generation areas or connected to the rest of the system through a tie-line that cannot withstand the total load of this area but the transmission lines connecting the loads of this area are sufficient to carry the loads within the area. In the second scenario, line impedances force the power to flow through some capacity limited lines and the linear programming optimisation problem will not converge without load curtailment. These scenarios cannot be detected by considering the power carrying capabilities of the transmission lines. Therefore, an intelligent search tool should be used to capture these events. The search space is bounded by the classified failure and success subspaces, which makes an intelligent search method to converge to these states with lower computation effort. In this work, BPSO search method is used to search for the failure states in the unclassified subspace.

A. Binary Particle Swarm Optimisation (BPSO)

Particle swarm optimisation (PSO) is a population-based intelligent search method, which has been proposed by Kennedy and Eberhart in [18]. Later, Kennedy and Eberhart proposed a discrete binary particle swarm optimisation to solve combinatorial optimisation problems [19]. PSO has been proven to be an effective optimisation technique and has, therefore, been used in the presented work. In this paper, BPSO is used to search for success and failure states rather than finding an optimum solution. In other words, BPSO searches for the states that have not been captured by the state space classification technique. The status of the generators and transmission lines are assumed to fall in two states (up states and down states) so that up states are represented by 1's and down states are represented by 0's. Moreover, the velocity update rule of the swarm is updated using two conflicting forces which are the probability of the states and the amount of the load curtailments. These two opposing forces are used to direct particles to search in a specific region of the state space. It is worth noting here that the size of the state space is 2^n , where n is the number of system components. Therefore, even for medium size systems, the number of states that may not be discovered by the state space classification technique (unclassified subspaces) could be very large and it is impractical to test the entire unclassified subspaces. Hence, a duality concept to compensate for the unclassified subspace is developed in this paper so that the necessity of searching for all failure states over the entire unclassified subspaces is eliminated.

BPSO based search technique searches through the state space and tests the visited states for a possibility of load curtailment by solving the linear programming problem. In performing the search process, for every iteration, particle velocity v_i or the direction of movement of particle i from position x_i can be governed by the following velocity update

rule [8], [9], [19]:

$$\begin{aligned} v_i^{k+1} = v_i^k &+ rand() \times c_1 \times (Ppbest_i - x_i^k) \\ &+ rand() \times c_2 \times (Pgbest - x_i^k) \\ &+ rand() \times c_3 \times (Cpbest_i - x_i^k) \\ &+ rand() \times c_4 \times (Cgbest - x_i^k) \end{aligned} \quad (9)$$

where k is the generation number, $rand()$ is a uniformly distributed random number between [0,1].

The best positions of particles and a group of particles from the probability point of view ($Ppbest$ and $Pgbest$) are the positions that have the highest probabilities. Also, the best positions of particles and a group of particles from the load curtailment point of view ($Cpbest$ and $Cgbest$) are the positions that have the highest load curtailment values.

The change in the positions of the particles can be defined by a sigmoid limiting transformation function and a uniformly distributed random number in [0,1] as following [8], [9], [14],

$$x_{id} = \begin{cases} 1, & rand() < S(v_{id}) \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

The sigmoid function of d^{th} 's component of particle i can be expressed as follows,

$$S(v_{id}) = \frac{1}{1 + e^{(-v_{id})}} \quad (11)$$

The probability of exploring new particles is limited by V_{max} which limits the ultimate probability of a bit in particle i to be zero or one. Choosing very large or very small values for V_{max} can limit the chance of exploring new vectors of particles. Therefore, a careful selection of V_{max} should be performed. In this work, V_{max} is adjusted according to the behaviour of the particles as described in the solution algorithm.

