

An Emission-Constrained Approach to Power System Expansion Planning

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Abstract—In view of recent regulations and amendments of emission reduction, electric utility industries have been required to reduce, monitor and control emissions from fossil fuelled generating units. This paper presents a method of power system expansion planning based on the sensitivity analysis, considering the constraints of CO_2 emissions. The method presented here uses a linearised power flow representation in the form of a linear programming (LP) model similar to those frequently used in power system studies to minimise the total operation cost and is applied to both generation and transmission expansion planning. The cost of the fuel, the cost/benefit of purchasing/selling emission allowances are combined to develop a piecewise linear objective function. This objective function is used to calculate the sensitivity of the operation with respect to emission limits. This work also utilises the concept of the shadow price to perform sensitivity analysis of the objective function value with respect to the operation constraints to identify the best cost effective generation and transmission expansion planning. The dual solution of the LP provides the shadow prices that are used for determining the sensitivity of the minimum cost with respect to power generation. Also, an explicit development of the linearised power flow is provided. The method is demonstrated on the IEEE Reliability Test System (IEEE-RTS).

Index Terms—Key words: Emission limits, allowances, sensitivity, linearised power flow, shadow price, dual solution.

NOMENCLATURE

HR_i	heat rate of the generating unit i ,
k_i, ℓ_i, m_i	coefficients of the heat rate of unit i ,
$P_i(t)$	real output power of the generating unit i at time t ,
Ea_i, Eb_i, Ec_i	coefficients of CO_2 emission rates,
a_i, b_i, c_i	CO_2 emission-cost coefficients of generating unit i ,
A_i, B_i, C_i	fuel-cost coefficients of generating unit i ,
$\alpha_i, \beta_i, \gamma_i$	coefficients of the combined cost function of generating unit i ,
P_k, Q_k	real and reactive power injected at bus k ,
V_k	voltage magnitude at bus k ,
G_{km}	conductance between buses k and m ,
B_{km}	susceptance between buses k and m ,
δ_{km}	angle difference between voltages of buses k and m ,
N_g	number of generators,
N_a	number of buses,
N_t	number of transmission lines,
B, G	susceptance and conductance sub matrices of Y_{bus} , $(N_a \times N_a)$,
b_{kk}	total susceptances of the shunt elements connected at bus k ,

g_{kk}	total conductances of the shunt elements connected at bus k ,
δ	vector of bus voltage angles, $(N_a \times 1)$,
V	vector of bus voltage magnitudes, $(N_a \times 1)$,
P_G, Q_G	vectors of real and reactive power of generation, $(N_g \times 1)$,
P_D, Q_D	vectors of real and reactive power load, $(N_a \times 1)$,
P_{loss}, Q_{loss}	vectors of real and reactive power losses, $(N_g \times 1)$,
P_G^{max}	vector of maximum available real power generation, $(N_g \times 1)$,
P_G^{min}	vector of minimum available real power generation, $(N_g \times 1)$,
Q_G^{max}	vector of maximum available reactive power generation, $(N_a \times 1)$,
Q_G^{min}	vector of minimum available reactive power generation, $(N_a \times 1)$,
V^{max}	vector of maximum allowable voltages, $(N_a \times 1)$,
V^{min}	vector of minimum allowable voltages, $(N_a \times 1)$,
b	a diagonal matrix of the transmission line admittances, $(N_t \times N_t)$,
F_f^{max}, F_r^{max}	vectors of forward and reverse flow capacities of the lines, $(N_t \times 1)$,
A	element-node incidence matrix, $(N_t \times N_a)$,
ECO_{2j}	total emission of CO_2 in tonne/t at power plant j ,
ECO_{2j}^{max}	cap on the CO_2 emission at power plant j ,
OC	operation cost,
C_i	the capacity of component i ,
$F(x)$	total generation cost when the system is at state x ,
X	set of all possible states,
E_i	emission from generating unit i ,
B', G'	susceptance and conductance sub matrices of Y_{bus} without including the susceptances and conductances of the lines, $(N_a \times N_a)$,
$\partial F(x) / \partial C_i$	shadow price of the objective function with respect to the capacity limits of the generating unit i , (π_{pgi}) ,
$\partial F(x) / \partial E_i$	shadow price of the objective function with respect to the emission limits of the generating unit i , (π_{Ei}) .

I. INTRODUCTION

Due to global warming and climate change, several acts, protocols and regulations have been introduced to reduce emissions. These acts and protocols limit the amount of the Carbon Oxides (CO_x), Sulfur Oxides (SO_x) and Nitrogen Oxides (NO_x), impose emission reduction laws and allow emission allowances trading (the action of buying or selling the authorised fixed amount of a pollutant on the open market). For instance, one of the main outcomes of these acts and protocols is emission trading in the free market. This trading makes the exchange of emission allowances more flexible and minimises the overall operational costs [1].

In view of these acts and protocols, there is an evolving body of research on minimum cost and minimum emission methods that are cognisant of these emerging factors. Several emission reduction auctions have been taking place. For instance, the first auction and trading on greenhouse emission allowances of California was on November 14, 2012 [2]. More than 8% reduction in the greenhouse gas emissions between 2008 and 2012 was achieved in the European Union Emissions Trading System and it is expected to reduce greenhouse gas emissions below 20% of the 1990 levels by 2020, starting from 2013 [3].

