

Multistage Optimal PMU Placement Considering Substation Infrastructure

Saleh Almasabi, *Student Member, IEEE*, and Joydeep Mitra, *Senior Member, IEEE*

Abstract—Phasor measurement units (PMUs) significantly benefit the operation and control of power systems. They offer precise phasor measurements with a high refresh rate. These measurements are utilized for wide-area measurement system to improve situational awareness and enhance the control infrastructure. In spite of these advantages, the industry has been reluctant to adopt PMU technology, largely due to the high cost of PMUs and the communication infrastructure. However, judicious selection of PMU locations through optimal placement of PMUs (OPP) enables the minimization of installation cost. There have been several approaches to solve the OPP problem most of which assume that the minimum number of PMUs achieves the minimum cost or consider the cost of PMUs without considering the communication infrastructure. This paper presents an OPP approach that considers both the communication infrastructure and the installation cost of PMUs. The proposed approach uses multistage installation where each stage is dependent on the cost set by the utility instead of the number of PMUs. Higher priority buses can be chosen under the proposed approach. An opposition-based elitist binary genetic algorithm is used to solve the OPP problem. The proposed approach is tested on the IEEE reliability test system, and on the IEEE 14-bus, 30-bus, and 118-bus test systems.

Index Terms—Binary genetic algorithm, critical buses, observability, optimal placement, phasor measurement units (PMUs), redundancy measurements.

I. INTRODUCTION

THE high accuracy of the time-synchronized measurements provided by phasor measurement units (PMUs) has advanced the operation and control of power systems. Moreover, PMUs offer a higher refresh rate compared to traditional measurements. Therefore, PMUs are now becoming an essential part of modern grid operation and control. Different applications of wide-area measurement system (WAMS) are improved by using PMUs instead of traditional measurements. For instance state estimation can become a noniterative process if PMUs are used for a complete observable network [1], [2]. However, due to the high cost of installing PMUs, replacing all traditional measurements is very unlikely in the near future. Therefore, researchers have proposed several techniques to solve the problem of optimal placement of PMUs (OPP).

Most OPP literature associates the PMU installation cost with the cost of the PMU unit. As a result, most of these techniques have been proposed to minimize the number of PMUs while considering the complete observability of the system [3]–[6]. Similarly, an integer linear programming (ILP)-based model was proposed for the OPP with minimum number of PMUs as an objective [7]. In [8], the OPP was solved for the minimum number of PMUs under controlled islanding conditions. Binary particle swarm optimization (BPSO) and exponential PSO were used in [6], [9] to obtain the optimum solution for the OPP problem. In [10], mixed integer linear programming was used to solve the OPP problem as a multi-objective problem. All the approaches mentioned above proposed minimizing the number of PMUs while achieving the desired observability conditions.

Application-based PMU placement, on the other hand, prioritizes buses for the installation of PMUs based on the chosen application. Most of these techniques are multistage approaches where PMU locations are determined in several stages to achieve complete observability of the network. Kumar and Thukaram [11] and Pal *et al.* [12] have suggested that the critical buses be ranked based on stability criteria, and then the PMUs be incrementally installed, enhancing the observability in the process. The OPP based on state estimation was solved in [13] using semidefinite programming and convex relaxation. In [14], ILP was used to solve the OPP problem and to detect bad data and critical measurements. In [15], the empirical observability Gramian was used to quantify and solve the OPP problem for dynamic state estimation. In [16], [17], ILP was used to enhance the reliability of observability by considering the probability of PMU failure in addition to the transmission line failure, while minimizing the number of PMUs. However, most of the application-based and multistage OPP approaches still use the traditional approach for OPP which assumes that minimizing the number of PMUs results in the lowest cost [11]–[21].

Realizing that the minimum number of PMUs does not guarantee the minimum installation cost, researchers started changing the scope of the OPP problem [22]–[25]. For instance, in [23], dual-use line relays were used to reduce the total cost of the OPP. Mohammadi *et al.* [24] incorporated the cost of the communication infrastructure into the OPP, where they have proposed to minimize the number of PMUs and the distance between the Phasor Data Concentrator (PDC) and PMUs. Nevertheless, they ignored the substation infrastructure and assumed that the minimum number of PMUs implies minimum cost. On the other hand, Rather *et al.* [26] highlighted the cost of substation infrastructure, and its effect on the OPP.

Manuscript received January 11, 2018; revised April 3, 2018 and May 22, 2018; accepted July 3, 2018. Paper 2018-IACC-0055.R2, presented at the 2017 IAS Annual Meeting, Cincinnati, OH, USA, Oct. 15, and approved for publication in the IEEE TRANSACTIONS ON INDUSTRY APPLICATIONS by the Industrial Automation and Control Committee of the IEEE Industry Applications Society. (Corresponding author: Saleh Almasabi.)

The authors are with the Department of Electrical and Computer Engineering, Michigan State University, East Lansing, MI 48824 USA (e-mail: almasabi@msu.edu; mitraj@msu.edu).

Digital Object Identifier 10.1109/TIA.2018.2862401

In [25], a channel-based OPP approach was used to enhance the voltage stability and measurement reliability with a flat cost per PMU.

Even with the optimization of PMU installation cost, the total investment cost required for complete observability is often quite significant. To address this issue, several multistage approaches were developed, where the PMUs are installed incrementally over time with a predetermined budget for each stage [20], [21], [27]. The predetermined budget can be based on the number of PMUs as in [20], [21] or on actual cost as in [27]. However, multistage PMU installation is likely to result in higher cost for the overall process. While the one-stage PMU installation treats the observability as a constraint, the multistage approach maximizes the observability at each stage, at the expense of the installation cost. In [27], the authors addressed this issue by relocating PMUs when needed after each stage. However, PMU relocation, at a flat rate, from a substation where the infrastructure has already been upgraded and prepared for PMU applications to a new substation, requires upgrading the new substation which can cost up to 95% of the PMU installation expense [28]. Moreover, other factors that affect the cost, in these multistage approaches such as the communication and substation infrastructures, were not considered.

In this paper, a multistage PMU placement strategy is proposed which considers two factors: substation cost (including PMU cost) and communication infrastructures. The prioritization of critical buses can be integrated with the proposed approach. This approach can be used in an incremental way where PMUs are installed in multiple stages under a constrained cost. Unlike most multistage approaches, this approach does not consider a predetermined number of PMUs at each stage. Instead, the multistage installation maximizes the network observability and prioritizes critical buses while remaining within a predetermined budget for each stage.

In OPP, buses having one or more of the following criteria: high voltage buses, high impact on transient stability, or sensitive loads, are considered critical buses [11], [12]. These buses are sometimes given higher priority to enhance system awareness from a stability perspective. Other researchers have proposed prioritizing buses based on different criteria such as reliability and state estimation [15]–[17]. Bus prioritization can be integrated with the proposed approach while considering both observability and the actual cost of the PMU installation. The major contributions of this work may be summarized as follows.

- It develops a comprehensive cost model for the OPP problem, including the cost of the PMUs as well as the infrastructure upgrade costs.
- It presents a flexible, multistage deployment plan, implemented over a period of time depending on the budget of the utility company.
- It affords the ability to prioritize PMU placement based on specific criteria such as bus criticality, thereby enabling application-based deployment.