B. The Proposed Power System Reliability Evaluation Using BPSO

Population-based intelligent search methods have been amply used in power system reliability studies. Particle swarm optimisation has been used as a searching tool for success or failure states in power system reliability evaluation in [11]–[14], [16], [17], [20]–[24]. The velocity update rule of the BPSO can be single objective or multi-objective. In [11]–[14], [16], [17] a single objective function has been used, which is the minimum load curtailment, to prune the success states from the state space. The use of multi-objective BPSO as a searching tool for failure states in composite system reliability evaluation has been proposed in [8], [9], [22]–[24]. One objective was used to maximise load curtailments while the other objective was used to maximise probabilities of the visited states. These two conflicting objective (forces) have been used to force the particles to search on entire state space. However, using two conflicting objective functions may not guarantee that the particles will not be trapped to one corner of the search space if the weighting factors are not chosen carefully. To circumvent this difficulty, an effective approach

that adjusts these weighting factors is presented in section IV-C.

One of the differences between using BPSO in power system reliability evaluation and Monte Carlo simulation is that the former evaluates the reliability indices by summing up the probabilities of the visited states, while in the latter, the reliability indices are evaluated by dividing the number of encountered failure states to the number of samples. It is well-known that Monte Carlo simulation is extremely time consuming in calculating the reliability indices for an acceptable accuracy. However, in performing Monte Carlo simulation, no need to visit all system states since it calculates the weighted values not the exact values. On the other hand, to calculate the reliability indices using BPSO, all failure states or success states have to be visited and tested, which is computationally expensive, if not impossible for large size systems. Therefore, another technique to overcome this drawback is necessary. New indices have been introduced in [8], [9] which are normalised loss of load probability index, $LOLP_{norm}$, and projected loss of load probability index, ($LOLP_{proj}$) to eliminate the necessity of visiting all the failure and success states. The definitions of these two indices are given in the following section.

1) *Calculation of the $LOLP_{norm}$ Index:* The definition of the $LOLP_{proj}$ and $LOLP_{norm}$ indices can be related to the dual value of the LOLP index (DLOLP). The $LOLP_{norm}$ index is the ratio between the sum of the probabilities of the failure states (or the LOLP index) to the sum of probabilities of the states encountered so far or $(1 + LOLP - DLOLP)$. The normalised LOLP ($LOLP_{norm}$) can be expressed as follows,

$$LOLP_{norm} = \frac{LOLP}{1 + LOLP - DLOLP} \quad (12)$$

2) *Calculation of the $LOLP_{proj}$ Index:* The $LOLP_{proj}$ index is the crossings between the tangents of the LOLP and DLOLP indices. It was found that this index converges faster than the $LOLP_{norm}$ index since it tracks the trajectories of the indices [9]. The work presented in [9] does not provide an explanation for the calculation of the $LOLP_{proj}$ index. For convenience, we provide a typical representation of the this index and develop an expression to determine its value.

The projected LOLP ($LOLP_{proj}$) is depicted in Fig. 2. Suppose that the $LOLP_{proj}$ index is updated every Δx iterations (e.g. every 100 iterations) where $x^{k+1} = x^k + \Delta x$. Then, the $LOLP_{proj}$ index can be calculated from the crossings of the tangents of the LOLP and DLOLP indices as follows,

$$LOLP_{proj} = LOLP|_{x^k} + \left(\frac{\Delta LOLP}{\Delta LOLP - \Delta DLOLP} \right) (LOLP|_{x^k} - DLOLP|_{x^k}) \quad (13)$$

where $LOLP|_{x^k}$ and $DLOLP|_{x^k}$ are the values of the LOLP and DLOLP indices at x^k respectively, $\Delta LOLP$ is the change in the LOLP index ($\Delta LOLP = LOLP|_{x^{k+1}} - LOLP|_{x^k}$) and $\Delta DLOLP$ is the change in the DLOLP index ($\Delta DLOLP = DLOLP|_{x^{k+1}} - DLOLP|_{x^k}$).