While electric utility industries have been focusing on the potential use of the clean energy sources such as wind and solar, in most industrialised countries the bulk of electric generation comes from fossil fuelled power plants. According to the Environmental Protection Agency (EPA), in 2012, the CO_2 emissions from combustion of fossil fuels account for about 38% of total CO_2 emissions in the U.S.A. [4]. In the UE-27 (the European Union (EU) of 27 Member States from January 1, 2007 - June 30, 2013), 40% of energy resources are used in generating electricity [5]. In 2008, 601.32 GW was generated from coal in China which is about 75.87% of the total electric energy generated [6].

Several methods have been proposed to reduce emissions from power generating units such as installing post-combustion cleaning equipment and carbon capturing [7], [8], switching to fuels with less emissions [9], increasing the penetration of the renewable energy [10], [11], and modifying existing dispatch strategies to include emissions.

A considerable amount of research has been conducted on economic/emission dispatching strategies. Solving economic-emission load dispatch with line capacity constraints was proposed in [12]. Subsequently, considerable research were reported on variations and extensions of these methods [13]–[20]. These variations use different means of accommodating emissions within the optimisation problem. Considering the effect of emission constraints on the reliability of composite power systems were introduced in [21], [22]. A considerable amount of research has focused on evolutionary techniques and multi-objective minimisation functions to minimise emissions and generation costs in generation expansion planning [14]–[17], [23]. Some of the proposed formulations also take into account the different emission reduction methods described above, and thereby provide for strategies that may allow utilities to forgo, defer, or minimise additional capital costs.

For example, [24] shows how a utility may avoid installation of new emission equipment by changing its commitment and dispatch schedules, or by switching to fuels with low emission (and high cost), or both. Planning decisions may comprise a trade-off between the cost of buying extra emission allowances or switching to fuels with low emission (and high cost), or both. Therefore, there is an emerging need to include emission considerations in operation and planning procedures. The work reported in [25] introduced a hybrid generation and transmission expansion planning method that considers the emission and reliability of power systems.

The work reported in this paper responds to the need of reducing emissions from power generating units including emission caps in the power system expansion planning. This paper proposes a method based on combining the economic-emission dispatch considering emission constraints and the cost of emission caps to perform sensitivity analysis that can be used in both planning and operation studies. The results of the sensitivity analysis can be used to determine marginal prices of the addition of new components. This work uses the shadow price concept to determine the sensitivity of the combined cost function with respect to the operation limits. Two sensitivity indices are proposed to provide a measure for the change of the objective function value with respect to the operation and emission limits. The uncertainty of the emission prices is also accommodated in performing the sensitivity analyses. Also, this paper uses a linearised load flow model that closely depicts the properties of the full AC load flow without iterations which was developed by the authors. This load flow model is incorporated with the linear programming optimisation problem to find the minimum operating cost. In addition to power balance and flow constraints normally used, this work introduces a suitably formulated set of emission constraints to initiate the action of selling or buying emission allowances. If the emission limit is reached, the planner should choose between buying emission allowances and investing in another power station that has available emission allowances. The proposed method is intended to identify the most suitable options to minimise the total operating cost. The method is applicable to any source that produces undesirable by-products that are capped by regulatory and other policies, as long as these by-products are a function of the output power. Also, this method can be extended to accommodate other limits such as limits on fuel sources which has been introduced in [25].

The remainder of this paper is organised as follows: section II shows the modelling of the fuel cost and emission functions, section III presents the modelling of the power grid, section IV presents the formulation of the optimisation problem, section V explains the inclusion of the uncertainties in the emission prices, section VI shows the sensitivity analysis of the cost function with respect to the emission allowances, section VII presents case studies and section VIII provides concluding remarks.

II. MODELLING OF FUEL COST AND EMISSION FUNCTIONS

This section presents a method to combine the fuel cost and the emission cost-benefit functions and to linearise this

combined function by a series of linear segments with different slopes for use in the objective function of the linear programming problem. This objective function is used to calculate the sensitivity of the operating cost with respect to emission limits. Instead of using multi-objective functions to model the cost of the fuel and the cost of buying extra emission allowances or selling the excess emission allowances, one objective function is proposed. Emissions are modelled as functions of the heat rate. Since the fuel costs of the generating units are functions of the heat rates, these objective functions are combined prior to using them in the optimisation problem.

A great variety of solution methods have been proposed in the literature to model the emission function of the thermal generating units. Some of these models are based on modifying the heat rate functions of the generating units and some others are based on multiplying the heat rates by appropriate emission factors [12]–[19], [23], [24], [26]–[31]. The heat rates themselves can be modelled in several forms. Emission rates have been modelled using a straight linear form as in [18], [26], combination of a straight line and two exponentials as in [13], a quadratic form as in [12], [14]–[16], [19], [23], [24], [27], [28], a quadratic and an exponential form as in [17], [29], [30], and multiplying the heat rate function with an emission factor as in [31]. In this work, the emissions are modelled based on the heat rates and linearising the quadratic form. The quadratic heat rate for a generating unit i can be expressed as:

$$HR_i = k_i + \ell_i P_i(t) + m_i P_i^2(t) \quad (\text{MMBtu/MWh}) \quad (1)$$

Since the heat rates are functions of the output power and the emissions are functions of the heat rates, the CO_2 emission of the generating unit i (ECO_{2i}) can be expressed as a function of the output power as follows,

$$ECO_{2i} = Ea_i + Eb_i P_i(t) + Ec_i P_i^2(t) \quad (\text{tonne/t}) \quad (2)$$

where t is the time horizon chosen by a utility.

