

Emergency Power Supply for Load Point Reliability

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ABSTRACT: Often, certain loads deemed critical by a customer are connected to the utility supply through an emergency power supply (also called backup or standby power supply) to provide continuity of service in the event of failure of the utility supply. This article describes the basic requirements and components of an emergency power supply. Since the purpose of an emergency power supply is essentially to enhance reliability of service to a load point, this article also shows how to assess service reliability and how to size the emergency power supply to provide a desired level of reliability.

KEYWORDS: emergency power supply, backup power supply, standby power supply, uninterruptible power supply, reliability, storage sizing.

1 Introduction

Utility electric power systems generally supply electric power of acceptable quality. It is, however, reasonable to assume that electric power from a single source will have interruptions which most of the customers can tolerate with some inconvenience but without any significant damage. Yet some customer loads, such as medical facilities, emergency lighting, data processing and communication centers, cannot tolerate interruptions in electric supply. Loss of power to medical facilities may result in loss of life, interruption of emergency lighting may lead to vandalism and communication and data processing centers can similarly have serious problems. For such loads, emergency and standby power systems [1, 2] are installed to provide electric power of acceptable quality and quantity to critical portions or all of the user facilities in an event of failure of normal electric supply.

Emergency and standby power systems are independent reserve sources of electric energy which, upon failure of the utility supply, can provide electric power of acceptable quality so that the concerned facilities may continue to operate satisfactorily. The difference between emergency and standby supplies is that the former comes into operation automatically, within a specified time following the failure of the normal supply. The appropriateness of installing an emergency or standby power system at a facility, as well as the configuration of the installed system, is determined by such factors as the criticality of the load, the nature of the load, and the justifiable cost

of reliability of the supply required by the load.

The following sections will briefly describe the required reliability concepts, the factors governing the choice and configuration of emergency and standby power systems, the components which constitute such systems, and methods of determining the reliability of such systems.

2 Reliability Metrics

In introducing reliability metrics, we will first introduce some basic reliability measures which apply to emergency and standby power supplies. For more detailed descriptions of reliability issues, see references [3–6].

Load interruption frequency: This is the mean number of interruptions per unit time. A more commonly used name is frequency of system failure.

Failure rate: This is the mean number of failures per unit up time. This may apply to a component or a system. The up time refers to the period of time during which the component or system is functional. For highly reliable systems, this is almost equal to the failure frequency.

Mean interruption time: This is the expected or long term average duration of a single interruption. It is more commonly known as mean down time.

Expected duration of load interruption: This is the average time of interruption during the interval under consideration. It is the product of load interruption frequency and mean interruption time.

Each facility or load has specific requirements in terms of acceptable interruption frequency or duration. For instance, a steel mill is very sensitive to interruption frequencies; a power failure during the processing of a job is likely to cause the job to be wasted and can be considerably expensive. Such a facility would require an emergency power supply which can come into operation almost immediately following failure of the primary supply. On the other hand, a chemical plant may use an electrolytic process which is not affected by frequent interruptions as long as the durations are brief, and the process continues as long as it receives energy. For such a plant an offline standby supply would probably be adequate. A facility which requires both frequency and duration of power interruptions to be extremely small is a hospital intensive care unit. For such a situation a highly reliable uninterruptible power supply is needed.

Higher reliability of supply obviously demands higher cost of installation, and the nature and criticality of the load determine the configuration of the emergency or

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standby power supply, the amount of reliability built into it, and whether it should be online or offline.

3 Constructing Emergency and Standby Power Systems

The equipment used in assembling emergency and standby power systems can be classified as sources and energy storage equipment, or switching equipment. Some of the commonly used sources and switching equipment will be briefly described here. A more thorough treatment is available in [1].

3.1 Power sources and energy storage equipment

Generators. Generators used in emergency and standby power systems may be engine driven or turbine driven. Engine driven generators normally use diesel, gasoline, natural gas or liquefied petroleum gas as fuels. Turbines use steam or gas as prime movers. These generators normally have low startup times, in the order of minutes or seconds, and capacities in the kilowatt to megawatt range. They are generally applied as offline units, and may be used in combination with electrical or mechanical energy storage devices.