The OPP is solved using an opposition-based elitist binary genetic algorithm (O-BEBGA) along with multisource Dijkstra algorithm. Multisource Dijkstra algorithm is integrated

into the O-BEBGA to optimize the network infrastructure cost. The Dijkstra algorithm is chosen for its efficiency in finding the shortest path compared to other graph algorithms [29]–[32]. This paper is organized as follows. Section II discusses the cost of installing PMUs and the communication infrastructure. The proposed approach is presented in section III. Sections IV and V present the case studies and the conclusion respectively.

II. PMU INSTALLATION COST

This section discusses the cost of PMU installation. It presents the cost model for upgrading a substation and the cost of the communication infrastructure for PMU installation.

A. Substation Infrastructure

Most of the cost is associated with the installation process of the PMU and not the PMUs themselves. In fact, the PMUs cost about 5% of the total installation cost [28]. Most of the cost is spent on upgrading the substation and communication infrastructures. A report recently published by the U.S. Department of Energy (DOE) showed that the PMU installation cost ranges from \$40,000 to \$180,000 per PMU [28].

The cost varies depending on the infrastructure support for the PMUs. Typically, PMUs need sufficient communication infrastructure to send the measurement data to the PDC. The substation infrastructure also needs to be sufficient to utilize the functionalities of PMUs. Formulating the installation cost for the PMUs is a complicated process. Although PMU installation requires the same infrastructure upgrades, such as communication, cyber-security, and other equipment upgrades, the approach to installing PMUs can differ depending on the utility and existing infrastructure support for the PMUs. For instance, a utility can install new stand-alone PMUs, or upgrade existing digital relays to enable PMU functionality [28]. Moreover, installing PMUs also depends on the availability of CTs and PTs [26]. Once the infrastructure of the substation is in place to support the PMUs, the installation cost can go down to 35% of the initial cost [28].

The proposed cost model of PMU installation in (1) considers the difference between prepared buses, where minimal upgrades are needed, and unprepared buses, by introducing g_i index. The index takes the value of 4.5 for unprepared buses and 1 for prepared buses. The model also includes the cost of adding additional measurement channels by including the cost of PTs and CTs.

$$\text{Cost} = \sum_{i=1}^N g_i (aP_i + b_i P_i) + K(P) \quad (1)$$

where

N number of buses in the system;

g_i prepared bus index;

a cost of installing PMU and basic upgrades at the substation;

b_i cost for installing additional PT or CT at substation i ;

$K(P)$ cost function for the communication infrastructure for PMUs;

$$P = [P_1, P_2, \dots, P_i, \dots, P_N]^T;$$

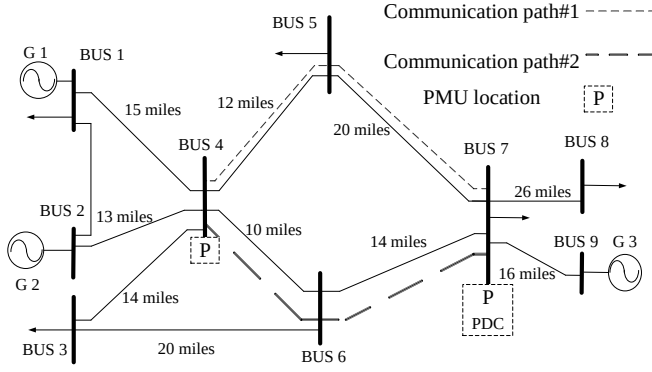


Fig. 1. Sample 9-bus system with possible communication paths

where P_i takes the value of zero or one and indicates if a PMU is to be installed at substation i .

B. Communication Infrastructure

The measurement data obtained by the PMUs are sent to the PDC, where the PDC sorts the data and processes it for other applications. Mohammadi *et al.* [24] have proposed to reduce the distance between the PMUs and the PDC to lower the total cost. In [24], it is also have proposed to place the PDC on a non-PMU bus to minimize the total communication distance. The work in [24], however, have not considered the cost of upgrading the substation for PMU installation. In this paper, the PDC is assumed to be installed at one of the substations where a PMU is to be placed. Then, the path connecting all PMUs is minimized to lower the communication infrastructure cost (2).

There can be several communication paths connecting all PMUs at different substations. Consider the 9-bus system in Fig. 1; the PMUs are placed at buses 4 and 7 to make the system observable. However, there are two communication paths to connect both PMUs with the PDC located at bus 7. As seen in Fig. 1, the first communication path is about 32 miles, and the second one is about 24 miles. Therefore, in order to minimize the communication infrastructure cost, the path connecting all PMUs with PDC needs to be minimized. The communication infrastructure is assumed to be passive optical network (PON) with optical ground wire (OPGW). The cost model in (2) is derived from [33], [34].

$$\min K(P) = \sum_{j=1}^n \text{len}_{i,j} \cdot \text{cc}_{i,j} + N_e \cdot P_j + N_b \cdot P_j + \text{PDC} \quad (2)$$

where n is the number of PMUs and $\text{len}_{i,j}$ is the length of the transmission line between buses i and j . The communication cost cc is either \$2,414 or \$0 per mile [35]. N_e represents the passive cost of the communication infrastructure such as the housing chassis, optical switch, wave filters. N_b , on the other hand, represents the cost per additional channel. N_e and N_b are assumed to be \$5,530 and \$125 respectively [33]. PDC is assumed to have a total cost of \$7,500 [36].

III. PROPOSED APPROACH

This section presents the approach for the multistage OPP. As discussed in the previous sections, the PMU installation cost plays a critical role in determining the installation process. In the multistage approach, maximizing the benefits of PMU installation takes higher priority over the cost function. This approach assumes that the utility sets a budget for the installation of PMUs and the first objective is to maximize the observability and priority buses while minimizing the installation cost and not exceeding the predetermined budget for the current stage.

A. Problem Statement

As mentioned earlier, the optimal placement for PMUs highly depends on the installation cost and available budget. In the proposed multistage approach, the observability is maximized, subject to the observability constraint described below, and the cost function is minimized. The observability in (3) needs to have enough redundancies for the desired observability conditions. For instance, under normal operating conditions, the observability constraint in (4) must be satisfied.

$$\mathcal{O} = H \times P \quad (3)$$

$$\mathcal{O} \geq I' \quad (4)$$

where P is a vector of length equal to the number of buses N , as described in section II-A; I' is a vector of length N with all its elements equal to 1; and H is an $N \times N$ connectivity matrix. The entries for P and H are defined in (5) and (6) respectively.

$$P_i = \begin{cases} 1, & \text{if a PMU is installed at bus } i \\ 0, & \text{if no PMU is at bus } i \end{cases} \quad (5)$$

$$h_{ij} = \begin{cases} 1, & \text{if } i = j \\ 1, & \text{if there is a branch} \\ & \text{connecting bus } i \text{ and bus } j \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

In the proposed approach, the observability function in (7) is treated as a higher level objective function, and is subjected to minimizing the cost function. The cost function in (9) is treated as the lower level objective function, subjected to the higher level objective (observability function). This setup allows maximizing the observability while minimizing the cost without violating the budget constraint, thereby reaching the optimal solution for the given budget. It should be noted that during the multistage process complete observability in (4) cannot be achieved; therefore the observability constraint is changed to the multistage condition in (8).

$$\max \sum_{i=1}^N \mathcal{O}'_i \quad (7)$$

$$\text{subject to} \quad \mathcal{O} \leq \mathbf{I} \quad \text{“multistage observability condition”} \quad (8)$$

$$\min \text{Cost} = \sum_{i=1}^N g_i(a_i P_i + b_i P_i) + K(P) \quad (9)$$

subject to

$$C \leq C_{\text{budget}}$$

1) *Priority Buses*: For application-based OPP schemes, some buses are prioritized for PMU installation regardless of their contribution to the overall observability. These schemes range from stability criteria to reliability, and many others [11], [12], [14], [16]. In the proposed OPP scheme, priority buses can be chosen using any criterion.