For a power generating plant j consists of n thermal generating units at time t , the total CO_2 emission (ECO_{2j}) is expressed as,

$$ECO_{2j} = \sum_{i=1}^n ECO_{2i}(P_i(t)) \quad (\text{tonne/t}) \quad (3)$$

Equation (2) can be easily expressed in terms of (\$ per unit of time (t)) by multiplying the coefficients of (2) by the price of the CO_2 emission (\$/tonne) as follows,

$$E_{ci} = a_i + b_i P_i(t) + c_i P_i^2(t) \quad (\$/t) \quad (4)$$

The emission cap-and-trade systems have been based on an annual time horizon and inter-year banking. However, the yearly allowances and emission auction periods can be converted into tonnes per unit time of CO_2 emission as it has been assumed in several research papers such as in [32], [33].

The fuel cost function of a generating unit i , F_{ci} , can be defined in a quadratic form as follows,

$$F_{ci} = A_i + B_i P_i(t) + C_i P_i^2(t) \quad (\$/t) \quad (5)$$

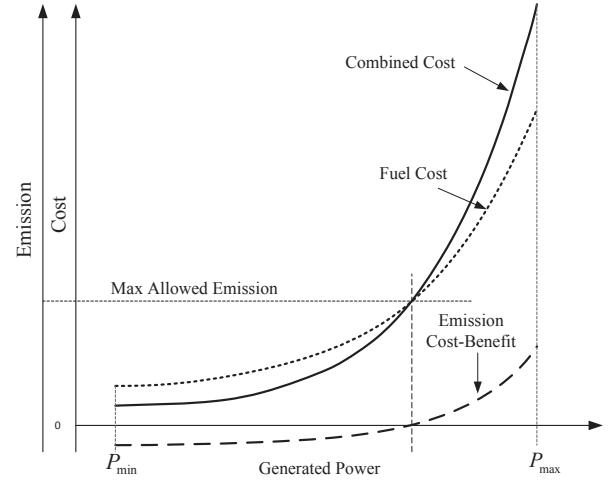


Fig. 1. The combined fuel cost and emission cost-benefit functions. The emission cost-benefit curve is shifted to cross the zeros cost line at the emission limit.

Now, the fuel cost function and the emission cost function have the same units and can be combined. The combined fuel-emission cost function can be expressed as following,

$$F_i = \alpha_i + \beta_i P_i(t) + \gamma_i P_i^2(t) \quad (\$/t) \quad (6)$$

The combined fuel-emission function is depicted in Fig.1.

In this paper, a linearisation model which was developed in [34] is adopted to linearise the cost objective function. Now, consider the typical nonlinear cost curve of a generating unit i as depicted in Fig. 2. Such cost curve can be approximated by a series of linear segments with different slopes. For instance, this cost curve is approximated with three linear segments of P_{i1} , P_{i2} , and P_{i3} , with slopes of h_1 , h_2 , and h_3 . Therefore, the cost function can be approximated as,

$$F_i(P_{Gi}) = F_i(P_{Gi}^{min}) + h_1 P_{i1} + h_2 P_{i2} + h_3 P_{i3} \quad (7)$$

where,

$$P_{Gi} = P_{Gi}^{min} + P_{i1} + P_{i2} + P_{i3} \quad (8)$$

and,

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \quad (9)$$

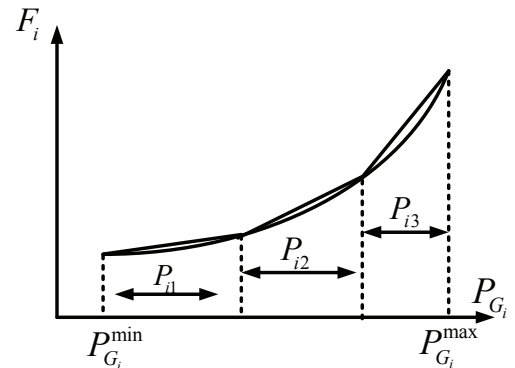


Fig. 2. Linearisation of the cost function curve

In reality, it is required for a generator starting to generate power from segment 2 only after generating all the power in segment 1. Since the cost function of generators are convex, when the objective function is to minimize generation cost, the linear program will automatically make the generators produce sequentially from segment 1, segment 2 and so on.

III. NETWORK MODELLING - LINEARISED LOAD FLOW

Load flow in power system networks plays a significant part of power system planning and operating studies. Incorporation of a load flow model and redispatch optimisation problem with minimising operation costs is commonly used in planning and operation decisions. Three types of load flow models have been commonly used in the literature in modelling power system networks which are, AC load flow, DC load flow and capacity or transportation load flow. The DC load flow ignores the effect of line resistances and the reactive power and assumes voltages magnitudes are 1 p.u. These assumptions make the DC load flow less accurate because in many scenarios the voltage collapse due to voltage limits and insufficient reactive power support can lead to under-voltage load shedding. The transportation load flow model ignores the physical parameters of the transmission lines and considers only the capacity of the lines. The full AC load flow model is the most accurate model but it is computationally expensive specially for the applications that require repetitive runs of load flow. In this paper, a linearised version of the AC load flow that closely represents power system networks that was developed by the authors in [35] is used in modelling power grids. This model can be considered as a modified version of the DC load flow model or a linearised form of the full AC load flow model. However, this model is not intended to replace the full AC load flow. The development of the linearised load flow model to accommodate minimum cost of fuel and emission are explained in the following subsections.