3) *Convergence of the $LOLP_{proj}$ Index:* The $LOLP_{norm}$ and $LOLP_{proj}$ indices are the same as the weighted LOLP index used in Monte Carlo simulation. However, $LOLP_{proj}$ converges to the approximate value of the LOLP faster than

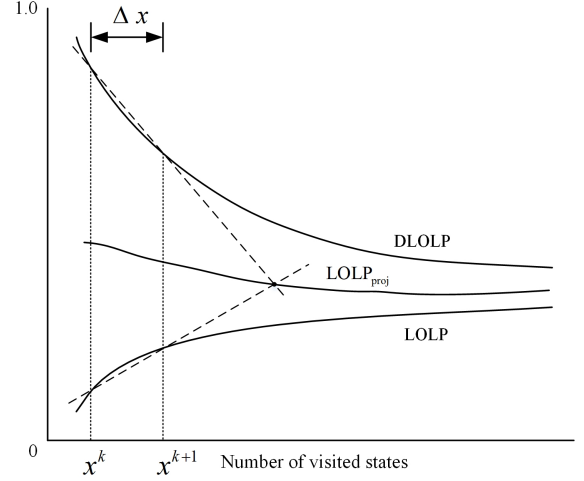


Fig. 2. Typical representation of the profile of the LOLP, DLOLP and $LOLP_{proj}$ indices

$LOLP_{norm}$ [9]. It can be noted that even though BPSO and Monte Carlo simulation both evaluate the weighted index, the number of states to be visited using BPSO is significantly less than the number of states to be visited using MCS [9]. However, the method presented in [9] is inapplicable to calculate the other reliability indices such as the energy and frequency and duration indices. Also, the method presented in [8], [9] has been applied on the entire state space and cannot be applied on a specific subspaces without introducing intelligent factors. A directed BPSO based search method is explained in section IV-C.

C. The Proposed Directed BPSO

By examining the state space, the probabilities of the success states are larger than the probabilities of the failure states due to the fact that power system components have very large probability of availability. Further, the amount of load curtailment increases as the number of failed components increases. If the swarm were to search in the subspaces that are dominated by success states, it would unlikely to encounter failure states and the opposite is true for the subspaces that are dominated by failure states. As mentioned previously, using two conflicting functions does not guarantee that the particles will search in the desired search space unless the weighting factors that are associated with the velocity update rule are chosen carefully. Fig. 3 and Fig. 4 show two situations in which the weighting factors of the two conflicting functions of the velocity update rule were not chosen correctly. Fig. 3-(a) and Fig. 4-(a) show the behaviour of the particles and Fig. 3-(b) and Fig. 4-(b) show the expected profile of the reliability indices. In Fig. 3, the swarm is trapped to the upper corner of the state space which is dominated by success states. In this case, the normalised LOLP, ($LOLP_{norm}$) as well as the projected LOLP, ($LOLP_{proj}$), will be under estimated. As the particles continue to search in this region, most of the visited states will be success states and more likely no failure states will be discovered. Therefore, no much change will be in the LOLP and the $1-DLOLP$ will be sharply increasing. On the

other hand, if the swarm were trapped into the lower corner which is dominated by failure states as shown in Fig. 4, most of the visited states will be failure states and it would unlikely to encounter success states. In this case, LOLP, ($LOLP_{norm}$) as well as the projected LOLP, ($LOLP_{proj}$), will be over estimated. The LOLP index will continue to increase whereas the 1-DLOLP index will remain almost constant.

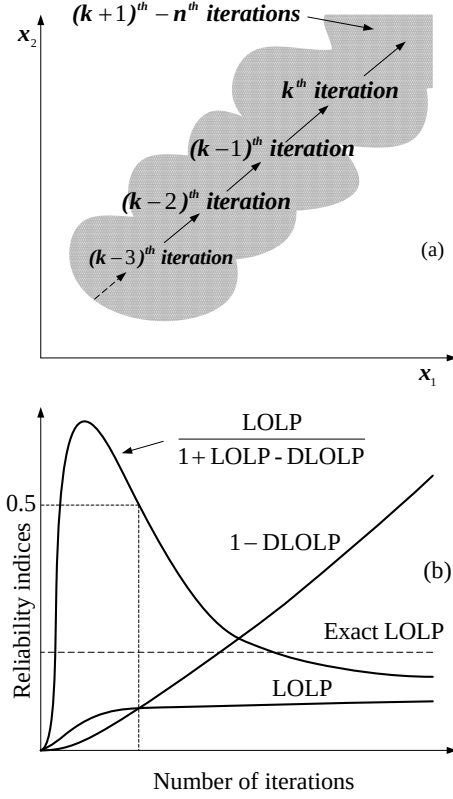


Fig. 3. Reliability indices profile in case the particles are trapped to the upper corner of the state space