Electrical energy storage devices. Batteries are the commonly used electrical energy storage devices. Normally, while commercial (utility) power is available, it is used to charge the battery through a rectifier. Upon failure of the utility supply, the energy stored in the battery is supplied to the critical load through an inverter. An inverter-battery system may be used as an offline supply, or it may be kept online, with an appropriate switching device which allows the load to draw power from the battery almost instantaneously following failure of the utility supply. The battery provides power to the load for a limited but known period of time, during which an alternative offline source may, if necessary, be brought into service if the utility supply is not restored within the period the battery power is known to be available.

Mechanical energy storage devices. Such a device generally consists of a rotating flywheel mounted on the connecting shaft of a motor-generator set. This is normally used online, with the motor continuously energized by the utility power, and the load connected to the generator in the motor-generator set. In the event of failure of the utility supply, the energy stored in the rotating flywheel continues to rotate the generator shaft for a few seconds, allowing time for an offline source to be brought into service if necessary. This kind of a mechanical energy storage system can be used in combination with other standby systems.

3.2 Switching devices

Switching devices are used to establish or change connections in or amongst circuits. The switching devices commonly used in emergency and standby power supplies are transfer switches and bypass switches.

Automatic transfer switch. This is a self-acting equipment for transferring one or more load conductor connections from one power source to another. Automatic transfer switches (ATS) may be mechanical or static (STS). Mechanical switches generally operate within tenths of a second. Static switches are more expensive but much faster.

Bypass switch. This is a device which is used in conjunction with an automatic transfer switch to provide a means of directly connecting load conductors to a power source and of disconnecting the automatic transfer switch. Bypass switches may be manual or synchronized.

3.3 Uninterruptible power supplies

An uninterruptible power supply (UPS) is a system designed to automatically come into operation, with negligible delay and transients, immediately following the failure of the utility supply, and continue to energize the load acceptably until normal supply is restored. An UPS generally includes generators or energy storage devices or combinations thereof, and automatic transfer switches and bypass switches. Redundancy may be incorporated by connecting two or more sources or storage devices in parallel, such that load is transferred simultaneously to all redundant systems. This increases the reliability of the UPS, since the likelihood of simultaneous failure of all the redundant systems is much lower than that of each individual system.

Reference [1] provides detailed descriptions of components and configurations of emergency and standby power systems. The following section describes and analyzes a typical configuration.

4 Reliability Analysis of Emergency and Standby Power Systems

Since the objective of emergency and standby power systems is to enhance the reliability of power supplied to a critical load, it is pertinent to briefly discuss some methods which can be used to determine the reliability of such systems and the extent to which such systems improve the reliability of power supplied to the critical load. Several methods of system reliability modeling and analysis are described in references (3)–(5). Some of these approaches will be illustrated in this section, by means

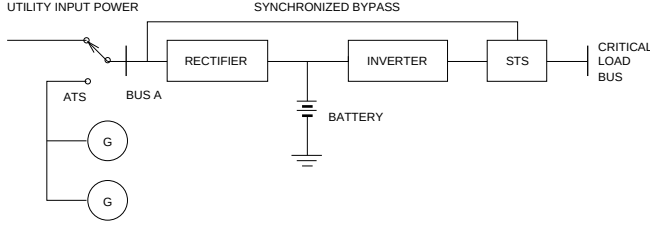


Figure 1: Parallel-supplied nonredundant uninterruptible power supply.

of an example. These techniques may be suitably adapted to analyze almost any emergency or standby power system.

4.1 Sample system description

Consider a sample system as shown in Fig. 1. The critical load normally receives power from utility input power through the UPS. A synchronized bypass and static switch protect the critical load in the event of an inverter failure. If voltage is lost to the critical load, the STS re-establishes voltage in less than one quarter of a cycle. This is considered continuous power for most loads.

When the generators are in standby mode, their failures remain undetected except during periodic inspections. Therefore, while starting, there is a probability p_s that a generator may fail to start.

Only one generating unit is taken out for planned maintenance. If a generator fails while the other is on planned maintenance, it is possible to accelerate the maintenance

Table 1: System data

Equipment/Supply	λ (f/y)	r (h/f)
Utility Supply, single circuit	0.53700	5.66
Generator (per hour of use)	0.00536	478.00
Inverter	1.25400	107.00
Rectifier	0.03800	39.00
ATS	0.00600	5.00
STS	0.08760	24.00
Battery	0.03130	24.00
Equipment Maintenance	freq (/y)	dur (h)
Generator	1.00	10.00
UPS	1.00	4.00
Other data:		
Battery can supply load for		4.0 h
Common mode failure of generators	(λ_{cm})	0.0
Acceleration factor for planned maintenance of generators	(α)	2.0
Probability of failure to start a generator	(p_s)	0.015

on the second generator by a factor of α .