The priority buses for the network are embedded in the observability function as bias using a priority vector R (10). The R vector has the length of the number of buses N . If all buses are treated equally then all elements of R are set to zero. The higher priority buses are determined based on the utility criteria, then arranged in descending order in a vector L . Then the priority bias is assigned using the following algorithm.

Procedure 1 Priority Vector R

Initialize priority buses (L), $R = \text{zeros}_{1 \times N}$

N = number of buses

M = maximum number of branches in (H)

kk = length of L

for $j = 1 : kk$

$i = l(j)$

if $i \neq 0$ then

$r_i = M(kk - j + 1)$

else

$r_i = 0$

endif

endfor

Maximizing the modified observability vector $\mathcal{O}'$ gives bias to the higher priority buses. However, this vector cannot be used to test the observability of the network since the R vector skews the observability. As a result, the skewed observability in (7) is used for optimizing the OPP, and the original observability in (3) is used as a constraint for observability testing.

$$h'_{ij} = \begin{cases} 1 + r_i, & \text{if } i = j \\ 1, & \text{if there is a branch} \\ & \text{connecting bus}(i) \text{ and bus } (j) \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

where

$\mathcal{O}' = H' \times P$;

r_i bus (i) priority index;

N number of buses.

The cost of the overall PMU installation can be reduced by considering the effect of zero-injection buses (ZIB). Considering the effect of ZIBs improves the overall observability of the system, thereby reducing the number of PMUs needed to achieve the observability constraint. The effect of ZIB can be summarized into two points. If all buses connected to a ZIB are observable, the ZIB is considered observable by applying KCL. Also, an unobservable bus, when connected to an observable ZIB, is considered observable only if all of the other buses connected to the ZIB are observable.

B. Algorithm

The optimal placement problem in subsection III-A is a discontinuous bi-level problem. It also involves optimizing the communication infrastructure cost $K(P)$ within the installation cost in (9). A multisource Dijkstra algorithm is used to obtain the shortest path connecting all PMUs and PDC. Since the proposed model involves optimizing three objective functions (2), (7) and (9), evolutionary algorithms are the appropriate tools for solving such a problem. The proposed algorithm uses an opposition-based elitist binary genetic algorithm (O-BEBGA) to solve the bi-level OPP in subsection III-A. The opposition element is added to the algorithm to enhance the overall performance since opposition-based methods have proven their superiority in terms of convergence speed and results [37]–[39].

1) *Multisource Dijkstra*: The design of the communication infrastructure involves finding the most cost effective path (2) between the PMUs and PDC. There are several algorithms that can be used to find this path such as Floyd-Warshall, Bellman-Ford, Johnson and Dijkstra algorithms. The Dijkstra algorithm is among the most efficient algorithms for single source undirected weighted graphs [29]. However, the communication network design problem is not a single source/destination problem; rather, it is a multisource single destination problem, or a single source/destination with a must pass nodes. The traditional Dijkstra algorithm can still be used to solve this problem. This entails using the Dijkstra algorithm n times to establish one communication line between two source nodes out of n source nodes. The multisource Dijkstra algorithm, on the other hand, can be used to pair up source nodes in one run.

The multisource Dijkstra is used to find the shortest paths P_{x1} connecting every source node s_i to the nearest source node s_j , where $(s_i, s_j \in S)$. This step generates a set $G = \{\phi_1, \phi_2, \dots, \phi_{n1}\}$ with n_1 subsets, where $n_1 = \text{floor}(N_s/2)$ and N_s is the number of source nodes. Each subset ϕ has at least two connected source nodes. The next step is to connect the subsets in G to each other. First, the weights for the paths in P_{x1} are set to zero. The multisource Dijkstra is then used to obtain new paths P_{x2} that connect the subsets in G . It should be noted that the new paths P_{x2} , may have redundant routes, however these redundant routes have zero weight. The process of finding new paths and updating their weights is repeated until all the subsets in G are connected with one path. This path is the union of all paths P_{xi} obtained from the multisource Dijkstra algorithm.

The sample graph in Fig. 2 demonstrates the implementation of multisource Dijkstra algorithm for the communication network design. The source nodes in S are a, d, f and j . The first loop of the multisource Dijkstra generates the G set with subsets ϕ_1 and ϕ_2 . The ϕ_1 subset contains the source nodes a and j ; the ϕ_2 subset contains source nodes d and f . The P_{x1} paths for the subsets in G are $\{a-k-j, d-f\}$. The next step is to connect the subsets ϕ_1 and ϕ_2 , which produce the path $P_{x2}=\{a-b-c-d\}$. The union of the paths P_{x1} and P_{x2} produces the shortest path connecting all the source nodes in S .

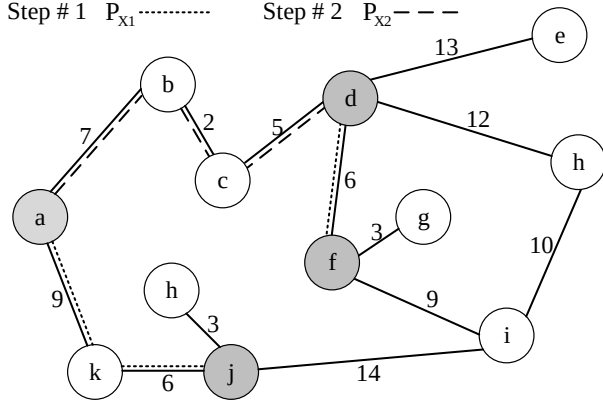


Fig. 2. Sample graph to demonstrate the implementation of multisource Dijkstra algorithm.

2) *O-BEBGA*: As discussed previously, multistage approaches can lead to higher cost for the overall installation of PMUs, since the solutions for each stage are often sub-optimal for the complete observability. This is because maximizing the observability at each stage increases the installation cost [27]. To overcome this issue, the *O-BEBGA* solves the OPP for the desired observability condition first. Then, the optimal solution for the complete observability (X_s) is used as optimal solution for the multistage installation. To enhance the performance further, the search space multistage installation is reduced to include only optimal location of PMUs in X_s .

The proposed algorithm solves the optimal placement problem in a parallel manner by initializing random candidates where each candidate x_i has the length of the number of buses N , thereby evaluating the candidate buses simultaneously instead of using systematic increments. By maximizing the observability function (higher objective), while minimizing the cost of PMU installation, the predetermined budget C_{budget}^k for each stage (k) is optimally utilized.

