

# A Fault-Tolerance Based Approach to Optimal PMU Placement

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**Abstract**—Phasor measurement units (PMUs) have offered significant advancements in power system operation and control. Despite these advantages, the industry has been slow in adopting PMU technology. This slow adoption is partially due to the high cost of installing PMUs, which motivates the optimal PMU placement (OPP) problem. The work presented in this paper is motivated by the idea that the value of a high cost asset can be increased by deploying it to improve network fault-tolerance. This strategy of deploying PMUs in the proximity of higher probability contingencies increases the likelihood of more effective remedial actions, both preventive and corrective. The proposed OPP therefore considers both the cost and the extent to which the PMU placements will benefit system observability in the presence of network vulnerabilities. A bi-level framework is used for optimizing the network vulnerability and the installation cost of PMUs. In the proposed framework, the installation cost of PMUs is treated as the primary objective, and the observability, which incorporates the vulnerability analysis, is treated as the secondary objective. The proposed approach can be used as a multi-stage installation process or as a single-stage installation process. This approach uses evolutionary algorithms to optimize and prioritize locations and installation cost of the PMUs. The proposed approach is tested on the IEEE 14-bus, IEEE 30-bus, and IEEE reliability test system.

**Index Terms**—Observability, optimal placement, phasor measurement units (PMU), reliability.

## I. INTRODUCTION

PHASOR measurement units (PMUs) are becoming an essential part of the modern grid. They provide time-stamped current and voltage measurements using the global positioning system (GPS). PMUs also provide higher accuracy and a higher refresh rate when compared to conventional measurements. As a result, PMUs are becoming a fundamental part of wide area monitoring systems (WAMS). However, due to the high cost of installation of PMUs, replacement of all traditional measurement equipment is quite unlikely to happen in the near future. Keeping in mind the high cost of PMUs, it is often difficult to determine the number of PMUs required and the most appropriate placement of these units. Researchers have tackled the question of optimal PMU placement (OPP) with different approaches.

Several researchers have proposed minimizing the number of PMUs, while considering the complete observability of the system [1]–[3]. An integer linear programming (ILP) based model is proposed for the OPP in [4]. In [5], OPP problem is solved under controlled islanding conditions. Binary particle swarm optimization (PSO) and exponential PSO are used in

[6], [7] to obtain solutions for the OPP problem. In [8], mixed integer linear programming is used to solve the OPP problem as a multi-objective approach. Conventional measurements and their effect on the OPP have been considered in [9]–[12], where the OPP problem is solved using semi-definite programming or ILP. Controlling the cost by limiting the number of PMU channels has also been explored in the literature [13]–[15]. These approaches [1]–[15] assume that the PMU installation cost is mainly determined by the cost of the PMUs; therefore the OPP solution is the same as minimizing the number of PMUs.

More recently, researchers have started changing the scope of the OPP problem [16]–[19]. For instance, the authors of [16] have introduced the idea of including the communication network cost in the OPP problem, by minimizing the number of PMUs and the distance between the Phasor Data Concentrator (PDC) and the PMUs. In [17] dual-use line relays were used to reduce of the total cost for the OPP. On the other hand, [18] has suggested that installing the minimum number of PMUs may or may not lead to a minimum cost. In other words, the total cost of the PMU placement depends not only on the PMU cost and the communication infrastructure but also the security upgrades and the availability of potential transformers (PTs) and current transformers (CTs). This is also supported by the fact that PMUs on average cost about 5% of the total installation cost [20] since most of the cost is spent on upgrading the infrastructure to utilize the PMUs effectively.

On the other hand, there have been studies that focused on other aspects, such as system stability, bad data detection, and state estimation. In [21], [22] the authors have suggested ranking the critical buses and then placing the PMUs incrementally while enhancing the observability in the process. State estimation based OPP problem is solved in [23] using semidefinite programming and convex relaxation. In [24], ILP is used to solve the OPP problem and detect bad data and critical measurements. The empirical observability Gramian is used to quantify and solve the OPP problem for dynamic state estimation. Most multi-stage OPP approaches use the number of PMUs as the limit for each given state [21], [22], [25]–[28].

Prior research [13], [29] has also examined the impact of PMU failures on system observability. In these works, failures of connected transmission lines have been treated as channel failures, thereby capturing the dependency of observability on transmission line failure probabilities. Our work, on the other hand, utilizes the vulnerability resulting from all possible system contingencies, including generator, transformer and transmission line failures. The OPP problem has been defined in such a manner as to minimize the expected impact of these contingencies as a secondary problem to minimizing the cost of PMU deployment including PMU and infrastructure costs.

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The concepts of reliability and fault-tolerance are closely related, yet distinct. While reliability is the ability of a system to perform its required function within a specific period, fault-tolerance is defined as the ability of a system to perform despite a failure (fault) event [30], [31]. Nevertheless, both concepts share some common metrics such as failure rate, repair rate, and availability. The proposed strategy of deploying PMUs in the proximity of higher probability contingencies increases the likelihood of more effective remedial actions, both preventive and corrective, thereby enhancing fault-tolerance. The work presented in this paper is motivated by the idea that the value of a high cost asset such as PMU can be increased by deploying it so as to improve network fault-tolerance.

A new criterion for PMU placement is proposed that incorporates fault-tolerance into the OPP framework while minimizing the installation cost of PMUs. Reliability indices along with conditional probabilities are used to determine the critical components and assess the network vulnerability. The results from the vulnerability analysis are integrated into the OPP problem. Therefore, critical buses are given higher priority while considering both the network connections and the installation cost of PMUs. This enhances the situational awareness of network vulnerabilities and enables better operational control during faults. The main contributions of the proposed approach are:

- It improves the fault-tolerance of the system by enhancing the observability of critical components, which are determined using vulnerability analysis. This analysis is based on the reliability indices of the composite system.
- It integrates the vulnerability analysis into the observability function. This integration enables for cost-efficient OPP solutions while considering vulnerable buses. This integration of critical buses can be expanded to include multiple criteria for application-based OPP approaches.
- It uses a cost model for the installation of PMUs which is derived from the industry [20], [32]. Unlike most application-based OPP approaches which try to minimize the number of PMUs as in [21]–[24], this work uses the installation cost to achieve a more realistic strategy.
- It can be used as a multi-stage process under a constrained budget, or as a single-stage process while minimizing the total cost.
- It can achieve the optimal solution with the maximum observability in the case of multiple optima due to the nature of the bi-level formulation.

This paper is organized as follows. Section II discusses the fault-tolerance and the vulnerability assessment. In Section III, the cost of installing PMUs is presented. The proposed approach is presented in Section IV. Sections V and VI present the case studies and the conclusion respectively.

## II. FAULT-TOLERANCE: VULNERABILITY ANALYSIS

The vulnerability analysis applied here has two primary steps. The first step is to identify the critical components of the system. The second step is to rank and validate monitoring these components. There are several approaches to assess the criticality of the system components. In [33], [34] Markov

cut-set method along with DC-optimal power flow (DC-OPF) is used to assess the reliability of composite systems and to determine the critical components of the system. In [35], the critical components of the system are identified based on the impact of component outages on the interruption cost under predetermined loading conditions and forced outages.

The network vulnerability is determined by using AC-optimal power flow (AC-OPF) and determining the reliability indices of the conditional probability for the composite system  $S$ , given that component  $c_k$  has failed (2). The system failure is considered to be a load curtailment event, which can be considered as a *conditional* loss of load probability (LOLP), conditioned by a contingency, as shown in (2).