If power fails at bus A, the battery can sustain the load for upto 4 hours.

Table 1 lists the data pertaining to the system. The failure rates (λ) and the failure durations (r) of all the components comprise the required data for analysis.

Two methods of analysis, and the determination thereby of failure rates and durations of supply at bus A and the critical load bus, will now be demonstrated.

4.2 Analytical method

The analytical method described here uses the concepts of Markov chains and Markov cut-sets. A detailed treatment of these techniques is available in [4]. The method as applied to the sample system is described in [7]. A summary of the method is presented here.

A *Markov chain* is a sequence of events consisting of the transition of a component or system of components amongst a set of states such that the state to which the component or system transits in the future depends only on the current state of the component or system and not on the states it has transited through in the past. If a system conforms to (or, more realistically, approximates) this behavior, then the theory of Markov chains may be applied to it. In this kind of analysis, information of interstate transition rates is used to determine the rate of transiting to failure states.

The transition rate from state i to state j is the mean rate of the system passing from state i to state j . In this paper, the durations of component states are assumed exponentially distributed, which means that the interstate transition rates are constant. This is explained in the next section.

The state probabilities can be obtained by solving

$$BP = C \quad (1)$$

where

B matrix obtained from A by replacing the elements of an arbitrarily selected row k by 1;

A matrix of transition rates such that the element $a_{ij} = \lambda_{ji}$ and $a_{ii} = -\sum_j a_{ji}$;

λ_{ij} constant transition rate from state i to j ;

P column vector whose i th term p_i is the steady-state probability of the system being in state i ;

C column vector with k th element equal to one and other elements set to zero.

Once the steady-state probabilities have been calculated, the reliability indices (4) can be computed using the following relationships.

Frequency of System Failure. The relationship for the frequency of system failure [9] is given by Eq. (2) or

Eq. (3)

$$f_f = \sum_{i \in (S-F)} p_i \times \sum_{j \in F} \lambda_{ij} \quad (2)$$

$$= \sum_{i \in F} p_i \times \sum_{j \in (S-F)} \lambda_{ij} \quad (3)$$

where f_f is the frequency of the system failure, S is the system state space and F is the subset of failed states.

Mean Down Time. The expected time of stay in F in one cycle is

$$r_f = P_f / f_f \quad (4)$$

where

$$P_f = \sum_{i \in F} p_i \quad (5)$$

A *cut set* is a set of components that, if removed from the system, results in loss of power to the load point. If this set does not contain any cut set as a subset, then it is called a *minimal* cut set. In this definition, component is used in a wider sense of hardware or a particular system condition. In general cut sets containing m components are called m -order cut sets. Generally upto second order cut sets are considered and the contribution from higher order cut sets is considered negligible. The following relationships are important in cut-set calculations.

1. Frequency and duration of a cut set

a. First order cut set [3]

$$f_{cs_i} = \lambda_i \quad (6)$$

$$r_{cs_i} = r_i \quad (7)$$

where

f_{cs_i}, r_{cs_i} frequency and mean duration of cut set i

λ_i, r_i failure rate and mean duration of component comprising cut set i

b. Second order cut set [3]

$$f_{cs_i} = \lambda_i \lambda_j (r_i + r_j) \quad (8)$$

$$r_{cs_i} = r_i r_j / (r_i + r_j) \quad (9)$$

where

λ_i, λ_j failure rates of components i and j comprising cut set i

r_i, r_j mean failure durations of components i and j comprising cut set i

c. Conditional second order cut set [3]

Assuming that component i fails given component j has failed,

$$f_{cs_i} = \lambda_i \lambda_j (r_j) \quad (10)$$

$$r_{cs_i} = r_i r_j / (r_i + r_j) \quad (11)$$

The terms failure frequency and failure rate are often used interchangeably. Strictly speaking failure rate is the mean number of failures per unit of the up time and failure frequency is the mean number of failures per unit of the total time. If the probability of system being up is close to unity, then these two quantities are very close. In general, failure frequency is less than failure rate.