The higher and lower objectives share the same decision variables, meaning there are no decision variables exclusive to one objective or the other. The proposed approach exploits this advantage to evaluate both objectives simultaneously without using different search spaces for each objective. The proposed approach uses a sorting function to handle the simultaneous evaluation in the same search space. This function sorts all candidates according to their feasibility and fitness of the higher and the lower objectives, as seen in Fig. 3. As a result, the algorithm is guided towards the optimal solution where the cost is minimized and the observability is maximized.

The overall flowchart for the proposed algorithm is shown in Fig. 4. The crossover and mutation probabilities are $P_c = 0.7$ and $P_m = 0.3$ respectively. The O_{rn} denotes the opposition random variable; the crossover and mutation random variables are denoted by C_{rn} and M_{rn} respectively. Double point crossover is used to generate the offspring population X_c , and single point mutation is used to generate the mutated population X_m . The algorithm uses dynamic opposition with probability of $P_o = 0.4$.

Candidates					Cost	$\mathcal{O}$	Rank
Bus 1	Bus 2	Bus 3	Bus 4	...			
1	0	1	1	...	62000	18.3	1
1	0	1	0	...	54000	14.6	2
1	1	0	0	...	55000	14.6	3
...	...	...	...	...	...	...	...
0	0	0	1	...	3000	2.9	n-2
0	0	1	1	...	2400	2.2	n-1
0	1	0	1	...	1400	1.4	n

Fig. 3. Sorting function.

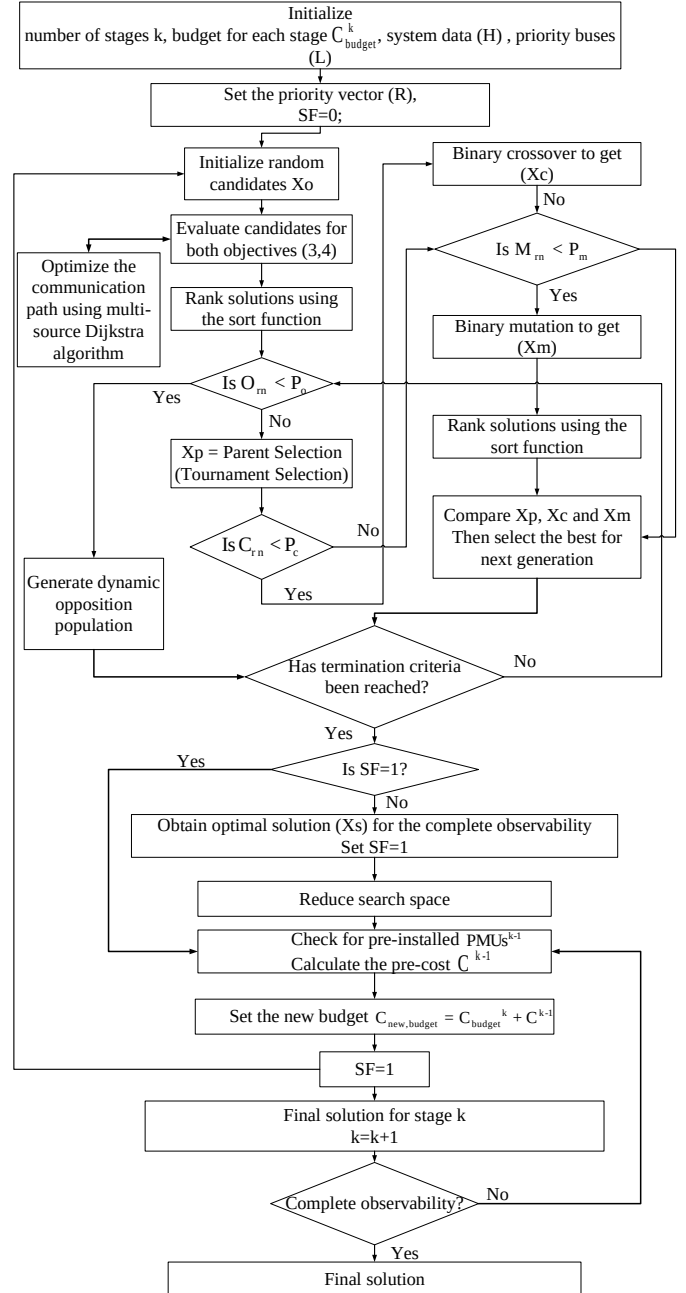


Fig. 4. Flow chart of the proposed *O-BEBGA* algorithm.

There are many variations of opposition techniques in the literature. The two most common are the global opposition and the dynamic opposition. In the proposed algorithm, a modified dynamic opposition is used to generate the opposition population when applicable. Instead of generating the total opposite of the chosen individual X_i , only a third of the variables in X_i are selected for the opposition process (11).

$$X_i^{\text{opp}} = X^{\min} + X^{\max} - X_i \quad (11)$$

The algorithm is terminated if the conditions in (12) are met. The terms α and β are constants; where $\gamma(1)$ indicates how much of the current population is feasible and $\gamma(2)$ indicates if the population is converging to an optimum. The variance of the cost ($\text{Var}[C]$) is used to determine $\gamma(2)$, where k is the index for the current population.

$$\gamma = \begin{cases} \gamma(1) = |\overline{\mathcal{O}} - \max(\mathcal{O})| \leq \alpha \\ \gamma(2) = |\text{Var}[C^{k-1}] - \text{Var}[C^k]| \end{cases} \quad (12)$$

C. Multistage Installation

In realistic scenarios, PMUs are installed in a multistage manner. The majority of existing multistage methods assume the minimum number of PMUs per stage. This assumption is unrealistic since PMU installation is restricted by the financial burden, substation infrastructure, and technical benefits at each stage. The proposed approach uses a financial capital C_{budget} , as the limit for each stage instead of using the number of PMUs as the limit.

At each stage k , the previously installed PMUs (PMU^{k-1}) are initiated, and the pre-installation cost C^{k-1} is calculated. The budget for each stage C_{budget}^k is set. Then the budget C_{budget}^k is modified to include the pre-installed PMUs cost C^{k-1} . The pre-installed PMU locations are maintained during the initialization of the random population X_o and during the mutation step. Since the pre-installed PMU locations are maintained for the initial and parent populations, the crossover population X_c maintains the pre-installed PMU locations by default.

$$C_{\text{new,budget}}^k = C_{\text{budget}}^k + C^{k-1} \quad (13)$$

1) *Complete observability*: Initially, the multistage approach cannot achieve complete observability, however as stages are added, or more money is added to the budget, the complete observability constraint in (14) is satisfied.

$$H \times P = \mathcal{O} \geq \tilde{\mathbf{I}} \quad (14)$$

2) *Observability under single line outage*: It is required to have at least two measurement redundancies for every bus in the network to achieve observability under a single line outage. Therefore, the observability constraint is changed to (15).

$$H \times P = \mathcal{O}_s \geq \tilde{\mathbf{I}} \quad (15)$$

$$\mathcal{O}_{s,i} = \begin{cases} \sum(a_{ij} \times p_i) \\ 2, & \text{if a PMU is installed at bus } i \end{cases} \quad (16)$$

where $\mathcal{O}_s$ is the observability vector for all buses in the system and $\tilde{\mathbf{I}}$ is a vector of length equal to the number of buses, with all entries equal to 2.