A. Linearised Load Flow Model

Starting with the well known real and reactive power injection equations at bus k ,

$$P_k = V_k \sum_{m \in k} V_m (G_{km} \cos \delta_{km} + B_{km} \sin \delta_{km}) \quad (10)$$

$$Q_k = V_k \sum_{m \in k} V_m (G_{km} \sin \delta_{km} - B_{km} \cos \delta_{km}) \quad (11)$$

where, $m \in k$ means the set of the buses connected to bus k .

If the voltage at bus k is assumed 1 p.u., for calculating the power injection equations, then equations (10) and (11) can be approximated as follows,

$$P_k \approx \sum_{m \in k} V_m (G_{km} \cos \delta_{km} + B_{km} \sin \delta_{km}) \quad (12)$$

$$Q_k \approx \sum_{m \in k} V_m (G_{km} \sin \delta_{km} - B_{km} \cos \delta_{km}). \quad (13)$$

It should be noted that, this assumption will not prevent the voltage at bus k from being calculated, rather it approximates the power injection at bus k . This is only an approximation

that enables the linearisation; it is not an assumption that the voltage magnitude equals 1 p.u. The voltage magnitudes at the other buses (V_m where $m \in k$) are calculated based on this assumption [35], [36].

Differences in bus voltage angles usually are very small and equations (12) and (13) can be further approximated by applying the cosine and sine rules. When δ is very small, $\cos(\delta) \approx 1$ and $\sin(\delta) \approx \delta$. Therefore,

$$P_k \approx \sum_{m \in k} (V_m G_{km} + V_m B_{km} \delta_{km}) \quad (14)$$

$$Q_k \approx \sum_{m \in k} (V_m G_{km} \delta_{km} - V_m B_{km}). \quad (15)$$

With further manipulation, the above equations can be rewritten as shown in equation (16).

$$\begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} -B' & G \\ -G' & -B \end{bmatrix} \begin{bmatrix} \delta \\ V \end{bmatrix} \quad (16)$$

The diagonal elements of B' and G' can be expressed as,

$$B'_{kk} = B_{kk} - b_{kk}$$

$$G'_{kk} = G_{kk} - g_{kk}$$

This model has been applied on several known systems and the results show its validity and it closely approximates the full AC load flow. For further details about this model, the readers are suggested to refer to [35], [36].

B. Inclusion of Losses

The developed linearised power flow model is lossless. In large power system with thousands of buses, real power loss can be substantial which is not advisable to be neglected.

If the term of $-\frac{\delta_{km}^2}{2}$ of the Tylor expansion of the cosine function is kept, while neglecting other higher order terms, then,

$$P_k \approx \sum_{m \in \Omega_k} (V_m G_{km} + B_{km} \delta_{km}) - \frac{1}{2} \sum_{m \in \Omega_k} G_{km} \delta_{km}^2 \quad (17)$$

Note that the first part of (17) describes the real power flow that other buses receive from bus k , while the second part indicates the real power loss in the power transition as follows,

$$P_{loss}^k = -\frac{1}{2} \sum_{m \in \Omega_k} G_{km} \delta_{km}^2 \quad (18)$$

Similarly, the reactive power losses can be expressed as,

$$Q_{loss}^k = \frac{1}{2} \sum_{m \in \Omega_k} B_{km} \delta_{km}^2 \quad (19)$$

Equations (18) and (19) explicitly indicate active and reactive power losses are quadratic forms of δ_{km} . G_{km} , real part of Y_{km} , is always non-positive, hence P_{loss}^k is a non-negative number. Ref. [37]–[39] described a piecewise method to linearise transmission line losses.

Note that (18) and (19) only include the losses produced by the series impedance of transmission lines. Losses produced from the shunt part of the transmission lines have already been considered in the original formulation expressed by (16).

IV. MINIMUM OPERATING COST

In this paper, the linear programming optimisation problem is incorporated with the linearised load flow to minimise the operating costs; fuel costs and emission cost-benefit. This section describes the formulation and incorporation of the CO_2 emissions in the objective function of minimum operation cost. In this paper, the emission allowances have been incorporated to determine the point at which the produced emission crosses the boundary of the maximum emission allowance. The objective function used in the optimal economic dispatch is to minimise the total operation cost. This objective function is subject to equality and inequality constraints of the power system operation limits. The equality constraints are the power balance at each bus and the inequality constraints are the capacity limits of generating units, power carrying capabilities of transmission lines, voltage and reactive power limits and emission limits. The sensitivity of the objective function value (cost) with respect to the emission limits is calculated by determining the values of the shadow prices.