This approach assesses the direct impact of each component on the system. Each component  $c_k$  in the system has a failure rate  $\lambda_k$  and a repair rate  $\mu_k$ . The probability of failure  $Q_k$  is shown in (1). To properly determine the criticality of a component  $c_k$  on the system, the failure rate of that component is inflated so that the probability of failure is almost one  $Q_k \approx 1$ . Then the reliability indices of the system are obtained by using AC-OPF along with Monte Carlo simulations. The results from this analysis represent the criticality of component  $c_k$  to the system.

$$Q_k = \frac{\lambda_k}{\lambda_k + \mu_k} \quad (1)$$

$$\Gamma_k = P_{k,f}(S_f|c_{k,f}) \quad (2)$$

where

$\Gamma_k$  criticality of component  $c_k$ ;

$P_{k,f}$  the probability of a system failure event given that component  $c_k$  has failed;

$S_f$  the event of the composite system failure;

$c_{k,f}$  the component  $c_k$  failure event.

However, since the purpose of this analysis is to prioritize the location of the PMUs for installation, ranking components by their criticality is not enough. The failure rate of the individual components needs to be considered. For instance, consider the 3-bus system in Fig. 1 which has one generator, two transmission lines and two loads. It can be observed that the generator and transmission line  $L_1$  are the critical components for this system. However, if these components are highly reliable and transmission line  $L_2$  is unreliable, monitoring the transmission line  $T_2$  takes priority over transmission line  $T_1$ . Therefore, it is not enough to consider the criticality of a component, but the reliability of each component needs to be considered too. The vulnerability indices in (3) consider both the criticality of each component and its failure rate. The pseudo algorithm for the vulnerability analysis is shown in Procedure 1.

$$V_k = \mathcal{N}(\Gamma_k Q_k) \quad (3)$$

where

$V_k$  the vulnerability index for component  $k$ ;

$Q_k$  the failure probability of component  $k$ ;

$\mathcal{N}$  the cumulative distribution function (CDF) for the normal distribution.

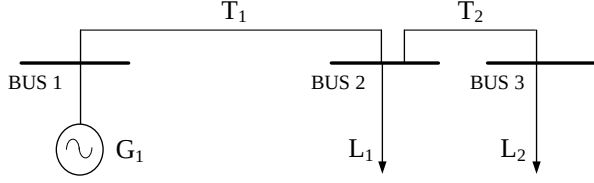


Fig. 1. 3-bus sample system

**Procedure 1** Network Vulnerability

Initialization load system and reliability data

 $N_c$  : the number of components**for**  $k = 1 : N_c$ Inflate the probability of failure  $Q_k$  for component  $c_k$ 

Perform Monte Carlo simulation

Obtain  $\Gamma_k$ Calculate  $V_k = \mathcal{N}(\Gamma_k Q_k)$ , and reset  $Q_k$ **endfor**

The results from this analysis are used to give the vulnerable buses higher priority by forming a bias matrix containing the vulnerability indices. The elements of the bias matrix are determined as in (4). When applying the vulnerability analysis to the sample system in Fig.1, the components of the system will get a vulnerability index  $V_k$ . These indices are used to build the bias matrix, a  $3 \times 3$  matrix in this case, where the elements of the bias matrix are determined using (4). These elements are incorporated into the observability in section IV-B.

$$v_{i,j} = V_k, \text{ if component } c_k \text{ is connecting bus } i \text{ to bus } j$$

$$v_{i,i} = \sum_{j=1}^{j=N} 1 + v_{i,j} - \sum_{j=1}^{j=N} H_{i,j} \quad (4)$$

where  $N$  is the number of buses and  $H$  is the  $N \times N$  connectivity matrix defined in (5).

$$H_{ij} = \begin{cases} 1, & \text{if a PMU is installed at bus } i \\ 1, & \text{if there is a branch connecting} \\ & \text{bus } i \text{ and bus } j \\ 0, & \text{else} \end{cases} \quad (5)$$

**III. PMU INSTALLATION COST**

Most of the cost is associated with the installation process and the substation infrastructure rather than the PMUs themselves. A report published by the U.S. Department of Energy (DOE) has showed that the PMU installation cost ranges from \$40,000 to \$180,000 per PMU [20]. The cost varies depending on the infrastructure support for the PMUs. Therefore, it stands to reason that installation cost should be considered instead of minimizing the number of PMUs.

Although PMU installation requires the same infrastructure upgrades, such as communication, cyber-security and other equipment upgrades, the approach to install them can differ depending on the utility and the existing infrastructure support for the PMUs. For instance, a utility can install new stand alone PMUs, or upgrade existing digital relays to enable PMU

functionality [20]. Moreover, installing PMUs also depends on the availability of CTs and PTs [18]. Once the substation's infrastructure is in place to support PMUs, the installation cost can go down to 35% of the initial cost [20]. This paper uses a model that includes the basic cost of each PMU unit in addition to the infrastructure cost, such as communication, security, PT and CT which may differ from one substation to another. The installation cost for PMUs is modeled as follows:

$$C = \sum_{i=1}^N (a_i + b_i + k(i)) P_i + \text{PDC} \quad (6)$$

where

 $a_i$  unit cost for PMU at substation  $i$  $b_i$  cost for installing additional channels at substation  $i$  $k(i)$  cost function for the communication network infrastructure at substation  $i$ ;

PDC cost of phase data concentrator.

The  $P_i$  terms form a vector  $P = [P_1, P_2, \dots, P_i, \dots, P_N]^T$ , where  $P_i$  assumes the value of zero or one indicating if a PMU is to be installed at substation  $i$ . The cost coefficient for the communication network  $k(i)$  at substation  $i$ , can take values between zero—if the communication infrastructure is sufficient for the installation—or \$8,000 per branch if the communication infrastructure are to be updated, as described in (7).

$$k(i) = \sum_{j=1}^n \text{len}_{i,j} \text{cc}_{i,j} \quad (7)$$

where  $n$  is the number of buses, and  $\text{len}_{i,j}$  is the transmission line between buses  $i$  and  $j$ . The communication cost  $\text{cc}$  is either \$8,000 or \$0.

The PMU installation cost model in (6) considers additional PMU channels and their associated costs, by considering the additional equipment needed for these channels such as PTs and CTs. It is assumed that the PMU will utilize all available channels. However, the cost of these channels is considered by the variable  $b_i$ , which differs from one substation to another depending on the number of channels and the type of available equipment. For instance, instead of using PTs for voltage measurements, capacitively coupled voltage transformers can be used. This can be accommodated by changing the value of the veritable  $b_i$  to correspond to the specific equipment available at substation  $i$ .

**IV. PROPOSED APPROACH**

This section presents the approach for the optimal placement of PMUs. As discussed in the previous section the cost plays a critical role in determining the optimal installation process. In the proposed approach, the cost is given a higher priority over the network and vulnerability. The approach assumes that the utility sets a budget for the installation of PMUs and the higher objective is to minimize the cost and to make sure the optimal placement does not exceed the budget. In the lower objective, the network observability is maximized while given a bias for the vulnerable buses or branches. This bias is used to prioritize the vulnerable buses for the OPP.