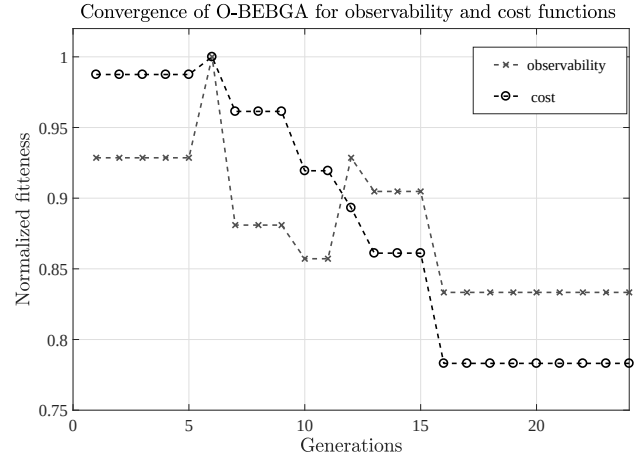


Fig. 5. Convergence of O-BEBGA for the normalized observability and cost functions.

3) *Single PMU outage*: In a single PMU outage, every bus needs to have two independent measurements, either by two different PMUs or if ZIB effect is considered through KCL and a PMU. Therefore, the observability constraint in (8) is changed to the following:

$$H \times P = \mathcal{O}_p \geq \tilde{\mathbf{I}} \quad (17)$$

$$\mathcal{O}_{p,i} = \sum(a_{ij} \times p_i) \quad (18)$$

IV. SIMULATION AND RESULTS

In this section, the proposed approach is tested on the IEEE reliability test system (RTS), IEEE 14-bus, 30-bus and 118-bus test systems. In subsection IV-A all buses are treated equally and no prioritization is given to any bus. The higher priority buses and other observability conditions are tested in subsection IV-B. The buses are divided into two categories: prepared buses and unprepared buses. The prepared buses are assumed to have sufficient infrastructure, require basic security, network upgrades, and cost 75% less than the unprepared buses [28].

The base cost per PMU is assumed to be \$40,000, and the cost per additional PT or CT is assumed to be \$2,380 [40]. The cost of PDC is assumed to be \$7,500. The length of the transmission lines are obtained from [41]. The communication links are assumed to be running along the transmission lines where the cost of the communication links is assumed to be \$2,414 per mile [35] or \$0 if the communication link already exists.

TABLE I
MULTISTAGE APPROACH, NOT CONSIDERING EFFECT OF ZIB

System	PMU locations	Stage budget C_{budget}^k	Cost	Remaining	Unprepared buses
First Stage ($k = 1$); Effect of ZIB Is Not Considered					
IEEE 14-bus	4, 5, 11	\$300,000	\$252,189	\$47,811	7, 9
IEEE RTS	15, 16, 21	\$350,000	\$213,522	\$136,478	10, 11, 17, 24
IEEE 30-bus	15, 16, 26	\$450,000	\$407,745	\$42,255	9, 12, 25, 27, 28
IEEE 118-bus	2, 5, 15, 19, 21, 30, 34, 45, 49, 66, 68, 77, 84, 89, 92, 105	\$4,000,000	\$3,910,268	\$89,732	—
Second Stage ($k = 2$); Effect of ZIB Is Not Considered					
IEEE 14-bus	4, 5, 8, 11, 13 ^a	\$200,000	\$121,656	\$78,344	7, 9
IEEE RTS	5, 6, 8, 15, 16, 21	\$350,000	\$286,227	\$63,773	10, 11, 17, 24
IEEE 30-bus	6, 7, 11, 15, 16, 22, 26	\$350,000	\$235,776	\$114,224	9, 12, 25, 27, 28
IEEE 118-bus	2, 5, 9, 15, 19, 21, 27, 30, 34, 40, 45, 49, 52, 56, 59, 66, 68, 71, 77, 80, 84, 89, 92, 105, 110	\$2,000,000	\$1,987,498	\$41,932	—
Third Stage ($k = 3$); Effect of ZIB Is Not Considered					
IEEE RTS	5, 6, 8, 9, 15, 16, 21, 23 ^a	\$350,000	\$266,245	\$83,755	10, 11, 17, 24
IEEE 30-bus	3, 6, 7, 11, 13, 15, 16, 20, 22, 26, 29 ^a	\$350,000	\$317,741	\$132,259	9, 12, 25, 27, 28
IEEE 118-bus	2, 5, 9, 12, 15, 19, 21, 27, 30, 31, 32, 34, 36, 40, 45, 49, 52, 56, 59, 63, 66, 68, 70, 71, 77, 80, 84, 86, 89, 92, 94, 100, 105, 110, 118 ^a	\$2,000,000	\$1,022,043	\$977,957	—

^aComplete observability is achieved.

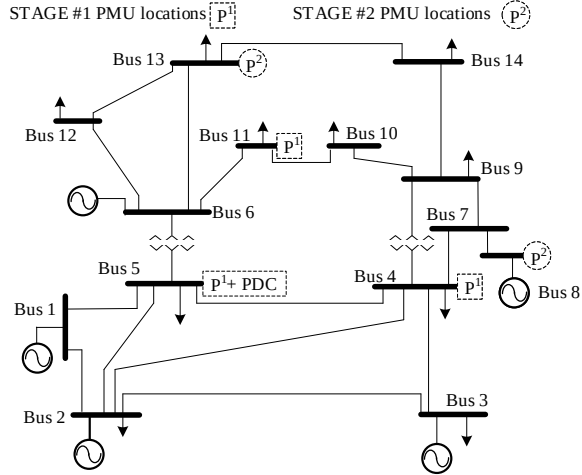


Fig. 6. OPP solution for the IEEE 14-bus; not considering effect of ZIB.

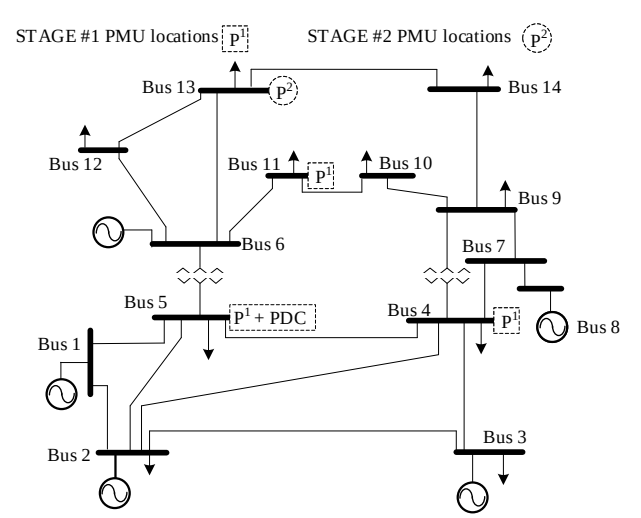


Fig. 7. OPP solution for the IEEE 14-bus; considering effect of ZIB.