A. Power Balance Constraints

Power balance equation is an equality equation that represents the sum of the complex power at the buses. The power balance equation can be derived from (16) as follows:

$$\begin{aligned} B'\delta - GV + P_G &= P_D \\ G'\delta + BV + Q_G &= Q_D \end{aligned} \quad (20)$$

B. Real and Reactive Power Constraints of the Generators

Real power limits of the generators are bounded by maximum and minimum operating values. Reactive power limits, however, can be constrained in two ways: constant maximum and minimum reactive power or using P-Q capability curve. The maximum reactive power produced or absorbed by a generator can be determined from the real power output according to the P-Q power capability curve. In this paper, for simplicity, the reactive power limits are bounded by constant maximum and minimum reactive power assigned for each generator. Therefore, the constraints of real and reactive power of the generators can be expressed as,

$$\begin{aligned} P_G^{min} &\leq P_G \leq P_G^{max} \\ Q_G^{min} &\leq Q_G \leq Q_G^{max} \end{aligned} \quad (21)$$

C. Voltage Limits Constraints

Voltage constraints are limited according to the allowed voltage fluctuations. The maximum voltage allowed in this paper is 1.05 p.u., V^{max} , and minimum voltage allowed is 0.95 p.u., V^{min} , as follows,

$$V^{min} \leq V \leq V^{max} \quad (22)$$

D. Line Current Limit

The current flowing in a line connecting buses k and m can be calculated from the voltage difference and branch impedance as follows:

$$\hat{I}_{km} = \frac{\hat{V}_k - \hat{V}_m}{\hat{Z}_{km}} \quad (23)$$

Equation (23) can be manipulated to produce a linearised expression for the line current. This approximation is based on the assumption that bus voltage angles are very small such that $\cos(\delta) \approx 1$ and $\sin(\delta) \approx \delta$ and line resistance is much smaller than line reactance, i.e., $R_{ik} \ll X_{ik}$. A piecewise linearisation of the current limit constraints in the linear programming problem is given in [35] as follows,

$$\begin{aligned} bA'\delta + bA'V &\leq F_f^{max} \\ -bA'\delta - bA'V &\leq F_r^{max} \end{aligned} \quad (24)$$

where

$$\begin{aligned} bA' &= bA \cos \theta_l \\ bA' &= bA \sin \theta_l \end{aligned}$$

where θ_l is approximation of the power factor angle [35].

E. CO_2 Emission Constraints

The emission constraints are used in this work to determine the point at which the action of buying or selling the emission allowances can be decided. Also, the shadow prices of emission constraints at each power plant are used in estimating the sensitivity of the objective function with respect to the emission limits. The emission constraints are derived from (3) as follows,

$$ECO_{2j} \leq ECO_{2j}^{max} \quad \forall j. \quad (25)$$

F. Linear Programming Formulation

The Linear Programming is used here to minimise the operation costs. The objective function is to minimise the combined fuel-emission costs as described in (7). From the above constraints, the minimisation problem can be formulated as,

$$\text{Operation Cost} = \min \left(\sum_{i=1}^{N_g} F_i \right) \quad (26)$$

Subject to:

shadow price

$$\begin{aligned} B'\delta - GV + P_G - P_{loss} &= P_D, & \pi_{pd} \\ G'\delta + BV + Q_G - Q_{loss} &= Q_D, & \pi_{qd} \\ P_G^{min} &\leq P_G \leq P_G^{max}, & \pi_{pg} \\ Q_G^{min} &\leq Q_G \leq Q_G^{max}, & \pi_{qg} \\ V^{min} &\leq V \leq V^{max}, & \pi_v \\ bA'\delta + bA'V &\leq F_f^{max}, & \pi_{tf} \\ -bA'\delta - bA'V &\leq F_r^{max}, & \pi_{tr} \\ \delta & & \text{unrestricted} \end{aligned} \quad (27)$$

and the emission constraints

$$ECO_{2j} \leq ECO_{2j}^{max} \quad \forall j, \quad \pi_E \quad (28)$$

The shadow prices of the constraints of (27) and (28) are defined as follows: π_{pd} and π_{qd} are for the bus real and reactive power load limits respectively, π_{pg} and π_{qg} are for the generators' real and reactive power limits respectively, π_v is for the bus voltage limits, π_{tf} and π_{tr} are for the transmission lines forward and reverse flow limits respectively and π_E is for the emission limits.

In the standard minimisation problem given by (26) and (27), all generation and network constraints have been taken into consideration. Also, in order to get a feasible solution for the standard problem, it has been assumed that one of the bus angles is zero in the constraints of (27).

V. UNCERTAINTIES IN THE EMISSION PRICES

Emission trading system and emission prices have been experiencing considerable fluctuations [40], [41]. In this work, a probability density function (PDF) for the emission prices is adapted which is provided in [40] as shown in Fig. 3.

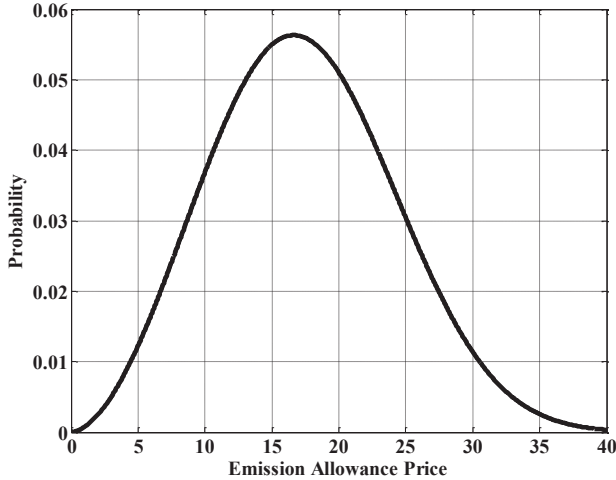


Fig. 3. Modelling of CO_2 Prices - reprinted from [40]

For the operating studies, the CO_2 emission prices can be assumed constant at some value for the operation time horizon. For the planning studies, which utilise the shadow price concept to calculate the sensitivity of the operation cost with respect to the operation limits, the probability density function of Fig. 3 is used to determine the CO_2 emission prices. Monte Carlo simulation has been used in this work to include the uncertainties of the emission prices.