2. Frequency and duration of interruption [3]

$$\begin{aligned} f_s &= \text{frequency of load interruption} \\ &= \sum_{\forall i} f_{cs_i} \end{aligned} \quad (12)$$

$$\begin{aligned} r_s &= \text{mean duration of load interruption} \\ &= \sum f_{cs_i} r_{cs_i} / f_s \end{aligned} \quad (13)$$

The theory concerning Markov chains and minimal cut-sets is dealt with in greater detail in [4]. These concepts are used as follows to analyze the sample system.

First, the combination of the utility supply and the two generators is analyzed for failure modes. Fig. 2 shows the possible states this combination can assume, and the transition rates between these states. Based on these transition rates, the probabilities and frequencies of occurrence of the failed states are determined.

Equations (1)–(5) can be used to determine the reliability indices of the utility-generator subsystem:

$$\begin{aligned} f_p &= \text{frequency of loss of power} \\ &= \text{frequency of states 3 and 8} \\ &= P_3(2\mu_g + \mu_s) + P_8(\mu_g + \mu_s + \frac{\alpha}{r_{mg}}) \end{aligned} \quad (14)$$

$$\begin{aligned} \lambda_p &= \text{failure rate} \\ &= f_p / (1 - P_3 - P_8) \\ &= 0.001576 \text{ failures/year} \end{aligned} \quad (15)$$

$$\begin{aligned} r_p &= \text{mean duration} \\ &= (P_3 + P_8) / f_p = 5.443 \text{ hours} \end{aligned} \quad (16)$$

Equations (6)–(13) are used for the remainder of the analysis. This analysis is summarized in Table 2. Some of the steps are described below.

The first step in analyzing the rest of the system involves computation of the rate and duration of power failure at bus A:

$$\lambda_A = \lambda_p + \lambda_{ATS} = 0.007576 \text{ f/y} \quad (17)$$

$$r_A = \frac{\lambda_p r_p + \lambda_{ATS} r_{ATS}}{\lambda_p + \lambda_{ATS}} = 5.092 \text{ h} \quad (18)$$

Having determined this, it is necessary to compute the frequency of the event that power loss at bus A will exceed 4 hours, which is the length of time the UPS can