The proposed algorithm is used with a population size of $3 \times N$, where N is the number of buses. The performance of the O-BEBGA is shown in Fig. 5. Although the algorithm maximizes the observability, the minimization of the PMU installation cost drives the observability to a cost effective solution. It should be noted that maximizing observability often increases the cost. However, there exist cases where the same or better observability can be achieved at a better cost, as is the case for generations 5 and 12 in Fig. 5.

A. No Priority Buses

The model in section II is used and no priority is given to any bus. The multistage OPP is performed as a two-stage process for the IEEE 14-bus and a three-stage process for the IEEE RTS, the IEEE 30-bus and the IEEE 118-bus. Each stage is treated independently budget-wise, meaning the remainder

of the budget from each stage is not added to the next stage budget. Complete observability is achieved for all systems within three stages. It should be noted that the number of stages in which complete observability is achieved depends on the budget specified by the utility.

The OPP is performed in two different cases. In the first case, ZIBs are treated as normal buses. The result of the multistage OPP is shown in Table I. The OPP solution for the IEEE 14-bus is shown in Fig. 6 and Fig. 7. The ZIB effect is considered in the second case as shown in Table II.

The proposed approach is compared with some of the recent approaches in OPP literature, as seen in Table III. These approaches include classical and evolutionary methods, mainly particle swarm optimization (PSO), Cellular Learning Automata (CLA) and binary imperialistic competition algo-

TABLE II
MULTISTAGE APPROACH, CONSIDERING EFFECT OF ZIB

System	PMU locations	Stage budget C_{budget}^k	Cost	Remaining	Unprepared buses
First Stage ($k = 1$); Effect of ZIB Is Considered					
IEEE 14-bus	4, 5, 11	\$300,000	\$252,189	\$47,811	7, 9
IEEE RTS	5, 20	\$400,000	\$352,366	\$47,634	10, 11, 17, 24
IEEE 30-bus	3, 4, 10, 15, 20	\$500,000	\$426,236	\$73,764	9, 12, 25, 27, 28
IEEE 118-bus	45, 49, 53, 72, 80, 84, 86, 94	\$2,500,000	\$2,357,860	\$142,140	—
Second Stage ($k = 2$); Effect of ZIB Is Considered					
IEEE 14-bus	4, 5, 11, 13 ^a	\$150,000	\$73,448	\$76,552	7, 9
IEEE RTS	2, 5, 14, 20, 21	\$350,000	\$222,514	\$127,486	10, 11, 17, 24
IEEE 30-bus	3, 4, 10, 13, 15, 20, 29	\$350,000	\$238,861	\$111,139	9, 12, 25, 27, 28
IEEE 118-bus	2, 8, 12, 19, 21, 27, 34, 37, 45, 49, 53, 56, 68, 72, 75, 77, 80, 84, 86, 92, 94	\$2,000,000	\$1,924,513	\$75,487	—
Third Stage ($k = 3$); Effect of ZIB Is Considered					
IEEE RTS	2, 5, 8, 14, 20, 21 ^a	\$350,000	\$142,932	\$207,068	10, 11, 17, 24
IEEE 30-bus	3, 4, 7, 10, 13, 15, 16, 20, 29 ^a	\$350,000	\$230,720	\$119,280	9, 12, 25, 27, 28
IEEE 118-bus	2, 8, 11, 12, 19, 21, 27, 31, 32, 34, 37, 40, 45, 49, 53, 56, 62, 68, 72, 75, 77, 80, 84, 86, 89, 92, 94, 100, 105, 110 ^a	\$2,000,000	\$1,924,513	\$75,487	—

^aComplete observability is achieved.

TABLE III
COST COMPARISON OF PROPOSED APPROACH WITH OTHER APPROACHES

System	Ref. [26] ^a	Ref. [3]	Ref. [42]	Ref. [43]	Ref. [24]	Ref. [6] ^b	Proposed
Under Normal Operating Conditions; Effect of ZIB Is Not Considered							
IEEE 14-bus	2, 8, 10, 13 \$554,650	2, 6, 7, 9 \$870,400	2, 6, 7, 9 \$870,400	—	—	2, 6, 8, 9 \$679,320	4, 5, 8, 11, 13 \$373,845
IEEE RTS	3, 4, 7, 10, 13, 16, 20, 21 \$1,342,300	2, 3, 8, 10, 16, 21, 23 \$1,222,100	2, 3, 8, 10, 16, 21, 23 \$1,222,100	—	—	—	5, 6, 8, 9, 15, 16, 21, 23 \$765,994
IEEE 30-bus	3, 6, 7, 11, 13, 15, 17, 20, 21, 24, 26, 30 \$1,236,400	1, 2, 6, 9, 10, 12, 15, 19, 25, 27 \$1,825,100	2, 3, 6, 9, 10, 12, 15, 19, 25, 27 \$1,802,100	—	—	—	3, 6, 7, 11, 13, 15, 16, 20, 22, 26, 29 \$961,262
IEEE 118-bus	—	—	1, 5, 9, 12, 13, 17, 21, 23, 26, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 71, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114 \$8,224,600	—	—	2, 5, 11, 12, 15, 17, 21, 24, 25, 28, 34, 37, 40, 45, 49, 52, 56, 62, 63, 68, 73, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114 \$7,988,600	2, 5, 9, 12, 15, 19, 21, 27, 30, 31, 32, 34, 36, 40, 45, 49, 52, 56, 59, 63, 66, 68, 70, 71, 77, 80, 84, 86, 89, 92, 94, 100, 105, 110, 118 \$6,890,379
Under Normal Operating Conditions; Effect of ZIB Is Considered							
IEEE 14-bus	2, 8, 10, 13 \$554,650	2, 6, 9 \$631,110	2, 6, 9 \$631,110	2, 6, 9 \$631,110	—	—	4, 5, 11 \$325,637
IEEE RTS	2, 8, 10, 15, 22, 23 \$1,076,000	2, 8, 10, 15, 20, 21 \$1,032,300	1, 2, 8, 16, 21, 23 \$941,410	—	—	—	2, 5, 8, 14, 20, 21 \$895,817
IEEE 30-bus	3, 7, 8, 10, 13, 15, 17, 19, 29 \$976,260	1, 2, 10, 12, 15, 19, 27 \$1,239,600	2, 3, 10, 12, 18, 24, 30 \$1,159,400	2, 4, 10, 12, 15, 18, 27 \$1,176,600	3, 7, 10, 12, 15, 20, 27 \$1,099,700	—	3, 4, 7, 10, 13, 15, 16, 20, 29 \$717,812
IEEE 118-bus	—	—	1, 6, 8, 12, 15, 17, 21, 25, 29, 34, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114 \$7,840,900	2, 8, 11, 12, 15, 19, 21, 27, 31, 32, 34, 40, 45, 49, 52, 56, 62, 65, 72, 77, 80, 85, 86, 90, 94, 101, 105, 110 \$7,523,100	2, 8, 11, 12, 17, 21, 24, 27, 29, 43, 47, 49, 52, 56, 62, 71, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110, 114 \$8,173,500	—	2, 8, 11, 12, 19, 21, 27, 31, 32, 34, 37, 40, 45, 49, 53, 56, 62, 68, 72, 75, 77, 80, 84, 86, 89, 92, 94, 100, 105, 110 \$6,063,617

^aHas two optimal solutions, one for minimum number of PMUs and the other for a cost model.