To control the BPSO to search in the unclassified subspace and to overcome the difficulty of choosing the weighting factors of the velocity update rule as well as the change in probability limit, we propose a technique that traces the trajectories of both the normalised LOLP and the number of the new discovered states (the mutation rate). After performing few iterations, if $LOLP_{norm}$ continues to increase, then the particles were trapped in a region where failure states are dominant. On the other hand, if $LOLP_{norm}$ continues to decrease, then the particles were trapped in a region where success states are dominant. Also, if the number of new discovered states is very small and the designated reliability index does not converge to the specified accuracy, then V_{max} is limiting the probability of bit change. Also, if the algorithm discovers that particles have entered the already classified success or failure subspaces, the weighting factors of the velocity update rule are adjusted accordingly as shown in Fig. 5. The procedures of evaluating the reliability indices and preventing the particles from searching in the already classified subspaces are described in detail in the solution algorithm.

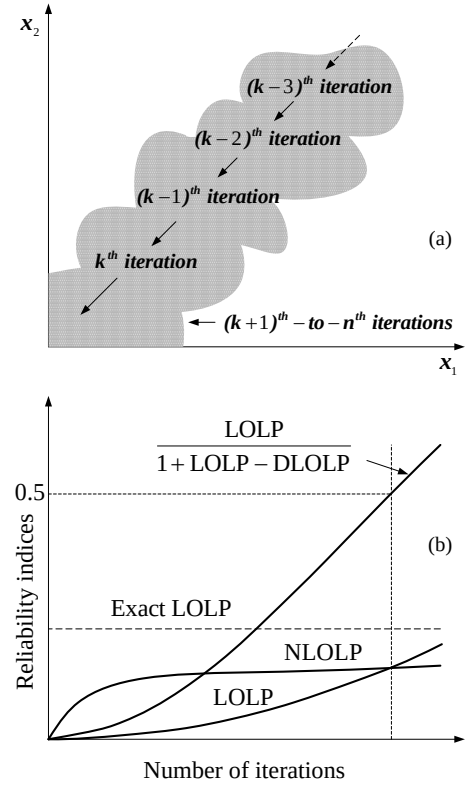


Fig. 4. Reliability indices profile in case the particles are trapped to the lower corner of the state space

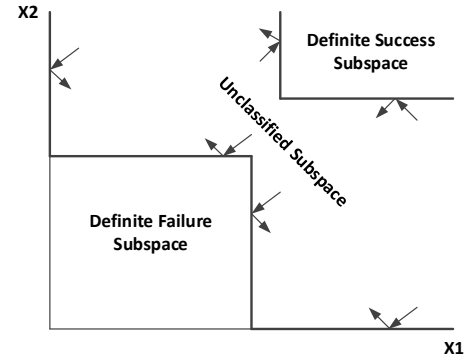


Fig. 5. Redirecting the swarm in case it enters the success or failure subspaces

V. THE PROPOSED SOLUTION ALGORITHM FOR POWER SYSTEM RELIABILITY EVALUATION

The solution algorithm explains the flow of the procedures of evaluating the reliability indices as well as the technique that is used to direct and prevent the particles from searching in the classified subspaces. The steps are explained as follows:

- 1) Perform the state space classification technique (sections III-A and III-B) to determine: (1) the boundaries of the success and failure subspaces and (2) determine the values of the reliability indices from the failure subspace as described in section II-B.
- 2) Initialise the positions and velocities of the particles, x_i

and v_i respectively. Positions of particles are initialised by using the forced outage rates of system components; components that are in the up state are represented by 1's and components in the down state are represented in 0's. The length of a particle string equals the number of components.