### A. Problem Statement

As discussed earlier, the OPP highly depends on the installation cost and the available budget. Therefore, the cost function in (8) is treated as higher level objective function. The observability function in (9) is treated as a lower level objective, subjected to the higher level objective (cost function). This bi-level setup allows maximizing the observability while minimizing the cost without violating the budget constraint, thereby reaching the optimal solution for the given budget. Moreover, this bi-level formulation enables for the optimal solution with the maximum observability to be selected in the case of multiple optimal solutions. The vulnerability of the network is incorporated into the observability function as bias for vulnerable buses or branches as in (12). These vulnerability indices are obtained using Procedure 1 and (4).

$$\min C = \sum_{i=1}^N (a_i + b_i + k(i))P_i + \text{PDC} \quad (8)$$

subject to

$$C \leq C_{\text{budget}} \quad (9)$$

where

$$\mathcal{O}' = H' \times B \quad (10)$$

$$B_i = \begin{cases} 1, & \text{if a PMU is installed at bus } i \\ 0, & \text{else} \end{cases} \quad (11)$$

$$H'_{ij} = \begin{cases} 1 + v_{i,i}, & \text{if a PMU is installed at bus } i \\ 1 + v_{i,j}, & \text{if there is a branch connecting bus } i \text{ to bus } j \\ 0, & \text{else} \end{cases} \quad (12)$$

$v_{i,j}$  the vulnerability element  $(i, j)$  in the bias matrix of (4);  
 $v_{i,i}$  the vulnerability element  $(i, i)$  in the bias matrix of (4);  
 $N$  number of buses.

Maximizing the modified observability vector  $\mathcal{O}'$  gives priority to the vulnerable elements in the network. However, this vector cannot be used to test the observability of the network since the observability is skewed by the vulnerability indices. Therefore, the constraints in (13) and (14) are added to the second objective function in (9). The constraint in (13) is used to give high priority to vulnerable buses and the constraint in (14) is used to check for the desired observability.

$$\frac{\mathcal{O}'}{\mathcal{O}} \geq \gamma \quad (13)$$

$$\mathcal{O} \geq \mathbf{1} \text{ "complete observability constraint"} \quad (14)$$

where

$$\mathcal{O} = H \times B$$

$\gamma$  priority index;

**1** a vector of length  $N$ , with all elements equal to 1;

**2** a vector of length  $N$ , with all elements equal to 2.

Zero Injection buses (ZIB) are buses with no load or generation units, and can be taken advantage of to reduce the number of PMUs required for achieving observability. Since the sum of all currents flowing to the ZIB is zero, Kirchhoff's current law can be used to determine the current flow through

the adjacent branches. The ZIB effect can be summarized as follows: A ZIB is considered observable if all adjacent buses of the ZIB are observable. An unobservable bus that is adjacent to a ZIB is considered observable if the ZIB and all of the other buses adjacent to the ZIB are observable.

### B. Algorithm

The optimal placement problem in subsection IV-A is a discontinuous bi-level problem. Therefore, evolutionary algorithms are the appropriate tools for solving such a problem. Several evolutionary algorithms can be used for solving the OPP problem; the proposed algorithm uses opposition-based elitist genetic algorithm (O-BEGA) to solve the bi-level OPP in subsection IV-A. The opposition element is added to the algorithm to enhance the overall performance since opposition-based methods have proven their superiority in terms of convergence-speed and results [36], [37]. The overall flowchart for proposed algorithm is shown in Fig. 2.

The proposed algorithm solves the OPP problem in a parallel manner by initializing random candidates where each candidate  $X$  has the length of the number of buses  $N$ . However, this approach necessitates using of higher objective (cost objective); otherwise, the solution would converge to installing the PMUs on all buses. The complexity rises from minimizing the cost (higher objective) while maximizing the observability function.

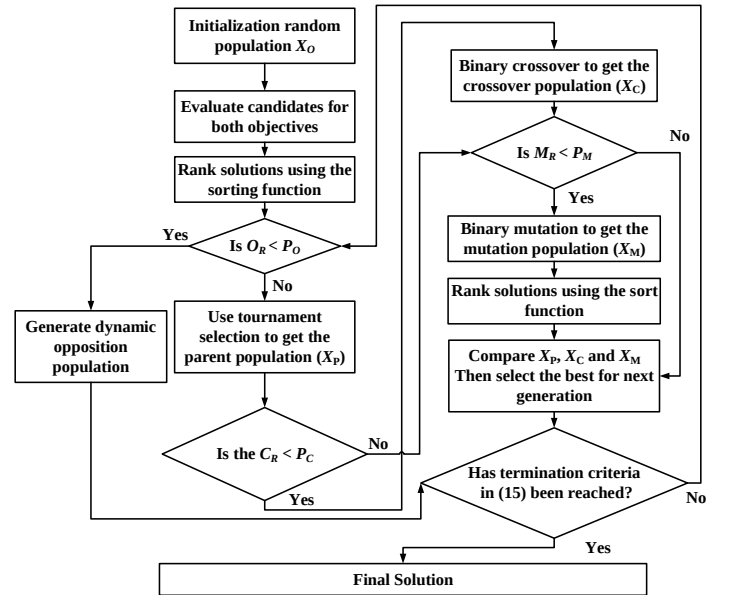


Fig. 2. Flowchart of the proposed algorithm

The higher and the lower objectives share the same decision variables, meaning there are no decision variables exclusive to one objective or the other. The proposed approach exploits this advantage to evaluate both objectives simultaneously without using different search spaces for each objective.

The proposed approach uses a sorting function to handle the simultaneous evaluation in the same search space. The sorting function sorts all candidates according to the feasibility, and fitness of both the higher and lower objectives, as shown in

Fig. 3. As a result, the algorithm is guided towards the optimal solution where the cost is minimized and the observability is maximized.

The algorithm starts by randomly generating an initial population  $X_O$ , and then evaluating and ranking this population. The dynamic opposition is used when the random variable  $O_R$  is lower than opposition probability ( $P_O = 0.4$ ); where the dynamic opposition is applied on a third of the randomly selected variables. The parent population ( $X_P$ ) is selected using tournament selection, where two random candidates are matched against each other, and the winner is selected as a parent candidate for generating the crossover population ( $X_C$ ). The probability of generating the crossover population  $P_C$  is 0.8, and the  $X_C$  is generated using binary double point crossover. The mutation population  $X_M$  is generated with a probability  $P_M$  of 0.1 using one-bit binary mutation, where one variable is randomly chosen and changed. The crossover and mutation steps are performed after comparing the probabilities of  $P_C$  and  $P_M$  against the random variables  $C_R$  and  $M_R$ . After all the populations ( $X_p, X_M, X_c$ ) have been generated, they are evaluated and ranked using the sorting function, then the termination criteria in (15) are checked. If the termination criteria have not been reached, the best among the current population is sent for the next iteration. The algorithm is summarized in Fig. 2

The algorithm is terminated if the conditions in (15) are met. In  $\Upsilon(1)$ , the mean of the population observability is measured against the member with the maximum observability in the current population. As the algorithm converges, the left-hand side of  $\Upsilon(1)$  will converge to zero. Since the member with the maximum observability is more likely to be feasible due to the nature of the OPP problem, measuring the population mean observability against the maximum observability serves as an effective measure of feasibility.