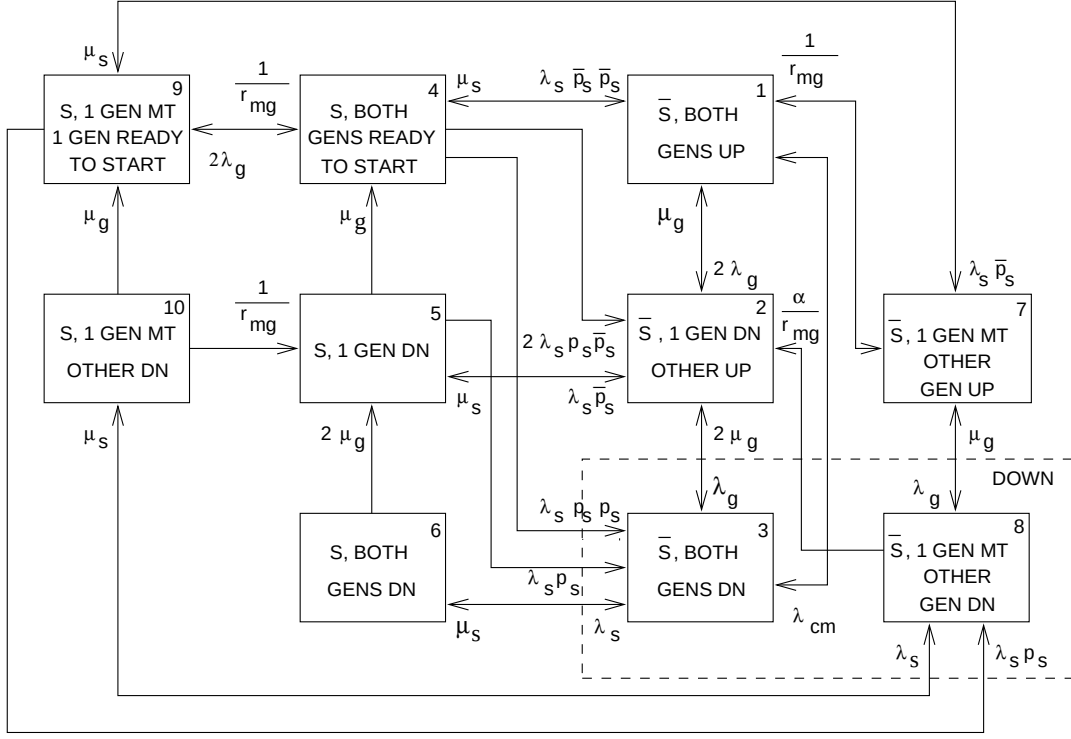


Figure 2: State transition diagram for utility supply and generators. S = utility power up; $\bar{S}$ = utility power failed; p_s = probability of failure to start the generator; $\bar{p}_s = (1 - p_s)$, i.e., probability of starting; λ_g, μ_g = failure rate of generator while running and repair rate of generator; λ_s, μ_s = frequency of need to start and shut down the generators. In this case, these parameters equal the frequency of loss and restoration rate of utility supply; λ_{mg}, r_{mg} = maintenance frequency and duration for a generator; λ_{cm} = common mode failure of generators; α = acceleration factor for planned maintenance.

Table 2: Summary of cut set analysis

	Cut Set	λ (f/y)	r (h/f)	λr
1.	Power loss at bus A > 4 h	0.003454	5.092	0.017588
2.1	Power loss at bus A (0.007576, 5.092)			
	and	0.000125	4.845	0.000606
2.2	Failure of [Inverter or battery or STS] (1.3729, 99.812)			
3.1	Maintenance on UPS (1.0, 4)			
	and	0.000003	2.240	0.000007
3.2	Power loss at bus A (0.007576, 5.092)			
4.1	Inverter failure(1.254, 107.0)			
	and	0.001643	19.603	0.032208
4.2	STS failure(0.0876, 24.0)			
	Σ	0.005225		0.050409

sustain the load. Assuming the failure duration to be exponentially distributed, the failure frequency of a cut set having duration $\geq c$ can be found as follows.

$$\begin{aligned} f_{cs}(\geq c) &= f_{cs}(\text{Prob. of event duration} \geq c) \\ &= f_{cs} \exp(-c/r_{cs}) \end{aligned} \quad (19)$$

$$r_{cs}(\geq c) = c + r_{cs} \quad (20)$$

Note that although the mean duration of failures greater than c hours is $c + r_{cs}$, the duration of actual loss is r_{cs} as the battery can supply power upto c hours.

The frequency and duration of event 2.2 (see Table 2): this event will occur if any one of the component inverter or battery or STS fails.

$$\lambda_{2.2} = 1.254 + 0.0313 + 0.0876 = 1.3729 \text{ f/y} \quad (21)$$

$$\begin{aligned}
r_{2.2} &= \frac{1.254 \times 107 + 0.0313 \times 24 + 0.0876 \times 24}{1.3729} \\
&= 99.812 \text{ h}
\end{aligned} \tag{22}$$

Now events 2.1 and 2.2 can be combined by using equations (8) and (9). Similarly events 3.1 and 3.2 can be combined using equations (10) and (11). It is assumed that a UPS will be taken out on planned maintenance once per year for 4 hours. Finally, the events 4.1 and 4.2 can be combined using equations (8) and (9).

The system indices, i.e., rate and duration of power loss at the Critical Load Bus (CLB) are determined to be

$$\lambda_{CLB} = \sum_{\text{cut-sets}} \lambda = 0.005225 \text{ f/y} \tag{23}$$

$$r_{CLB} = \frac{\sum \lambda r}{\sum \lambda} = 9.648 \text{ h} \tag{24}$$

In addition to determining the means of the failure rates and durations, it may be useful to be aware of their variances. If all distributions are assumed exponential (see below), then the standard deviations of all the up times and down times would equal the corresponding mean up times and down times. This implies that the standard deviations of the failure rates would also equal the corresponding means computed.

4.3 Monte Carlo simulation

This is another method of determining reliability indices. This method is described in detail in [4], while an application of this method to the sample system is given in [8]. A summary of the method is presented here.

The reliability indices of an actual physical system can be estimated by collecting data on the occurrence of failures and the durations of repair. The Monte Carlo method mimics the failure and repair history of the components and the system by using the probability distributions of the component state durations. Statistics are then collected and indices estimated using statistical inference.

Though there are different ways of implementing Monte carlo simulations, the technique most appropriate for such a system which includes dependent failures is the *next event* method. This is a sequential simulation method which proceeds by generating a sequence of events using random numbers and probability distributions of random variables representing component state durations. A flowchart is given in Fig. 3.

The input data consists of the failure rate (λ) and mean down time (r) of every component. The failure rate is the reciprocal of the mean up time. The mean down time is the reciprocal of the repair rate (μ). The failure and repair rates, λ and μ , of a component will be used to determine how long the component will remain in the 'UP' state and the 'DOWN' state.