- 3) Check if there are identical particles; if so, discard the identical ones and save the rest of the particles in a temporary array vector by converting the binary numbers into decimal numbers. The use of the decimal numbers instead of binary numbers reduces the memory requirement.
- 4) Check if there are particles that already exist in the database; if so, set probabilities and load curtailments of the existing particles to zeros to decrease the chance of revisiting these states. Save the rest in the database and go to the next step.
- 5) Compute the exact probability of each particle which represents the probability of a system state. If the probability of a particle is less than a threshold ε discard this particle, otherwise go to the next step. In this work, ε is set to 10^{-15} .
- 6) Set system parameters and update the status of the generators and transmission lines for every particle. Solve the linear programming optimisation problem to check if there are load curtailments. Update the reliability indices.
- 7) Determine and update personal and global best positions from the probability and load curtailment of states perspectives. Update the velocities of the particles using (9) and update the positions of the particles using (10) and (11).
- 8) Check the normalised LOLP ($LOLP_{norm}$) whether it is increasing or decreasing and adjust c_1 , c_2 , c_3 and c_4 accordingly. If it is increasing, increase the values of c_1 and c_2 and decrease the values of c_3 and c_4 . Perform the opposite if $LOLP_{norm}$ is decreasing. In this work, the amount of increase or decrease of these weighting factors is adjusted according to the amount of change of the $LOLP_{norm}$ index. Any strategy can be used to adjust these factors. However, the optimum adjustment of these factors is out of the scope of this work. The amount of change of these factors is related to the amount of change of the $LOLP_{norm}$ index ($\Delta LOLP_{norm}$). Since the changes in the $LOLP_{norm}$ index have large values at the beginning of the simulation and then decrease exponentially as shown in Fig. 6, the change in the weighting factors can be expressed in an exponential form as follows,

$$c_{(1,2)-new} = c_{(1,2)-old} \times e^{(100 \times \Delta LOLP_{norm})} \quad (14)$$

$$c_{(3,4)-new} = c_{(3,4)-old} \times e^{(-100 \times \Delta LOLP_{norm})} \quad (15)$$

- 9) Check if there are particles that have entered the success and/or failure subspaces. If a particle has entered the success subspace, redirect the particle by maximising the load curtailment term in the velocity update rule (ignoring the maximum probability term) by setting c_1 and c_2

to zeros. If a particle has entered the failure subspace, redirect the particle by maximising the probability term in the velocity update rule (ignoring the maximum load curtailment term) by setting c_3 and c_4 to zeros.

- 10) Check for convergence. If the stopping criterion has been met, stop; otherwise go to the next step.
- 11) Determine the number of newly discovered states since the last few iterations. If the number of the new discovered states is less than the threshold, adjust V_{max} and then go to step 2; otherwise go directly to step 2. The threshold is chosen to be 80% of the total number of particles for the first 10 iterations, 50% for the next 10 iterations and 20% thereafter. V_{max} is adjusted according to the direction of the $LOLP_{norm}$ index. If the particles are searching around the success subspace, adjust V_{max} to a large number. On the other hand, if the particles are searching around the failure subspace, adjust V_{max} to a small number. In this work, the large and small numbers are chosen as 4.0 and 2.0, respectively. Again, the optimum value of V_{max} to better improve the behavior of the search method is out of the scope of this work.

VI. CASE STUDIES

The proposed method was applied on three test systems, the IEEE RTS [26], the modified version of the IEEE RTS and the Saskatchewan Power Corporation (SPC) in Canada [27]. The reason of choosing these systems is because these systems have different loading levels, line capacities, sizes and components availabilities. For instance, the available capacity margin of the IEEE RTS is less than that of the SPC which is expected to make it more stressful than the SPC system. Moreover, the annualised indices of the IEEE RTS at the mean load are zeros whereas for the SPC mean loads causes some load curtailments. In terms of availability, the probability of all components are being in the up state of the IEEE RTS and the Modified IEEE RTS is 0.230441 and for the SPC system is 0.443064.