In  $\Upsilon(2)$ , the mean of the installation cost is measured against the member with the maximum installation cost in the current population. Notice that the member with the maximum installation cost is more likely to represent a feasible yet sub-optimal solution. Nevertheless, as the population converges, this member will start drifting towards the mean of the population—since all the population members drift toward the optimal solution upon convergence. Therefore, the left-hand side of  $\Upsilon(2)$  tends to zero as the algorithm progresses.

The terms  $\alpha$  and  $\beta$  are constants. These constants are inversely proportional to the size of the system. Therefore, the  $\alpha$  and  $\beta$  values that work for a big system will work for a smaller system, but the converse may not hold. The  $\alpha$  and  $\beta$  are also proportional to the size of the population; the smaller the size of the population the smaller these constants must be.

$$\Upsilon = \begin{cases} \Upsilon(1) = |\bar{O}' - \max(O')| \leq \alpha \\ \Upsilon(2) = |\bar{C} - \max(C)| \leq \beta \end{cases} \quad (15)$$

### C. Single-Stage Installation

In a single-stage installation process, the OPP is solved for achieving the desired observability within one budget period. In other words, it is assumed that there are enough resources to deploy the PMUs at the required locations in a relatively short

| Variables |       |       |     |       | Cost  | $\mathcal{O}'$ | Rank |
|-----------|-------|-------|-----|-------|-------|----------------|------|
| Bus 1     | Bus 2 | Bus 3 | ... | Bus N |       |                |      |
| 0         | 1     | 1     | ... | 1     | 62000 | 17.4           | 1    |
| 0         | 1     | 0     | ... | 0     | 54000 | 14.4           | 2    |
| 0         | 1     | 0     | ... | 1     | 55000 | 13.2           | 3    |
| ...       | ...   | ...   | ... | ...   | ...   | ...            | ...  |
| 1         | 0     | 0     | ... | 0     | 3000  | 3.9            | N-2  |
| 0         | 1     | 0     | ... | 0     | 2400  | 2.6            | N-1  |
| 0         | 0     | 0     | ... | 1     | 1400  | 1.4            | N    |

Fig. 3. Sorting function

period. It should be noted that most of the OPP approaches are single-stage processes. The desired observability such as normal conditions, single line outage or a single PMU outage.

In any of the desired observability conditions the priority index  $\gamma$  of (13) is used. This index gamma assumes values in the interval  $[1, 2]$ . When  $\gamma$  is set to 1, all buses are treated equally from the vulnerability perspective. When  $\gamma$  is greater than 1, higher priority is given to the vulnerable buses. For the complete observability under normal conditions, the placement of PMUs must satisfy the observability constraint in (14).

As for observability under a single line outage, it is necessary to have at least two measurement redundancies for every bus in the network. Therefore, the observability constraint in (14) is changed to (16).

$$H \times B = \mathcal{O}^s \geq 2 \quad (16)$$

where  $\mathcal{O}^s$  is the observability vector for all buses in the system under single line outage contingency, as shown in (17).

$$\mathcal{O}_i^s = \begin{cases} 2, & \text{if a PMU is installed at bus } i \\ \sum (a_{ij} \times b_i), & \text{otherwise} \end{cases} \quad (17)$$

In a single PMU outage, every bus needs to have at least two independent measurements. Either by two different PMUs, or if ZIB effect is considered, through KCL and a PMU. Therefore, the observability constraint in (14) is changed to (18).

$$H \times B = \mathcal{O}^p \geq 2 \quad (18)$$

where  $\mathcal{O}^p$  is the observability vector for all buses in the system under single PMU outage contingency, as shown in (19).

$$\mathcal{O}_i^p = \sum (a_{ij} \times b_i) \quad (19)$$

### D. Multi-Stage Installation

In multi-stage installation OPP approaches, a more realistic scenario is assumed, where existing resources may not be enough to achieve the desired observability in one budget period. Therefore, the desired observability is achieved over several stages or years due to limited resources or budget.

The majority of existing multi-stage methods assume the minimum number of PMUs per stage. This assumption is unrealistic since PMU installation is restricted by the financial burden, infrastructure of the substations, and technical benefits at each stage.

The proposed approach uses a financial capital  $C_{budget}$ , as the limit for each stage instead of using the number of PMUs as the limit. The buses are prioritized using the vulnerability indices as in (12). The same bi-level model in section IV-A is

used, except the modified observability function  $\mathcal{O}'$  is treated as the higher objective, and the cost objective is treated as the lower objective. In the final stage, the objectives are switched back to the original formulation with the observability as the lower objective and the installation cost as the higher one. The observability constraint in (14) is changed to the following.

$$\mathcal{O} \leq 1$$

At each stage  $k$ , the pre-installed PMUs ( $\text{PMU}^{k-1}$ ) are initiated, and the pre-installation cost  $C^{k-1}$  is calculated using (6). The budget for each stage  $C_{budget}^k$  is set. Then, the budget  $C_{budget}^k$  is modified to include the pre-installed PMUs cost  $C^{k-1}$ . The pre-installed PMU locations are maintained for the initial random population  $X_O$  and for the mutation population  $X_M$ . Since the pre-installed PMU locations are maintained for the initial and parent populations, the crossover population  $X_C$  maintains the pre-installed PMU locations by default. The multi-stage approach is summarized in Procedure 2.

$$C_{new,budget}^k = C_{budget}^k + C^{k-1} \quad (20)$$

### Procedure 2 Multi-Stage Installation

Initialization number of stages ( $n$ ),  $C_{budget}$  for each stage

for  $k = 1 : \text{number of stages}$

set  $\text{PMU}^{k-1}$ ,  $C_{budget}^k$

calculate  $C^{k-1}$ ,  $C_{new,budget}^k$  using (20)

if  $k = \text{number of stages}$

Switch the higher and lower objectives

endif

Optimize the PMUs installation using the O-BEGA in section IV-B

$k = k + 1$

endfor

## V. SIMULATION AND RESULTS

This section illustrates the application of the proposed OPP approach. In the first part of this section, the OPP approach is presented and compared with other approaches in the literature. Then, the OPP solutions for the cost criteria are presented. The second part of this section presents the vulnerability analysis and the OPP solution when incorporating this analysis. The algorithm is implemented in MATLAB, and the computation times for all three cases are under 2 seconds on a single processor.

### A. Considering Installation Cost

In this section, the analysis is done without including the vulnerability analysis. The model in section III is used, and no bias is given to any bus. The buses are divided into two categories: prepared buses and unprepared buses. The prepared buses are assumed to have sufficient infrastructure and require basic security and network upgrades, which are covered under the basic PMU installation. The unprepared buses, on the other hand, require more upgrades such as additional PTs and CTs for the added measurements. Where as the prepared buses are assumed to cost 35% of the initial cost. The base cost per