^bHas multiple optimal solutions, only the solution with the minimum cost is presented.

algorithm (BICA). The results show that the proposed approach achieves better overall cost for the OPP. This enhancement is partially due to the comprehensive installation cost model of the proposed approach, which considers the substation and communication infrastructure. Most of the other methods have not considered such a comprehensive model.

B. Using Priority Buses

The multistage PMU installation is performed on the IEEE 14-bus test system with predetermined priority buses. The higher priority buses for the IEEE 14-bus are chosen to be the

high voltage buses, $L = [1, 2]$. The installation is performed as a four-stage process.

The first stage has a budget limit of \$400,000, and the remaining stages have budget limits of \$500,000 each. The first and second stages are used to achieve complete observability under normal conditions (14). The third stage is used to achieve observability for single line outage contingencies (16). The single PMU outage in (18) is chosen as the desired observability for the final stage. The results in Table IV show the optimal PMU installation at each stage. The results show a comparison between treating all buses equally. The results

show that prioritizing buses can drive up the PMU installation cost as seen in Table IV.

TABLE IV
MULTISTAGE PMU INSTALLATION FOR THE IEEE 14-BUS (USING PRIORITY BUSES), NOT CONSIDERING EFFECT OF ZIB

	All buses are treated equally $L = [\phi]$	Priority buses are used $L = [1, 2]$	Unprepared buses
Stage One $\mathcal{O} \leq \hat{\mathbf{I}}$	4, 6, 8 \$219,799	1, 2 \$185,777	7, 9
Stage Two $\mathcal{O} \geq \hat{\mathbf{I}}$	4, 5, 6, 8, 11, 13, 14 \$300,966	1, 2, 8, 10, 13 \$491,476	7, 9
Stage Three $\mathcal{O}_s \geq \hat{\mathbf{I}}$	2, 4, 5, 6, 8, 10, 11, 13, 14 \$133,860	1, 2, 4, 6, 8, 10, 11, 13, 14 \$255,764	7, 9
Stage Four $\mathcal{O}_p \geq \hat{\mathbf{I}}$	2, 4, 5, 6, 8, 10, 11, 13, 14 ^a \$0 ^b	1, 2, 4, 6, 8, 10, 11, 13, 14 ^a \$0 ^b	7, 9

^aNo additional installation of PMUs.

^bObservability already achieved at the previous stage.

V. CONCLUSION

In this paper, a new approach for multistage OPP is presented, where a realistic model for the installation of PMUs and the communication infrastructure is considered. This approach considers a practical OPP installation process, where the PMUs are installed over the span of a number of years in a multistage manner, which is determined by the budget of the utility. Unlike the multistage OPP in the literature, the proposed approach can be scaled to achieve different observability conditions such as a single PMU outage. Moreover, this approach can be used to handle application-based OPP problems, where certain buses are given higher priority based on the utility criteria. The effectiveness of the proposed approach is demonstrated on the IEEE 14-bus, IEEE RTS, IEEE 30-bus and IEEE 118-bus test systems.

REFERENCES

- [1] M. Göl and A. Abur, "A fast decoupled state estimator for systems measured by PMUs," *IEEE Trans. Power Sys.*, vol. 30, no. 5, pp. 2766–2771, 2015.
- [2] A. K. Singh and B. C. Pal, "Decentralized dynamic state estimation in power systems using unscented transformation," *IEEE Trans. Power Sys.*, vol. 29, no. 2, pp. 794–804, 2014.
- [3] S. Chakrabarti and E. Kyriakides, "Optimal placement of phasor measurement units for power system observability," *IEEE Trans. Power Sys.*, vol. 23, pp. 1433–1440, Aug 2008.
- [4] B. Gou, "Generalized integer linear programming formulation for optimal PMU placement," *IEEE Trans. Power Sys.*, vol. 23, pp. 1099–1104, Aug 2008.
- [5] N. M. Manousakis, G. N. Korres, and P. S. Georgilakis, "Taxonomy of PMU placement methodologies," *IEEE Trans. Power Sys.*, vol. 27, pp. 1070–1077, May 2012.
- [6] T. K. Maji and P. Acharjee, "Multiple solutions of optimal PMU placement using exponential binary PSO algorithm for smart grid applications," *IEEE Trans. Ind. App.*, vol. PP, no. 99, pp. 1–1, 2017.
- [7] F. Aminifar, A. Khodaei, M. Fotuhi-Firuzabad, and M. Shahidehpour, "Contingency-constrained PMU placement in power networks," *IEEE Trans. Power Sys.*, vol. 25, no. 1, pp. 516–523, 2010.
- [8] L. Huang, Y. Sun, J. Xu, W. Gao, J. Zhang, and Z. Wu, "Optimal PMU placement considering controlled islanding of power system," *IEEE Trans. Power Sys.*, vol. 29, pp. 742–755, March 2014.
- [9] M. Hajian, A. M. Ranjbar, T. Amraee, and B. Mozafari, "Optimal placement of PMUs to maintain network observability using a modified BPSO algorithm," *Int. J. Elect. Power Energy Syst.*, vol. 33, no. 1, pp. 28–34, 2011.
- [10] J. Aghaei, A. Baharvandi, A. Rabiee, and M. A. Akbari, "Probabilistic PMU placement in electric power networks: An milp-based multiobjective model," *IEEE Trans. Ind. Informat.*, vol. 11, pp. 332–341, April 2015.
- [11] V. S. S. Kumar and D. Thukaram, "Approach for multistage placement of phasor measurement units based on stability criteria," *IEEE Trans. Power Sys.*, vol. 31, pp. 2714–2725, July 2016.
- [12] A. Pal, G. A. Sanchez-Ayala, V. A. Centeno, and J. S. Thorp, "A PMU placement scheme ensuring real-time monitoring of critical buses of the network," *IEEE Trans. Power Del.*, vol. 29, pp. 510–517, April 2014.
- [13] V. Kekatos, G. B. Giannakis, and B. Wollenberg, "Optimal placement of phasor measurement units via convex relaxation," *IEEE Trans. Power Sys.*, vol. 27, pp. 1521–1530, Aug 2012.
- [14] B. Gou and R. G. Kavasseri, "Unified PMU placement for observability and bad data detection in state estimation," *IEEE Trans. Power Sys.*, vol. 29, pp. 2573–2580, Nov 2014.
- [15] J. Qi, K. Sun, and W. Kang, "Optimal PMU placement for power system dynamic state estimation by using empirical observability gramian," *IEEE Trans. Power Sys.*, vol. 30, pp. 2041–2054, July 2015.
- [16] Y. Wang, C. Wang, W. Li, J. Li, and F. Lin, "Reliability-based incremental PMU placement," *IEEE Trans. Power Sys.*, vol. 29, pp. 2744–2752, Nov 2014.
- [17] O. Gomez, M. A. Rios, and G. Anders, "Reliability-based phasor measurement unit placement in power systems considering transmission line outages and channel limits," *IET Generation, Transmission & Distribution*, vol. 8, no. 1, pp. 121–130, 2014.
- [18] A. S. Costa, A. Albuquerque, and D. Bez, "An estimation fusion method for including phasor measurements into power system real-time modeling," *IEEE Trans. Power Sys.*, vol. 28, pp. 1910–1920, May 2013.
- [19] R. Sodhi, S. C. Srivastava, and S. N. Singh, "Multi-criteria decision-making approach for multistage optimal placement of phasor measurement units," *IET Generat. Transmiss. Distrib.*, vol. 5, pp. 181–190, February 2011.
- [20] D. Dua, S. Dambhare, R. K. Gajbhiye, and S. A. Soman, "Optimal multistage scheduling of PMU placement: An ILP approach," *IEEE Trans. Power Del.*, vol. 23, pp. 1812–1820, Oct 2008.
- [21] F. Aminifar, M. Fotuhi-Firuzabad, M. Shahidehpour, and A. Khodaei, "Probabilistic multistage PMU placement in electric power systems," *IEEE Trans. Power Del.*, vol. 26, pp. 841–849, April 2011.
- [22] S. Almasabi and J. Mitra, "Multi-stage optimal PMU placement including substation infrastructure," in *2017 IEEE Industry Applications Society Annual Meeting*, pp. 1–8, Oct. 2017.
- [23] A. Pal, A. K. S. Vullikanti, and S. Ravi, "A PMU placement scheme considering realistic costs and modern trends in relaying," *IEEE Trans. Power Sys.*, vol. 32, no. 1, pp. 552–561, 2017.
- [24] M. B. Mohammadi, R. A. Hooshmand, and F. H. Fesharaki, "A new approach for optimal placement of PMUs and their required communication infrastructure in order to minimize the cost of the wams," *IEEE Trans. Smart Grid*, vol. 7, pp. 84–93, Jan 2016.
- [25] M. Esmaili and M. Ghamsari-Yazdel, "Voltage stability-constrained optimal simultaneous placement of PMUs and channels enhancing measurement reliability and redundancy," *IEEE Power and Energy Technol. Sys. J.*, vol. 4, pp. 32–39, June 2017.
- [26] Z. H. Rather, Z. Chen, P. Thøgersen, P. Lund, and B. Kirby, "Realistic approach for phasor measurement unit placement: Consideration of practical hidden costs," *IEEE Trans. Power Del.*, vol. 30, pp. 3–15, Feb 2015.
- [27] M. H. F. Wen, J. Xu, and V. O. K. Li, "Optimal multistage PMU placement for wide-area monitoring," *IEEE Trans. Power Sys.*, vol. 28, pp. 4134–4143, Nov 2013.
- [28] U. S. Department of Energy, Office of Electricity Delivery and Energy Reliability, "Factors affecting PMU installation costs." https://www.smartgrid.gov/document/factors_affecting_pmu_installation_costs.html, 2014. [Online, accessed: 6- Aug.- 2016].
- [29] T. H. Cormen, *Introduction to algorithms*. MIT press, 2009.
- [30] B. LI, S.-f. ZHENG, J.-d. CAO, X.-m. LIAN, and K.-q. LI, "Multi-source shortest distances algorithm based on road network hierarchical strategy [j]," *Computer Engineering*, vol. 12, p. 015, 2008.
- [31] S.-w. LI, F. WU, J. ZHANG, and C.-q. ZHU, "Selecting the shortest one from multi-source and multi-channel paths," *J. Geomatics Science and Tech.*, vol. 5, p. 018, 2010.