A. IEEE RTS

The IEEE RTS System has been extensively used for proving of concepts of power system reliability studies. The IEEE RTS consists of 24 buses, 38 transmission lines/transformers (33 transmission lines and 5 transformers) and 32 generating units on 10 buses. The total generation of this system is 3405 MW and total peak load is 2850 MW. System reliability parameters and system data including generation and transmission limits and load profile are given in [26].

B. Modified IEEE RTS

The modified IEEE RTS System is the same as the original IEEE RTS except that the generation is doubled and the loads are multiplied by a factor of 1.8. The reason of this modification is because the transmission lines of the original system have high power carrying capabilities in comparison with the generation and loading levels. Therefore, this modification will make the transmission lines more stressed. Also, this modification will help in evaluating the proposed method.

C. The SPC System

The SPC system is the network of the Saskatchewan Power Corporation in Canada [27]. The system consists of 45 buses, 29 generating units on 8 buses and 71 transmission lines/transformers. Four of the 45 buses are used to represent equivalent assistance from the Manitoba Hydro System. One of these four buses is a fictitious bus that represents the power import from Manitoba Hydro System which is 300 MW. This bus is connected to the other three buses and the sum of the 300 MW is represented by three generating units each of which 100 MW. The total generation is 2530 MW and total load is 1802.5 MW. System data are given in [27].

D. BPSO Parameters

Selection of the parameters of the binary particle swarm optimisation method in power system reliability studies depends on several factors such as the size of the system, the search space, component probabilities, etc. However, the BPSO used in this paper is not a classical PSO since it is controlled and bounded BPSO. In this work, all the parameters are dynamically adjusted as the simulation progresses except the number of particles. The weighting factors (c_1 , c_2 , c_3 and c_4) and probability limit (V_{max}) associated with velocity update rule are initialised with some values and during the simulation these values are dynamically adjusted as shown in the solution algorithm (section V). The number of particles used with the BPSO search method in this work was selected based on several previous studies. It was found that selecting large number of particles increases the number of repeated states and slows down the calculation, and selecting small number of particles limits the discovery of new states. The values of these parameters used in this work are given as follows: (1) the weighting factors c_1 , c_2 , c_3 and c_4 were initialised by having a flat value of 2.0, (2) velocities were initialised by setting V_{max} to 2.2 (probability limits are between 0.9 and 0.1) and (3) the number of particles were chosen 50 particles.

VII. RESULTS AND DISCUSSION

The proposed method was applied on the previously mentioned test systems. Evaluation of the annual as well as the annualised (assuming peak load) indices was conducted on the three test systems. The first stage of the analyses is to apply the proposed state space classification technique to classify the state space into success, failure and unclassified subspaces. The second stage is to apply the proposed dynamically directed BPSO search method on the unclassified subspaces if there is any.

Table , Table I, Table II and Table III show the annual and the annualised indices of the IEEE RTS and the modified IEEE RTS, respectively. Table IV shows the annualised indices of the SPC system. The annual indices of the SPC system are not provided because they have very small values. From these tables, it has been found that in calculating the annual indices of the IEEE RTS and the modified IEEE RTS (Table and Table II) and the annualised indices of the original IEEE RTS (Table I), the state space classification technique was able to classify the state space into failure

TABLE
ANNUAL INDICES OF THE IEEE RTS

Algorithm		LOLP	EDNS MW/yr	LOEE MWh/yr	LOLE h/yr
Proposed	SSC*	0.00117	0.16374	1430.465	10.22112
	BPSO**	—	—	—	—
	Sum	0.00117	0.16374	1430.465	10.22112
Monte Carlo		0.00119	0.16412	1433.752	10.39584

*: state space classification; **: binary particle swarm optimisation

and success subspaces and hence no BPSO was performed (terminated after few iterations). For these cases, the reliability indices are determined from the classified failure subspace. On the other hand, in calculating the annualised indices of the modified IEEE RTS (Table III) and the SPC system (Table IV), the state space classification technique was not able to classify the state space into only failure and success subspaces rather it produced unclassified subspaces. Therefore, the proposed dynamically directed BPSO search method was used to estimate the reliability indices from the unclassified subspaces. The final values of the reliability indices for these cases are the sum of the values of the indices that were calculated using the state space classification technique (from the failure subspace) and the values that were calculated using the proposed BPSO (unclassified subspaces). To validate the results of the proposed method, Monte Carlo simulation was used to calculate the same indices and the results are shown in the last rows of Table , Table I, Table II, Table III and Table IV.