TABLE I  
COMPARISON OF THE PROPOSED SOLUTION WITH OTHER SOLUTIONS:  
EFFECT OF ZIB IS NOT CONSIDERED

| System                                                             | Ref. [38]                          | Ref. [1]                                          | Ref. [18] <sup>b</sup>                 | Proposed                                                       |
|--------------------------------------------------------------------|------------------------------------|---------------------------------------------------|----------------------------------------|----------------------------------------------------------------|
| Under Normal Operating Conditions; Effect of ZIB Is Not Considered |                                    |                                                   |                                        |                                                                |
| IEEE 14-bus                                                        | 4 PMUs<br>$\sum \mathcal{O} = 19$  | 4 PMUs<br>$\sum \mathcal{O} = 16$                 | 4 PMUs<br>$\sum \mathcal{O} = 19$      | 4 PMUs<br>$\sum \mathcal{O} = 19$                              |
| IEEE 30-bus                                                        | 10 PMUs<br>$\sum \mathcal{O} = 50$ | 10 PMUs<br>$\sum \mathcal{O} = 50$                | 10 PMUs<br>$\sum \mathcal{O} = 47$     | 10 PMUs<br>$\sum \mathcal{O} = 52$                             |
| IEEE RTS                                                           | 7 PMUs<br>$\sum \mathcal{O} = 34$  | 7 PMUs<br>$\sum \mathcal{O} = 34$                 | 7 PMUs<br>$\sum \mathcal{O} = 29$      | 7 PMUs<br>$\sum \mathcal{O} = 34$                              |
| Single Line Outage; Effect of ZIB Is Not Considered                |                                    |                                                   |                                        |                                                                |
| IEEE 14-bus                                                        | —                                  | 7 PMUs <sup>a</sup><br>$\sum \mathcal{O} = 23^a$  | 7 PMUs<br>$\sum \mathcal{O} = 25$      | 7 PMUs<br>$\sum \mathcal{O} = 25$                              |
| IEEE 30-bus                                                        | —                                  | 15 PMUs <sup>a</sup><br>$\sum \mathcal{O} = 62^a$ | 16 PMUs<br>$\sum \mathcal{O} = 59, 54$ | 16 PMUs, 15 PMUs <sup>a</sup><br>$\sum \mathcal{O} = 60^a, 62$ |
| IEEE RTS                                                           | —                                  | 11 PMUs <sup>a</sup><br>$\sum \mathcal{O} = 45^a$ | 12 PMUs<br>$\sum \mathcal{O} = 45, 48$ | 12 PMUs, 11 PMUs <sup>a</sup><br>$\sum \mathcal{O} = 57^a, 49$ |
| Single PMU Outage; Effect of ZIB Is Not Considered                 |                                    |                                                   |                                        |                                                                |
| IEEE 14-bus                                                        | —                                  | —                                                 | 9 PMUs<br>$\sum \mathcal{O} = 36, 37$  | 9 PMUs<br>$\sum \mathcal{O} = 39$                              |
| IEEE 30-bus                                                        | —                                  | —                                                 | 21 PMUs<br>$\sum \mathcal{O} = 80, 81$ | 21 PMUs<br>$\sum \mathcal{O} = 85$                             |
| IEEE RTS                                                           | —                                  | —                                                 | 14 PMUs<br>$\sum \mathcal{O} = 65$     | 14 PMUs<br>$\sum \mathcal{O} = 65$                             |

<sup>a</sup> Ignores radial buses in contingency cases.

<sup>b</sup> Has multiple optimal solutions, maximum observability is shown.

TABLE II  
COMPARISON OF THE PROPOSED SOLUTION WITH OTHER SOLUTIONS:  
EFFECT OF ZIB IS CONSIDERED

| System                                                         | Ref. [38]                          | Ref. [4]                              | Ref. [1]                                          | Ref. [39]                          | Ref. [18] <sup>b</sup>                 | Proposed                           |
|----------------------------------------------------------------|------------------------------------|---------------------------------------|---------------------------------------------------|------------------------------------|----------------------------------------|------------------------------------|
| Under Normal Operating Conditions; Effect of ZIB Is Considered |                                    |                                       |                                                   |                                    |                                        |                                    |
| IEEE 14-bus                                                    | 3 PMUs<br>$\sum \mathcal{O} = 16$  | 3 PMUs<br>$\sum \mathcal{O} = 16$     | 3 PMUs<br>$\sum \mathcal{O} = 16$                 | 3 PMUs<br>$\sum \mathcal{O} = 16$  | 3 PMUs<br>$\sum \mathcal{O} = 16$      | 3 PMUs<br>$\sum \mathcal{O} = 16$  |
| IEEE 30-bus                                                    | 7 PMUs<br>$\sum \mathcal{O} = 47$  | 7 PMUs<br>$\sum \mathcal{O} = 44, 47$ | 7 PMUs<br>$\sum \mathcal{O} = 53$                 | 7 PMUs<br>$\sum \mathcal{O} = 55$  | 7 PMUs<br>$\sum \mathcal{O} = 55, 56$  | 7 PMUs<br>$\sum \mathcal{O} = 56$  |
| IEEE RTS                                                       | 6 PMUs<br>$\sum \mathcal{O} = 38$  | —                                     | 6 PMUs<br>$\sum \mathcal{O} = 34$                 | —                                  | 6 PMUs<br>$\sum \mathcal{O} = 33, 36$  | 6 PMUs<br>$\sum \mathcal{O} = 40$  |
| Single Line Outage; Effect of ZIB Is Considered                |                                    |                                       |                                                   |                                    |                                        |                                    |
| IEEE 14-bus                                                    | —                                  | 7 PMUs<br>$\sum \mathcal{O} = 27$     | 7 PMUs <sup>a</sup><br>$\sum \mathcal{O} = 35^a$  | —                                  | 7 PMUs<br>$\sum \mathcal{O} = 25, 25$  | 7 PMUs<br>$\sum \mathcal{O} = 27$  |
| IEEE 30-bus                                                    | —                                  | 13 PMUs<br>$\sum \mathcal{O} = 57$    | 10 PMUs <sup>a</sup><br>$\sum \mathcal{O} = 55^a$ | 13 PMUs<br>$\sum \mathcal{O} = 59$ | 13 PMUs<br>$\sum \mathcal{O} = 58, 61$ | 13 PMUs<br>$\sum \mathcal{O} = 66$ |
| IEEE RTS                                                       | —                                  | —                                     | 9 PMUs <sup>a</sup><br>$\sum \mathcal{O} = 52^a$  | —                                  | 10 PMUs<br>$\sum \mathcal{O} = 50, 52$ | 8 PMUs<br>$\sum \mathcal{O} = 46$  |
| Single PMU Outage; Effect of ZIB Is Considered                 |                                    |                                       |                                                   |                                    |                                        |                                    |
| IEEE 14-bus                                                    | 7 PMUs<br>$\sum \mathcal{O} = 33$  | 7 PMUs<br>$\sum \mathcal{O} = 35$     | —                                                 | 7 PMUs<br>$\sum \mathcal{O} = 35$  | 7 PMUs<br>$\sum \mathcal{O} = 33, 35$  | 7 PMUs<br>$\sum \mathcal{O} = 35$  |
| IEEE 30-bus                                                    | 15 PMUs<br>$\sum \mathcal{O} = 61$ | 15 PMUs<br>$\sum \mathcal{O} = 63$    | —                                                 | 14 PMUs<br>$\sum \mathcal{O} = 61$ | 15 PMUs<br>$\sum \mathcal{O} = 65, 69$ | 14 PMUs<br>$\sum \mathcal{O} = 72$ |
| IEEE RTS                                                       | 13 PMUs<br>$\sum \mathcal{O} = 62$ | —                                     | —                                                 | —                                  | 12 PMUs<br>$\sum \mathcal{O} = 59$     | 11 PMUs<br>$\sum \mathcal{O} = 61$ |

<sup>a</sup> Ignores radial buses in contingency cases.

<sup>b</sup> Has multiple optimal solutions, maximum observability is shown.