Simulation could be started from any system state, but it is customary to begin simulation with all the components in the UP state.

The time to the next event is generated by using the *inverse of probability distribution* method. This is explained as follows.

The transition times of the components are assumed to be exponentially distributed:

$$f(t) = \rho e^{-\rho t} \tag{25}$$

where ρ is the transition rate. The mean transition time is, therefore,

$$\int_0^{\infty} f(t) dt = \frac{1}{\rho} \tag{26}$$

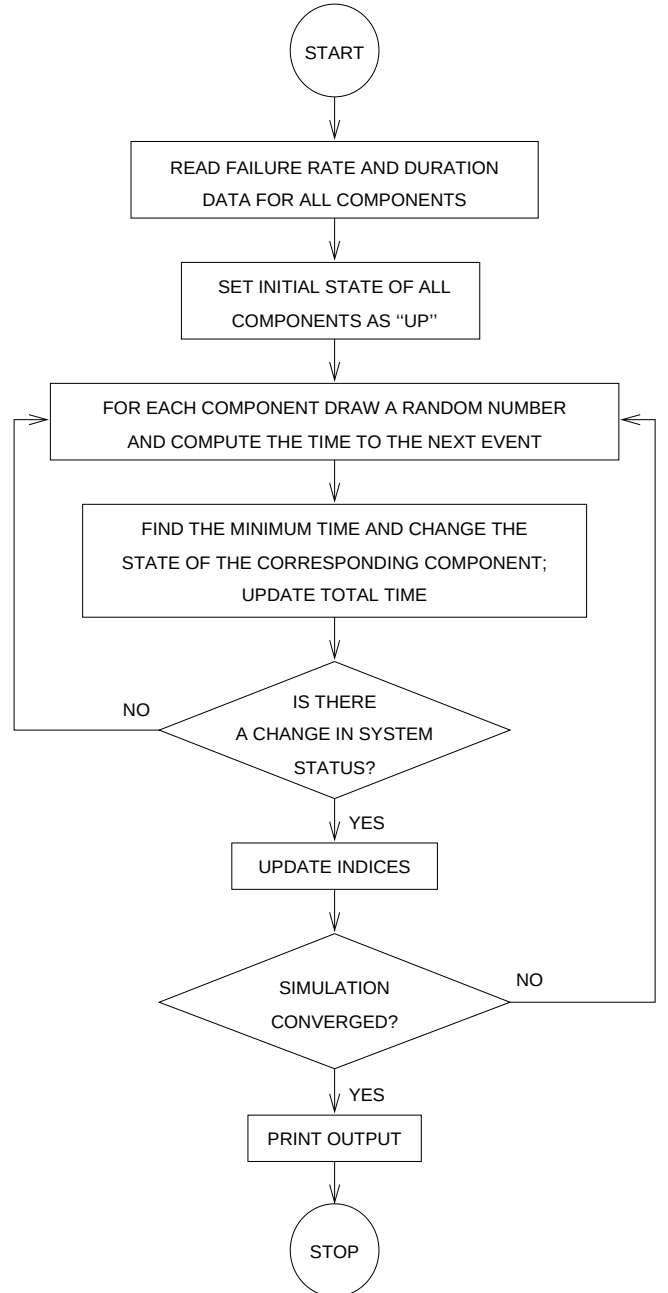


Figure 3: Flowchart for next-event simulation.

This means that if, for instance, a component is UP, then, regardless of how long it has been in the UP state, the expected time to the next failure is $1/\lambda$, i.e., the mean up time.

The *probability distribution function* of the transition time T would be

$$F(t) = P(T \leq t) = \int_0^t \rho e^{-\rho\tau} d\tau = 1 - e^{-\rho t} \quad (27)$$

Now $F(T)$ can be regarded as a random variable, uniformly distributed between 0 and 1. This means that the function

$$S(T) = 1 - F(T) = e^{-\rho t} \quad (28)$$

is also uniformly distributed between 0 and 1. So if a random number R_n is generated, $0 \leq R_n \leq 1$, it can be associated with the event that the next transition occurs after time t_r , given by

$$R_n = e^{-\rho t_r}, \quad \text{i.e.,} \quad t_r = -\frac{\ln(R_n)}{\rho} \quad (29)$$

This method is used to determine the time to the next transition for every component, using λ or μ for ρ , depending on whether the component is UP or DOWN.