TABLE I
ANNUALISED INDICES OF THE IEEE RTS

Algorithm		LOLP	EDNS MW/yr	LOEE MWh/yr	LOLE h/yr
Proposed	SSC*	0.08429	14.67784	128225.60	736.360
	BPSO**	—	—	—	—
	Sum	0.08429	14.67784	128225.60	736.360
Monte Carlo		0.08451	14.59761	127524.72	738.279

*: state space classification; **: binary particle swarm optimisation

TABLE II
ANNUAL INDICES OF THE MODIFIED IEEE RTS

Algorithm		LOLP	EDNS MW/yr	LOEE MWh/yr	LOLE h/yr
Proposed	SSC*	0.00106	0.07395	645.90125	9.26016
	BPSO**	—	—	—	—
	Sum	0.00106	0.07395	645.90125	9.26016
Monte Carlo		0.00105	0.07436	649.61330	9.17280

*: state space classification; **: binary particle swarm optimisation

Table V, Table VI and Table VII show the reduction in the computation time of the three test systems respectively. The calculation spent by the proposed algorithm is represented as a percentage of the time of evaluating the same indices using Monte Carlo simulation. The average base time using Monte

TABLE III
ANNUALISED INDICES OF THE MODIFIED IEEE RTS

Algorithm		LOLP	EDNS MW/yr	LOEE MWh/yr	LOLE h/yr
Proposed	SSC*	0.06039	10.31755	90134.083	517.259
	BPSO**	0.00976	0.12171	1063.256	85.263
	Sum	0.07015	10.43926	91197.375	612.830
Monte Carlo		0.07017	10.44237	91224.549	613.005

*: state space classification; **: binary particle swarm optimisation

TABLE IV
ANNUALISED INDICES OF THE SPC SYSTEM

Algorithm		LOLP	EDNS MW/yr	LOEE MWh/yr	LOLE h/yr
Proposed	SSC*	0.000535	0.0364205	319.0436	4.6866
	BPSO**	0.000555	0.0026805	23.4812	4.8618
	Sum	0.00109	0.039101	342.5248	9.5484
Monte Carlo		0.00108	0.039098	341.5600	9.4436

*: state space classification; **: binary particle swarm optimisation

TABLE V
REDUCTION IN THE COMPUTATIONAL TIME OF THE IEEE RTS

Category	Time Spent by		Total Time (seconds)	Time Reduction (%)
	SSC* (seconds)	BPSO** (seconds)		
Annual	28.66	0	28.66	97.56
Annualised	144.28	0	144.28	87.72

*: state space classification; **: binary particle swarm optimisation

TABLE VI
REDUCTION IN THE COMPUTATIONAL TIME OF THE MODIFIED IEEE RTS

Category	Time Spent by		Total Time (seconds)	Time Reduction (%)
	SSC* (seconds)	BPSO** (seconds)		
Annual	55.60	0	55.60	95.27
Annualised	336.15	409.75	745.90	36.52

*: state space classification; **: binary particle swarm optimisation

TABLE VII
REDUCTION IN THE COMPUTATIONAL TIME OF THE SPC SYSTEM

Category	Time Spent by		Total Time (seconds)	Time Reduction (%)
	SSC* (seconds)	BPSO** (seconds)		
Annual	—	—	—	—
Annualised	287.50	721.50	1009.0	30.41

*: state space classification; **: binary particle swarm optimisation

Carlo simulation was found 1175 seconds for the IEEE RTS and the modified IEEE RTS and 1450 seconds for the SPC system.