PMU is assumed to be \$40,000, and the cost per additional PT or CT is assumed to be \$4,000. New communication lines and software upgrades are assumed to cost \$8,000, and PDC are assumed to cost \$8,000 [32].

Three test systems are used for the analysis: the IEEE 14-bus, IEEE 30-bus and IEEE RTS systems. First, the bi-level optimization approach is tested using minimum number of PMUs as the cost function. The results are compared with other approaches as seen in Table I and Table II. The proposed algorithm always achieve better or comparable performance with the existing OPP schemes.

There can be several optimal solutions that satisfy the observability conditions while requiring same number of PMUs. However, these optimal solutions may have different observability levels (redundancies)  $\sum \mathcal{O}$ . The higher the number of redundancies, the better the solution is, as long as the cost of the solution is the same. For instance, the IEEE 14-bus single PMU outage in Table I shows two optimal solutions.

TABLE III  
COST FROM THE PROPOSED APPROACH: EFFECT OF ZIB IS NOT CONSIDERED

|                                                                                             | Minimum number of PMUs                                                          |                            | Proposed                                                                       |                          | Unprepared buses  |
|---------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------|----------------------------|--------------------------------------------------------------------------------|--------------------------|-------------------|
|                                                                                             | PMU locations                                                                   | Cost From PMU installation | PMU locations                                                                  | Cost of PMU installation |                   |
| Under Normal Operating Conditions; Effect of ZIB Is Not Considered ( $\mathcal{O} \geq 1$ ) |                                                                                 |                            |                                                                                |                          |                   |
| IEEE 14-bus                                                                                 | 2, 6, 7, 9                                                                      | \$224,800                  | 2, 8, 10, 13                                                                   | \$81,200                 | 7, 9              |
| IEEE RTS                                                                                    | 2, 3, 8, 10, 16, 21, 23                                                         | \$239,600                  | 2, 3, 5, 7, 14, 16, 21, 23                                                     | \$168,000                | 10, 11, 17, 24    |
| IEEE 30-bus                                                                                 | 2, 4, 6, 9, 10, 12, 15, 19, 25, 27                                              | \$510,600                  | 3, 5, 8, 11, 13, 15, 17, 19, 22, 26, 30                                        | \$215,600                | 9, 12, 25, 27, 28 |
| Single Line Outage; Effect of ZIB Is Not Considered ( $\mathcal{O}^s \geq 2$ )              |                                                                                 |                            |                                                                                |                          |                   |
| IEEE 14-bus                                                                                 | 1, 3, 6, 8, 9, 10, 13                                                           | \$216,400                  | 1, 3, 6, 7, 10, 12, 14                                                         | \$184,800                | 7, 9              |
| IEEE RTS                                                                                    | 1, 2, 3, 4, 7, 10, 11, 12<br>15, 16, 20, 21, 22                                 | \$437,800                  | 3, 4, 5, 6, 7, 9, 13, 14,<br>15, 18, 20, 22                                    | \$247,800                | 10, 11, 17, 24    |
| IEEE 30-bus                                                                                 | 2, 3, 7, 8, 10, 11, 12, 13, 15<br>16, 19, 22, 24, 26, 27, 29                    | \$488,200                  | 1, 3, 5, 6, 8, 10, 11, 13, 14, 15,<br>16, 19, 21, 24, 26, 29, 30               | \$345,800                | 9, 12, 25, 27, 28 |
| Single PMU Outage; Effect of ZIB Is Not Considered ( $\mathcal{O}^p \geq 2$ )               |                                                                                 |                            |                                                                                |                          |                   |
| IEEE 14-bus                                                                                 | 2, 4, 5, 6, 7, 8, 9, 11, 13                                                     | \$329,800                  | 1, 2, 3, 7, 8, 10, 11, 12, 13, 14                                              | \$243,600                | 7, 9              |
| IEEE RTS                                                                                    | 1, 2, 3, 7, 8, 9, 10, 11,<br>15, 16, 17, 20, 21, 23                             | \$527,400                  | 1, 2, 3, 4, 5, 6, 7, 8, 13, 14<br>15, 16, 19, 21, 22, 23                       | \$333,200                | 10, 11, 17, 24    |
| IEEE 30-bus                                                                                 | 2, 3, 4, 6, 7, 9, 10, 11, 12, 13, 15, 16,<br>18, 20, 22, 24, 25, 26, 27, 28, 29 | \$789,200                  | 1, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15,<br>16, 18, 20, 22, 24, 25, 26, 29, 30 | \$640,000                | 9, 12, 25, 27, 28 |

TABLE IV  
COST FROM THE PROPOSED APPROACH: EFFECT OF ZIB IS CONSIDERED

|                                                                                         | Minimum number of PMUs                             |                          | Proposed                                               |                          | Unprepared buses  |
|-----------------------------------------------------------------------------------------|----------------------------------------------------|--------------------------|--------------------------------------------------------|--------------------------|-------------------|
|                                                                                         | PMU locations                                      | Cost of PMU installation | PMU locations                                          | Cost of PMU installation |                   |
| Under Normal Operating Conditions; Effect of ZIB Is Considered ( $\mathcal{O} \geq 1$ ) |                                                    |                          |                                                        |                          |                   |
| IEEE 14-bus                                                                             | 2, 6, 9                                            | \$140,800                | 2, 8, 10, 13                                           | \$81,200                 | 7, 9              |
| IEEE RTS                                                                                | 2, 8, 10, 16, 21, 23                               | \$234,000                | 2, 5, 7, 16, 22, 23                                    | \$123,200                | 10, 11, 17, 24    |
| IEEE 30-bus                                                                             | 2, 4, 10, 12, 19, 24, 27                           | \$314,600                | 3, 5, 10, 13, 15, 17, 19, 29                           | \$163,800                | 9, 12, 25, 27, 28 |
| Single Line Outage; Effect of ZIB Is Considered ( $\mathcal{O}^s \geq 2$ )              |                                                    |                          |                                                        |                          |                   |
| IEEE 14-bus                                                                             | 1, 3, 6, 8, 9, 11, 13                              | \$216,400                | 1, 3, 6, 8, 10, 12, 14                                 | \$138,600                | 7, 9              |
| IEEE RTS                                                                                | 1, 2, 7, 9, 10, 16, 20, 21                         | \$260,600                | 2, 5, 6, 7, 9, 16, 20, 21                              | \$170,800                | 10, 11, 17, 24    |
| IEEE 30-bus                                                                             | 1, 2, 4, 7, 10, 11, 12, 13, 15, 16, 19, 23, 26, 30 | \$330,600                | 1, 3, 5, 10, 11, 13, 14, 16, 18, 19, 23, 26, 30        | \$256,200                | 9, 12, 25, 27, 28 |
| Single PMU Outage; Effect of ZIB Is Considered ( $\mathcal{O}^p \geq 2$ )               |                                                    |                          |                                                        |                          |                   |
| IEEE 14-bus                                                                             | 2, 4, 5, 6, 9, 11, 13                              | \$227,600                | 1, 2, 3, 8, 10, 11, 12, 13, 14                         | \$179,200                | 7, 9              |
| IEEE RTS                                                                                | 1, 2, 3, 7, 8, 9, 10, 18, 19, 20, 21               | \$322,200                | 1, 2, 4, 5, 6, 7, 8, 13, 15, 16, 19, 21                | \$250,600                | 10, 11, 17, 24    |
| IEEE 30-bus                                                                             | 1, 2, 4, 6, 7, 10, 12, 13 15, 16, 18, 19, 24, 27   | \$460,200                | 1, 3, 5, 7, 10, 11, 12, 13, 15, 16, 18, 19, 23, 26, 29 | \$386,600                | 9, 12, 25, 27, 28 |

Both solutions require the same number of PMUs (7 PMUs); however the proposed approach achieves a better observability ( $\sum \mathcal{O} = 39$ ) with PMU locations  $\{2, 4, 5, 6, 7, 8, 9, 10, 13\}$ . The optimal solution of [18] achieves an observability of ( $\sum \mathcal{O} = 37$ ) with PMU locations  $\{1, 2, 4, 6, 7, 8, 9, 10, 13\}$ .