VI. SENSITIVITY ANALYSIS AND THE CONCEPT OF THE SHADOW PRICE

Sensitivity analyses of the operating costs with respect to the operating limits are based on calculating the amount of change of the objective function with respect to the operating constraints. The analyses are performed by casting the dispatch operation as an optimisation problem, through minimising

the operating cost. One of the advantages of the sensitivity analysis is that it allows planners to distribute the available resources in a cost-effect paradigm. In the research reported in this paper, a shadow-price based method is proposed for performing the sensitivity analysis. Shadow prices are used here as indicators of the area or component in which the investment will reduce the operating cost.

A. Shadow Price Theory and Description

Shadow price approach was initially proposed by the former Soviet Union economist *Conte Petrovic* in the late 1930s when he was applying linear programming technique to maximise the output of some products [42]. Several definitions have been reported in the literature for the shadow price [43]. For instance, shadow price is defined as the sensitivity of the value of the objective function to the change in the right hand side of the linear programming problem constraints [43]. From a primal-dual perspective, it can be defined as the primal (x) and dual (y) solutions of the linear programming problem. From the optimisation point of view; however, shadow price can be interpreted as the rate of change in the objective function for an infinitesimal change in the right-hand side of the linear programming problem. From a geometric perspective, shadow price can be understood as the subgradients of the objective function along the dimension of resource provisioning changes. As it is seen in this section, shadow price approach has numerous applications in different areas due to its multi-facet nature. It has been used in [44] to forecast the short-term transmission congestion. It has also been used in the evaluation of some construction projects and management [45]. However, as it was mentioned earlier, shadow price is used here as a decision-making tool for minimum generation cost.

In general, suppose a system has m constraints with n variables, the standard *maximum* linear programming problem can be formulated as follows:

$$\max(Z) = \sum_{j=1}^n c_j x_j \quad (29)$$

Subject to the following constraints,

$$\sum_{j=1}^n a_{ij} x_j \leq b_i \quad (30)$$

$$x_j \geq 0 \quad (31)$$

The matrix of the coefficients of the constraints is,

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \quad (32)$$

The right hand side vector b of the constraints consists of m constants,

$$b = (b_1, b_2, \dots, b_m)^T \quad (33)$$

The row vector of the objective function c consists of n coefficients

$$c = (c_1, c_2, \dots, c_n)^T \quad (34)$$

The dual of this standard *maximum* problem is the standard *minimum* problem, that is,

$$\min(W) = \sum_{i=1}^m b_i y_i \quad (35)$$

Subject to the following constraints,

$$\sum_{j=1}^n a_{ij} y_j \geq c_j \quad (36)$$

$$y_i \geq 0 \quad (37)$$

If the slack variables of equations (30) and (31) are imported, the standard linear programming will have the following form,

$$\max(Z) = CX \quad (38)$$

With the equality constraints,

$$AX = b \quad (39)$$

$$X \geq 0 \quad (40)$$

The optimal and feasible solution of (35) can be expressed as,

$$X_b = \hat{B}^{-1}b \quad (41)$$

where $\hat{B}$ is the optimal basis at the optimal and feasible solution and X_b is the basic variable sub-vector. In the standard problem of (30), the significance of the dual variables y_i is described as the i^{th} shadow price. In other words, the y_i of the dual problem are interpreted here as the shadow prices of the linear constraints b_i .

Fig. 4 shows an example for the sensitivity of maximising the output power of power plant consists of two generating units (P_{G1} and P_{G2}) with respect to the change in the emission limit. EG_1 and EG_2 are the emissions from the two generating units and E^{max} is the maximum allowed emission at this power plant.

Due to space limitations, in this work, only the effects of the generation capacity and the transmission lines' power carrying capabilities are provided. The effects of the other parameters such as voltage and reactive power limits can be easily included.

B. Sensitivity of the objective functions with respect to the capacity limits of the generating units

The sensitivity of the objective functions with respect to the capacity limits of the generating units determines the reduction in the operating cost for each unit (1 MW) increase in the capacity limits which can be expressed as,

$$\partial OC / \partial C_i = \sum_{x \in X} P(x) \partial F(x) / \partial C_i \quad (42)$$

From (42), the generating units are ranked based on their effect on the overall operating cost. Planners may consider investments in the high ranked power plants to decrease overall operating cost.