In evaluating the annual and annualised indices of the IEEE RTS (Table I and Table II) and the annual indices of modified IEEE RTS (Table I), the state space classification technique was able to classify the entire state space into success and failure subspaces (no unclassified subspaces). This can be

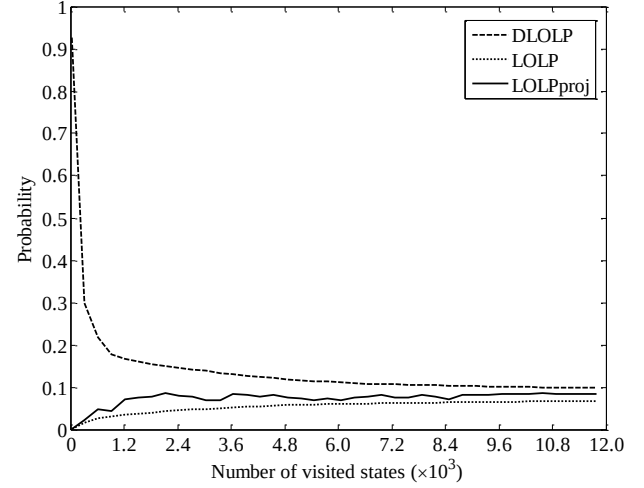


Fig. 6. Profiles of the reliability indices of the IEEE RTS

attributed to the following: (1) the loading level of IEEE RTS is very low in comparison with the capacity of the transmission lines and (2) the transmission lines are very reliable. In other words, the two scenarios that mentioned in section IV are very unlikely to occur. Therefore, for these cases, the state space classification technique produces very accurate results.

In evaluating the annualised indices of the modified IEEE RTS (Table III) and the SPC system (Table IV), some failure states were not discovered by the state space classification technique. From Table III and Table IV, the unclassified subspaces have significant effects on the LOLP and LOLE indices and very small effects on the EDNS and LOEE indices. This can be seen by comparing the values in the second, third and fifth rows of Table III and Table IV. These effects can be interpreted as following: (1) the unclassified subspaces have failure states with relatively high probability but with relatively small load curtailments and (2) failure states that are located far from the origin (all components are in the down state) have high probability and small load curtailments. Therefore, if only the EDNS and LOEE indices are important to the planner/operator, the calculated values of these indices using the state space classification technique would be enough for these cases.

Fig. 6 shows the profiles of the LOLP, DLOLP and the projected LOLP indices of the IEEE RTS as the simulation progresses. As the number of visited states increases, the estimated LOLP and the estimated DLOLP becomes close to each other. If the simulation were to continue to make the particles visit the entire state space which is impractical, these two indices (LOLP and DLOLP) would converge to the same value. Also, Fig. 6 shows some changes in the LOLP_{proj} index specially at the early stages of the searching process. These changes are due to the adjustments of the weighting factors of the velocity update rule as well as the adjustments in the velocity limit, V_{max} .

VIII. CONCLUSION

This paper has proposed a state space classification technique in conjunction with a dynamically directed Particle Swarm Optimisation search method. The state space classification technique is developed based on evaluating the adequacy of the available power generation and the power carrying capabilities of transmission lines to meet the load to classify the state space into success, failure and unclassified subspaces. A dynamically directed binary particle swarm optimisation (BPSO) search method is used to search for the failure states in the unclassified subspace. The effectiveness of the proposed method was demonstrated on the IEEE RTS, the modified IEEE RTS and the Saskatchewan Power Corporation (SPC) in Canada. The state space classification technique has resulted in reducing the computation burden significantly and in some cases it was able to classify the entire state space into success and failure subspaces. In some other cases, the proposed dynamically directed binary particle swarm optimisation has efficiently searched for failure states in the unclassified subspaces. Also, the reliability indices obtained by the proposed method correspond closely with those obtained using Monte Carlo simulation, while requiring lower computational burden.

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