The reliability indices are estimated as follows. At any time t , the mean failure rate is given by

$$\lambda_t = \frac{1}{t}(\text{number of failures till time } t) \quad (30)$$

and the mean down time is given by

$$r_t = \frac{1}{t}(\text{total time spent in failed state}) \quad (31)$$

The values of λ and r at the instant the simulation converges are the reliability indices for the system as obtained from the Monte Carlo method. The simulation is said to have converged when the indices attain stable values. This ‘stabilization’ of the value of an index i is measured by its standard error, defined as:

$$\eta\epsilon = \frac{\sigma_i}{\sqrt{n_c}} \quad (32)$$

where σ_i = standard deviation of the index i
 n_c = number of cycles simulated

Convergence is said to occur when the standard error drops below a prespecified fraction, ϵ , of the index i , i.e., when

$$\eta \leq \epsilon i \quad (33)$$

If, for instance, the mean down time r is chosen as the index to converge upon, then, after every system restoration simulated, the following relation is tested for validity:

$$\frac{\sigma_r}{\sqrt{n_c}} \leq \epsilon r \quad (34)$$

If this criterion is satisfied, the simulation is said to have converged.

Simulation is advantageous in that it not only allows the computation of indices at various points in the system, but also permits the accumulation of data pertaining to the distribution of these indices, thereby affording a better understanding of the system behavior.

For an emergency power system, for instance, statistics may be collected for failure frequency and duration at various points in the system, the annual incidence rates for failures, as well as for the variances of these indices. Table 3 shows the estimates obtained for the following statistics using Monte Carlo simulation, and compares them with the corresponding values as determined by the analytical method described earlier.

5 Improvement in Reliability of Supply

The above discussions illustrate not only the methods of analysis, but also that the reliability of the supply to the critical load bus is significantly improved by the presence of the uninterruptible power supply. Observe that the utility supply has a failure rate of 0.537 failures/year, while the supply to the critical load bus has a failure rate of only 0.005 failures/year. This demonstrates the role and objective of an emergency power supply.

It stands to reason, therefore, that it should be possible to select the size of storage so as to meet a desired reliability target. Here, the term *storage* is used in a general sense, to include both electrical storage (such as a battery, possibly as part of an UPS), and fuel (such as diesel, such as that stored in the tank of a diesel generator). In determining the appropriate size of backup storage for a particular application, one generally selects the power rating on the basis of the load to be supported, and the energy rating on the basis of the period of time one expects to have to support the load in the event of a failure of the primary supply. The energy rating on UPS units is often provided in Ampere-hours (Ah); in case of fuel storage, the size of the tank often determines the energy capacity, but this can often be supplemented with additional storage on site.

Reference [10] presents a method of selecting the size of a storage device in such a manner as to meet a reliability target. This method of sizing backup storage equipment

Table 3: Reliability indices of sample system

Index	Simulated		Calculated	
	Mean	SD	Mean	SD
λ_p (f/y)	0.001352	0.001497	0.001576	0.001576
r_p (h/f)	5.028	5.503	5.443	5.443
λ_A (f/y)	0.007193	0.006851	0.007576	0.007576
r_A (h/f)	5.216	5.201	5.092	5.092
λ_{CLB} (f/y)	0.004868	0.004888	0.005225	0.005225
r_{CLB} (h/f)	9.7135	13.3312	9.648	9.648

might be considered appropriate for facilities that have loads requiring very high levels of supply reliability.

5.1 Sizing storage to reliability target

Consider a system that is supplied by a primary source (such as a utility) of availability A_0 . Here, *availability* implies the steady state probability that power will be served to the load [11]. Assume that of the system load there is a part P_C (where P_C is the power requirement) that is considered critical. It is intended to add sufficient storage to increase the availability of power to this critical load to A_1 . It is clear that the power capacity of the required storage unit should be at least P_C . What remains to be determined is the energy capacity.

Define the fraction κ such that

$$\kappa = \frac{1 - A_1}{1 - A_0}. \quad (35)$$

This fraction κ , also known as the *unavailability reduction ratio*, can be understood as follows. Suppose $A_0 = 0.9999$ and the intention is to increase the availability of power to the critical load by ‘an additional 9’, i.e., to $A_1 = 0.99999$; then

$$\kappa = \frac{0.00001}{0.0001} = 0.1.$$

In developing the method for determining the required energy capacity, consider the following analysis. Denote

- S_F event that the primary supply has failed;
- L event that the critical load experiences failure of power supply;
- t_A length of time for which the storage unit can support the critical load in the event of failure of the primary supply;
- R random variable representing the down time (outage duration) of the primary supply;
- $f_R(r)$ probability density function of R .