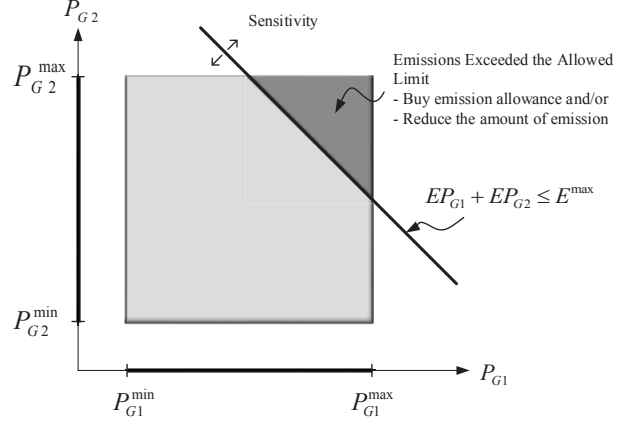


Fig. 4. Sensitivity of an objective function with respect to emission limits

C. Sensitivity of the objective functions with respect to the emission limits

The sensitivity analysis of the minimum cost objective function with respect to the emission limits determines whether planners should buy extra emission allowances at the power plants that have reached their emission limits or to invest in the other power plants that are operating within the allowed emission limits. For the power plants that have reached the emission limits, the sensitivity analysis estimates the amount of the reduction in the operating cost for each unit (1 tonne) increase in the emission limits which can be expressed as,

$$\partial OC / \partial E_i = \sum_{x \in X} P(x) \partial F(x) / \partial E_i \quad (43)$$

Similarly, from (43), the generating units are ranked based on their effect on the overall operating cost. Planners may consider buying extra emission allowances in the high ranked power plants to decrease overall operating cost.

VII. CASE STUDIES

This section illustrates the application of the proposed method on the IEEE RTS. The IEEE RTS system consists of 24 buses, 33 transmission lines, 5 transformers and 32 generating units. Total installed generating capacity is 3,405 MW and system peak load is 2,850 MW. The data of this system is given in [46].

The emission rates were calculated from the heat rates which are given in [46]. The emission coefficients were calculated from the emission rates based on 2nd order polynomials.

To show the results of the optimisation problem for minimum cost using the proposed objective function, one case scenario for a fixed price for the CO_2 emission is provided which is assumed \$10 per tonne. For planning studies, the uncertainties in the CO_2 emission prices are considered.

A. A Stopping Criterion and Solution Method

A stopping criterion was applied on the sensitivity indices which are given in sections VI-B and VI-B. The stopping criterion considering these two indices can be expressed as,

$$\sigma = \max(\sigma_1, \sigma_2) \quad (44)$$

where σ is a pre-specified tolerances,

$$\sigma_i = \frac{\sqrt{V(f_i(x))}}{f_i(x)} \quad (45)$$

where $V(\cdot)$ is the variance function, $f_i(x)$ is a sensitivity index and i is an index, i.e., $i = 1$ is for the sensitivity of the objective functions with respect to the capacity limits of the generating units and $i = 2$ is for the sensitivity of the objective functions with respect to the emission limits.

During the simulation, if σ is less than or equal to the specified tolerance, the algorithm is terminated; otherwise, the simulation continues. The flowchart of the simulation process is depicted in Fig. 5.

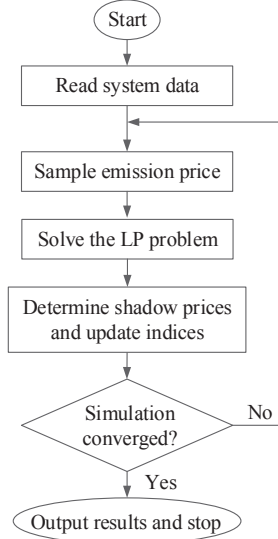


Fig. 5. Flowchart of the simulation procedures

The simulations were performed using MATLAB and the simulations converged after 13,200 samples. The machine specifications are: Intel(R), Core(TM), 2 Duo CPU E8400 @ 3.00 GHz with RAM 4.00 GB. The simulation time was 5 minutes and 23 seconds.

B. Operational Applications - Testing the Objective Function

In this part, two case studies were performed. The first case study was performed without imposing emission limits on the generating units. In the second case study, the amount of emission was reduced by 10% from the maximum produced emissions from each generating unit. In the first case study, two scenarios were considered which are economic dispatch and minimum emission dispatch. For the economic dispatch, the emissions neither included in the objective function nor are they included in the constraints. The objective function in

this case is minimisation of the fuel cost function of (5). The results of economic dispatch are shown in the second column of table I. For the minimum emission dispatch, the emissions are used in the objective function and no emission constraints were included. The objective function for minimum emission dispatch is minimisation of the emission cost function (4). The results of minimum emission dispatch is shown in the third column of table I. As a comparison between these two scenarios, the total generation cost for the economic dispatch scenario is 63,226.58 \$/h whereas the total generation cost for minimum emission dispatch scenario is 75,037.22 \$/h as shown in table II. As a comparison between using the linearised load flow model and the full AC load flow model, the total generation cost using the full AC load flow for the base case is 63352.21 \$/h.

In the second case study, the emission limits were reduced by 10% of the maximum allowable emission at each generating power plant to represent the imposed reduction requirements. In this case study, the objective function is the minimisation of the combined fuel-emission cost function of (6). The emission limits which are 90% of the maximum produced emission were used to combine and adjust the economic dispatch objective function and the minimum emission dispatch objective function. For this case study, the price of CO_2 emissions is considered \$10 per tonne. The results of the combined economic-emission dispatch are shown in the fourth column of table I. As a comparison between this case study and the first case study, the total generation cost is 68,161.05 \$/h as shown in table II. It can be seen that the total generation cost in the case of combined economic-emission dispatch falls between the total costs of economic dispatch and minimum emission dispatch.

TABLE I
OPTIMAL GENERATION LEVELS USING THE THREE DIFFERENT OBJECTIVE FUNCTIONS.