- [32] P. W. Eklund, S. Kirkby, and S. Pollitt, "A dynamic multi-source dijkstra's algorithm for vehicle routing," in *Intelligent Info. Syst.*, pp. 329–333, IEEE, 1996.
- [33] J. Chen, L. Wosinska, C. M. Machuca, and M. Jaeger, "Cost vs. reliability performance study of fiber access network architectures," *IEEE Communications Magazine*, vol. 48, pp. 56–65, Feb. 2010.
- [34] M. K. Weldon and F. Zane, "The economics of fiber to the home revisited," *Bell Labs Tech. J.*, vol. 8, pp. 181–206, July 2003.
- [35] J. Aotong, "High quality 24 core OPGW fiber optical cable." https://www.alibaba.com/product-detail/high-quality-24-core-OPGW-fiber_60369466427.html?s=p, 2017. [Online; accessed 20-Jul-2017].
- [36] SEL, "Station phasor data concentrator." <https://selinc.com/products/3573/>, 2017. [Online; accessed 20-Jul-2017].
- [37] S. Rahnamayan, H. R. Tizhoosh, and M. M. Salama, "Opposition versus randomness in soft computing techniques," *Applied Soft Computing*, vol. 8, no. 2, pp. 906–918, 2008.
- [38] S. Rahnamayan, H. R. Tizhoosh, and M. M. Salama, "Opposition-based differential evolution," *IEEE Trans. Evol. Comput.*, vol. 12, no. 1, pp. 64–79, 2008.
- [39] S. Almasabi, F. T. Alharbi, and J. Mitra, "Opposition-based elitist real genetic algorithm for optimal power flow," in *2016 North American Power Symposium (NAPS)*, (Denver, CO), pp. 1–6, Sept 2016.
- [40] HUAYI, "Current transformer price, current transformer manufacturer product." https://www.alibaba.com/product-detail/33kV-35kV-66kV-69kV-110kV-132kV_60646001002.html?spm=a2700.7724838.2017115.15.Dn2x6f&s=p, 2017. [Online; accessed 20-Jul-2017].
- [41] H. Plittgen, "Computational cycle time evaluation for steady state power flow calculations," *Report Prepared for Thomson-CSF, Division Simulateurs*.
- [42] B. S. Roy, A. Sinha, and A. Pradhan, "An optimal PMU placement technique for power system observability," *Int. Journal of Elec. Power Energy Sys.*, vol. 42, no. 1, pp. 71–77, 2012.
- [43] S. M. Mazhari, H. Monsef, H. Lesani, and A. Fereidunian, "A multi-objective PMU placement method considering measurement redundancy

and observability value under contingencies," *IEEE Trans. Power Sys.*, vol. 28, no. 3, pp. 2136–2146, 2013.



Saleh Almasabi (S'11) received the B.E. degree in Electrical and Electronics engineering from King Fahd University of Petroleum & Minerals (KFUPM), Saudi Arabia in 2008. Received M.S. from Wayne State University, Detroit, Michigan in 2014. He is currently pursuing PhD at the Department of Electrical and Computer Engineering, Michigan State University, East Lansing, Michigan. His research interests are power system reliability, PMU applications and state estimation.



Joydeep Mitra (S'94–M'97–SM'02) received the B.Tech. (Hons.) degree in electrical engineering from the Indian Institute of Technology Kharagpur, Kharagpur, India, in 1989, and the Ph.D. degree in electrical engineering from Texas A&M University, College Station, TX, USA, in 1997. He is currently an Associate Professor of electrical engineering with Michigan State University, East Lansing, MI, USA, the Director of the Energy Reliability and Security Laboratory, and a Senior Faculty Associate with the Institute of Public Utilities. His research interests include reliability, planning, stability, and control of power systems.