The proposed approach always achieve higher redundancy level because of the bi-level formulation, where the observability is treated as a lower objective and is being maximized, while the being subjected to the higher objective (minimize the cost). Other OPP approaches treat the observability as a constraint and try to minimize the cost (number of PMUs) as follows:

$$\begin{aligned} \min C &= \text{number of PMUs} \\ \text{s.t. } \mathcal{O} &\geq 1 \end{aligned} \quad (21)$$

Using the formulation in (21) should lead to an optimal solution. However, in the presence of multiple optimal solutions, this formulation may not lead to the solution with the higher number of redundancies as is evident from the results in Table I and Table II.

After verifying the performance of the bi-level optimization, the cost model in section (III) is used for the OPP. The PMU placement is performed for two different cases. In the first case, the PMUs are placed under normal conditions, under a single line outage and single PMU outage. Also, ZIBs are treated as normal buses. The result of the placement is shown

in Table III. The ZIB effect is considered in the second case under normal operation, single line outage and single PMU outage as shown in Table IV.

The results show that the using minimum number of PMUs does not necessitate a lower installation cost, especially when each substation has its own cost factors. For instance, consider the IEEE RTS solution for normal conditions in Table III. When the minimum number of PMUs is used as the objective, the installation cost is about \$239,600 with a total of a 7 installed PMUs. On the other hand, when the installation cost is used as the objective more PMUs are installed (8 PMUs) at a lower cost (\$81,200). The same phenomenon can be observed for the IEEE 14-bus case for normal conditions in Table IV.

The impact of different approaches (minimum number of PMUs approach and the proposed approach) on the level of redundancy of the solution can be observed from the cost. Consider the IEEE 14-bus under normal conditions in Table III; both solutions have the same number of PMUs (4 PMUs). However, the cost obtained by the proposed solution of  $\{2, 8, 10, 13\}$  with an observability of  $\sum \mathcal{O} = 14$  is much lesser compared to the other solution of  $\{2, 6, 7, 9\}$  which uses more channels and has a higher observability ( $\sum \mathcal{O} = 19$ ).

### B. Vulnerability Analysis

In this section, the vulnerability indices obtained from the vulnerability analysis are used to give bias to the vulnerable

buses. Monte Carlo simulation along with AC-OPF is used to obtain the conditional reliability for the IEEE RTS system [40]. Since the system failure is considered to be a load curtailment event, the loss of load probability (LOLP) is used as the indicator of the criticality of components. The results in Table V show the *conditional* LOLP of the transmission lines. The results also show the vulnerability indices of these components. The results from the conditional probability show that transmission lines  $T_{10}$  and  $T_{11}$  are highly critical to the system operation. Transmission line number  $T_{10}$  connects buses {6} and {10}, and transmission line number  $T_{11}$  connects buses {7} and {8} as seen in Fig. 4. To account for the reliability of the transmission lines: the probability of failure is multiplied by the conditional reliability indices. Then the CDF of the normal distribution is used to obtain the network vulnerability indices (V.I.) as in (3). The results are shown in Table V.

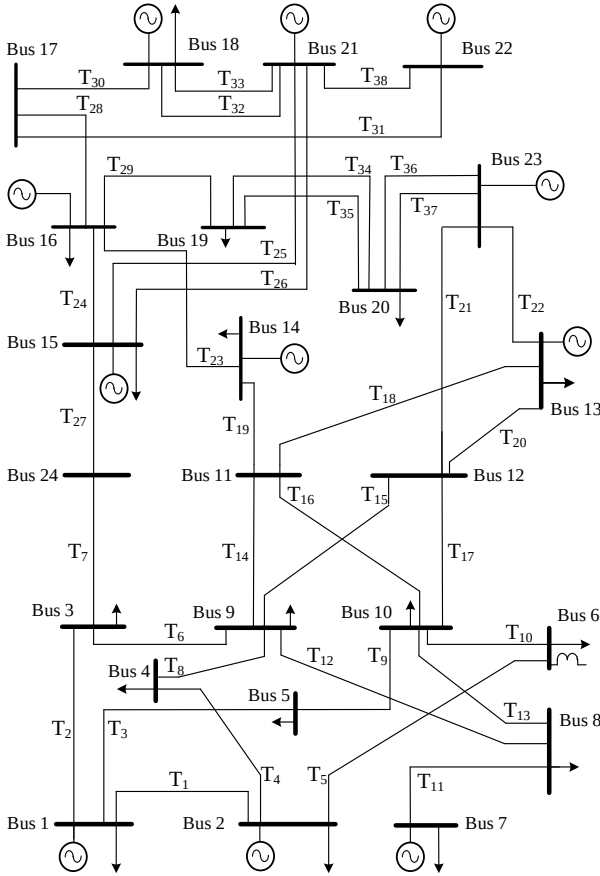


Fig. 4. IEEE RTS system

The vulnerability indices show that transmission lines  $T_{10}$  and  $T_{11}$  are still highly critical to the system operation. Therefore, the buses connected to these transmission lines are critical. The vulnerability indices are integrated into the observability function as described in subsection IV-A. By integrating the vulnerability indices into the observability, the OPP is solved to generate a more resilient solution while minimizing the total installation cost.

1) *Single-Stage Installation*: For the single-stage installation, the PMU placement is considered under all observability conditions. The results for the PMU placement are shown in Table VI and Table VII. The results show that higher priority

TABLE V  
CONDITIONAL RELIABILITY INDICES –VULNERABILITY INDICES (V.I.)