	Minimum Fuel Cost	Minimum Emission	Minimum Cost and Emission with %10 emission reduction
P_{G1} (MW)	184.00	159.00	184.00
P_{G2} (MW)	184.00	159.00	184.00
P_{G7} (MW)	241.00	253.00	212.00
P_{G13} (MW)	207.00	438.00	207.00
P_{G15} (MW)	167.00	179.00	196.00
P_{G16} (MW)	155.00	131.00	155.00
P_{G18} (MW)	400.00	400.00	400.00
P_{G21} (MW)	400.00	400.00	400.00
P_{G22} (MW)	300.00	300.00	300.00
P_{G23} (MW)	660.00	480.00	660.00

From Table I, the two nuclear power plants (400 MW each) at buses 18 and 21 and the six hydro driven generating units (50 MW each) at bus 22 are operating at their maximum capacity limits due to the fact that they have low operating costs and no CO_2 emissions are considered. Also, the four fossil coal driven generating units (76 MW each, two at bus 1 and two at bus 2) operate at their minimum capacity limits

TABLE II
SUMMARY OF THE OPTIMAL GENERATION LEVELS AND COSTS FOR THE
THREE SCENARIOS

	Minimum Fuel Cost	Minimum Emission	Minimum Cost and Emission
Tot. Cost (\$/hr)	63,226.58	75037.22	68161.05
Tot. Gen. (MW)	2896.77	2898.57	2898.04
Tot. losses. (MW)	47.92	48.57	48.04

for the minimum emission dispatch because these generating units have the highest CO_2 emission rates.

The accuracy of the results obtained using the proposed linearised load flow model depends strongly on the number of linearising segments. The results shown in Table I and Table II were obtained using 10 linear segments of the cost function and the power flow in the lines. As a comparison, Table III shows the results obtained using the full AC load flow, 10 segments linearised load flow, 3 segments linearised load flow and DC load flow models.

TABLE III
TOTAL COST, TOTAL GENERATION, AND TOTAL LOSSES OF THE IEEE
RTS OBTAINED USING DIFFERENT LOAD FLOW MODELS

Item and Method	AC load flow	Linearised load flow		DC load flow
		10 segments	3 segments	
Tot. Gen. (MW)	2896.77	2897.92	2901.27	2850.00
Tot. Losses (MW)	46.77	47.92	51.27	0.00
Cost (\$/hr)	63352.21	63,226.58	63591.47	61001.37

C. Planning Applications

In this part, Monte Carlo simulation was used to account for the uncertainties in the prices of the emissions and the loads. The uncertainties in the CO_2 prices are modelled using the probability density function of Fig. 3. The uncertainties in the loads are modelled by applying load clustering technique with 20 clusters for the load profile of the IEEE-RTS. The results of the sensitivity analyses are shown in table IV. In this table, the sign (#) denotes the bus number and (G) denotes the capacity, i.e., #13G197 means bus 13 generating unit capacity 197 MW.

From table IV it is clear that some generating units have zero effects on the objective function and that can be attributed to the fact that these units have high operating costs and they are hitting the minimum generation constraints in the linear programming problem. The 12 MW generating units at bus 15 have the highest effect on the minimum emission dispatch. On the other hand, the 20 MW generating units at buses 1 and 2 have the highest effect on the minimum economic-emission dispatch. If a planner is considering buying extra emission allowances, the 20 MW generating units at buses 1 and 2 are the best candidates.

VIII. CONCLUSION

Most of the emission studies in the literature have focused on minimising the combined economic-emission objective

TABLE IV
SENSITIVITY OF THE THREE OBJECTIVE FUNCTIONS WITH RESPECT TO
THE CAPACITY AND EMISSION CONSTRAINTS

Bus No. & Gen. Cap.	Minimum Fuel Cost	Minimum Emission	Minimum Cost and Emission with %10 emission reduction
#1G20	-0.00000	-14.39950	-3.62120
#1G76	-0.14271	-11.78807	-3.02142
#2G20	-0.00000	-14.39038	-3.62120
#2G76	-0.14539	-11.77895	-3.06183
#7G100	-0.00000	-0.81057	-0.19552
#13G197	-0.00000	-0.23727	-0.11343
#15G12	-0.00000	-52.67485	-6.22476
#15G155	-0.25506	-1.10304	-0.16579
#16G155	-0.26963	-1.10211	-0.17131
#23G155	-0.24946	-1.10009	-0.17403
#23G350	-0.21818	-0.07378	-0.09801

function and introducing new concepts in solving multi-objective functions. The goal has been to minimise emissions and fuel costs to accommodate for the emission reduction requirements. Minimising the emission is one solution among many methods such as utilising fuels with lower emissions, exploring new techniques to reduce the amount of emissions, etc. Redispatching the existing generators can help in minimising the emissions. The need to establish strategies to include the emission reduction requirements in power system planning is necessary. Inclusion of minimum emission dispatch in power system planning studies as described in this paper helps guide planning strategies with respect to emission allowance prices.

This work has introduced a framework for including the prices of emission allowances in optimally redispatching power generation. The maximum emission allowed at a certain power plant was used to combine the fuel cost function and the emission cost-benefit function. Also, the maximum allowed emission was used as index to distinguish between selling or buying emission allowances.

Sensitivity analyses based on the shadow price concept were used to determine the best investment locations and alternatives to reduce the operation cost. The sensitivity analysis can be used in the planning phase as a decision making tool to choose between buying emission allowances and investing in another power station that has available emission allowances.

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