| Transmission Line Number | Conditional LOLP, $\Gamma_k$ | V.I.    | Transmission Line Number | Conditional LOLP, $\Gamma_k$ | V.I.    |
|--------------------------|------------------------------|---------|--------------------------|------------------------------|---------|
| 1                        | 0.1034                       | 0.40822 | 20                       | 0.1105                       | 0.42403 |
| 2                        | 0.1061                       | 0.40822 | 21                       | 0.1141                       | 0.45381 |
| 3                        | 0.1072                       | 0.4006  | 22                       | 0.1040                       | 0.43729 |
| 4                        | 0.1058                       | 0.41097 | 23                       | 0.1125                       | 0.42119 |
| 5                        | 0.1074                       | 0.42925 | 24                       | 0.1067                       | 0.40651 |
| 6                        | 0.1069                       | 0.40984 | 25                       | 0.1096                       | 0.42547 |
| 7                        | 0.1187                       | 0.65915 | 26                       | 0.1086                       | 0.42466 |
| 8                        | 0.1046                       | 0.40461 | 27                       | 0.1156                       | 0.43029 |
| 9                        | 0.1088                       | 0.40344 | 28                       | 0.1093                       | 0.41242 |
| 10                       | 1.0000                       | 0.99987 | 29                       | 0.1068                       | 0.40864 |
| 11                       | 1.0000                       | 0.82297 | 30                       | 0.1050                       | 0.40338 |
| 12                       | 0.1082                       | 0.42223 | 31                       | 0.1134                       | 0.45758 |
| 13                       | 0.1105                       | 0.42403 | 32                       | 0.1047                       | 0.40929 |
| 14                       | 0.1053                       | 0.62489 | 33                       | 0.1038                       | 0.40868 |
| 15                       | 0.1075                       | 0.63061 | 34                       | 0.1048                       | 0.41548 |
| 16                       | 0.1074                       | 0.63035 | 35                       | 0.1057                       | 0.41614 |
| 17                       | 0.1126                       | 0.64372 | 36                       | 0.1111                       | 0.41149 |
| 18                       | 0.1057                       | 0.42027 | 37                       | 0.1052                       | 0.40759 |
| 19                       | 0.1054                       | 0.41798 | 38                       | 0.1065                       | 0.43133 |

buses are monitored directly by the PMUs. However, the OPP cost is higher than normal PMU installation. This is due to the constraint on the priority index  $\gamma$ , which forces the OPP solution to a more secure and resilient solution, where the fault-tolerance of the system is enhanced. For instance, for normal conditions the critical components are directly monitored when the priority index is used by placing PMUs on buses {6, 9}. A better option, from fault-tolerance perspective, is to use buses {8, 10}, however this option has a much higher cost.

TABLE VI  
COST COMPARISON OF THE OPP; EFFECT OF ZIB IS NOT CONSIDERED

|                                                              | $\gamma = 1$                                                         | $\gamma = 1.5$                                                      | Unprepared buses |
|--------------------------------------------------------------|----------------------------------------------------------------------|---------------------------------------------------------------------|------------------|
| Normal Operating Conditions; Effect of ZIB Is Not Considered |                                                                      |                                                                     |                  |
| IEEE RTS                                                     | 2, 3, 5, 7, 14, 16, 21, 23<br>\$168,000                              | 3, 5, 6, 7, 9, 16, 21, 23<br>\$170,800                              | 10, 11, 17, 24   |
| Single Line Outage; Effect of ZIB Is Not Considered          |                                                                      |                                                                     |                  |
| IEEE RTS                                                     | 3, 4, 5, 6, 7, 9, 13, 14, 15, 18, 20, 22<br>\$247,800                | 2, 3, 5, 7, 9, 10, 13, 14, 15, 18, 20, 22<br>\$337,600              | 10, 11, 17, 24   |
| Single PMU Outage; Effect of ZIB Is Not Considered           |                                                                      |                                                                     |                  |
| IEEE RTS                                                     | 1, 2, 3, 5, 6, 7, 9, 13, 14, 15, 16, 18, 19, 21, 22, 23<br>\$329,000 | 1, 2, 3, 5, 6, 7, 8, 9, 12, 14, 15, 16, 19, 21, 22, 23<br>\$338,800 | 10, 11, 17, 24   |

TABLE VII  
COST COMPARISON THE OPP; EFFECT OF ZIB IS CONSIDERED

|                                                              | $\gamma = 1$                                      | $\gamma = 1.5$                                               | Unprepared buses |
|--------------------------------------------------------------|---------------------------------------------------|--------------------------------------------------------------|------------------|
| Normal Operating Conditions; Effect of ZIB Is Not Considered |                                                   |                                                              |                  |
| IEEE RTS                                                     | 1, 2, 8, 14, 20, 21<br>\$123,200                  | 5, 6, 7, 9, 20, 21, 22, 23<br>\$162,400                      | 10, 11, 17, 24   |
| Single Line Outage; Effect of ZIB Is Considered              |                                                   |                                                              |                  |
| IEEE RTS                                                     | 2, 5, 6, 7, 9, 16, 20, 21<br>\$170,800            | 3, 4, 5, 6, 7, 9, 16, 20, 21<br>\$190,400                    | 11, 12, 17, 24   |
| Single PMU Outage; Effect of ZIB Is Considered               |                                                   |                                                              |                  |
| IEEE RTS                                                     | 1, 2, 5, 6, 7, 9, 15, 16, 19, 21, 23<br>\$228,200 | 1, 2, 5, 6, 7, 8, 9, 14, 16, 20, 21, 22, 23, 24<br>\$339,400 | 11, 12, 17, 24   |

2) *Multi-Stage Installation*: The multi-stage PMU installation is performed as a three-stage process; each stage has its own budget. All the stages have a budget limit of \$70,000, where each stage is treated independently budget-wise. The results in Table VIII show the optimal PMU installation at each stage. The third stage presents the complete observability solution for both cases. The results show a comparison between treating all buses equally and using vulnerability indices. The results show that prioritizing vulnerable buses, produce a better observability and a more resilient OPP solution.

TABLE VIII  
MULTI-STAGE OPP; EFFECT OF ZIB IS NOT CONSIDERED

|                                       | All buses are treated equally                                                      | Vulnerability indices are used                                                    | Unprepared buses  |
|---------------------------------------|------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-------------------|
| Stage one<br>$C_{budget}^1 = \$7e4$   | 16, 21, 23<br>$\mathcal{O} = 13, \mathcal{O}' = 24.35$<br>\$64,400                 | 9, 21, 23<br>$\mathcal{O} = 13, \mathcal{O}' = 27.39$<br>\$65,800                 | 10, 11,<br>17, 24 |
| Stage two<br>$C_{budget}^2 = \$7e4$   | 1, 3, 8, 16, 21, 23<br>$\mathcal{O} = 25, \mathcal{O}' = 41.29$<br>\$63,000        | 2, 3, 8, 9, 21, 23<br>$\mathcal{O} = 19, \mathcal{O}' = 44.33$<br>\$63,000        | 10, 11,<br>17, 24 |
| Stage Three<br>$C_{budget}^3 = \$7e4$ | 1, 2, 3, 8, 14, 16, 21, 23<br>$\mathcal{O} = 32, \mathcal{O}' = 50.39$<br>\$40,600 | 2, 3, 5, 8, 9, 16, 21, 23<br>$\mathcal{O} = 34, \mathcal{O}' = 55.97$<br>\$42,000 | 10, 11,<br>17, 24 |

## VI. CONCLUSION

In this paper, a new approach for application-based OPP is presented. Traditionally, for application-based OPP approaches, the objective is to minimize the number of PMUs. This paper, however, considers the installation cost of PMUs, the observability, and the vulnerability of the system. The fault-tolerance aspect is enhanced through the prioritization of the critical buses for the installation of PMUs, where the criticality of the buses is quantified and integrated into the observability function using vulnerability analysis.

The proposed approach can be used as a single-stage or a multi-stage installation. The bi-level formulation enables the optimal solution with the maximum observability to be selected in the case of multiple optima. Moreover, the integration of the vulnerability indices into the observability function allows for a solution that can attain a trade-off between the technical benefits and the cost-effectiveness of the OPP. This process is controlled by a priority index, which can be determined by the utility at the planning stage. This incorporation of priority indices into the observability function can be used for other application-based OPP approaches or for multiple criteria application-based OPP approaches.

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