Then the event L occurs when the primary supply is down for a period longer than t_A , and its probability is given by

$$\begin{aligned} P\{L\} &= P\{\{R > t_A\} \cap S_F\} \\ &= P\{R > t_A | S_F\} P\{S_F\} \\ &= \left(\int_{t_A}^{\infty} f_R(r) dr \right) P\{S_F\}. \end{aligned} \quad (36)$$

In (36), $P\{L\}$ is clearly $1 - A_1$ and $P\{S_F\} = 1 - A_0$. It is also clear from (35) and (36) that

$$\int_{t_A}^{\infty} f_R(r) dr = \kappa. \quad (37)$$

Equation (37) forms the basic relationship from which the required storage capacity can be determined. However, the storage device itself may have a certain probability of failure. In order to compensate for this, the storage

device should have an energy capacity that enables it to provide the required power (P_C) for a period of time t_S that is given by

$$t_S = \frac{t_A}{A_S}, \quad (38)$$

where t_A is given by (37) and A_S is the availability of the storage device. So the power capacity of the selected storage unit should be at least P_C , and the energy capacity should be at least $P_C t_S$.

5.2 Solution of the integral equation

In many instances, the failure rate of the primary supply is constant, and the down time R is exponentially distributed [11]. For exponentially distributed R , the probability density function can be expressed in the form

$$f_R(r) = \frac{1}{\bar{r}} \exp\left(-\frac{r}{\bar{r}}\right), \quad r \geq 0, \quad (39)$$

where $\bar{r}$ is the expectation (mean) of R . Then the solution of the integral equation (37) provides

$$t_A = -\bar{r} \ln \kappa. \quad (40)$$

So, for instance, if one were to use $\kappa = 0.1$ as in the example above, then a reasonably reliable storage unit ($A_S = 0.9$) would have to be capable of supporting the critical load for about 2.5 times the mean down time of the primary supply.

In instances where the failure rate is not constant, R follows other distributions, mostly Weibull or lognormal [11].

For a Weibull distribution with shape parameter β , the probability density function can be expressed in the form [11]

$$f_R(r) = \frac{\beta r^{\beta-1}}{r'^{\beta}} \exp\left[-\left(\frac{r}{r'}\right)^{\beta}\right], \quad r \geq 0, \quad (41)$$

where $r' = \bar{r}/\Gamma(1 + 1/\beta)$, and t_A can be obtained from

$$t_A = r' (-\ln \kappa)^{1/\beta}. \quad (42)$$

Note that for $\beta = 1$ the Weibull distribution degenerates to the exponential, and this is borne out by (41) and (42).

When R follows a lognormal distribution, there is no closed form solution. In case of R following a lognormal distribution with shape parameter β , the probability density function of R may be expressed in the form [11]

$$f_R(r) = \frac{1}{\sqrt{2\pi}\beta r} \exp\left[-\frac{1}{2\beta^2} \left(\frac{\beta^2}{2} + \ln \frac{r}{\bar{r}}\right)^2\right], \quad r \geq 0. \quad (43)$$

Then the solution of the integral equation given by (37) is obtained from

$$\Phi\left(\frac{1}{\beta} \left[\frac{\beta^2}{2} + \ln \frac{t_A}{\bar{r}}\right]\right) = 1 - \kappa, \quad (44)$$

where the function Φ is the cumulative distribution function of the standard normal variate, i.e.,

$$\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} \exp(-t^2/2) dt.$$

In order to solve (44), one would refer to a table of the standard normal distribution to look up the value of z corresponding to $\Phi(z) = 1 - \kappa$, and then determine t_A from

$$\frac{1}{\beta} \left(\frac{\beta^2}{2} + \ln \frac{t_A}{\bar{r}} \right) = z \Rightarrow t_A = \bar{r} \exp \left(\beta z - \frac{\beta^2}{2} \right). \quad (45)$$

5.3 An alternative reliability target

Instead of defining a reliability target in terms of availability, one may prefer to use a target in terms of mean down time. For instance, one may desire to reduce the mean down time of supply to the critical load to $\kappa \bar{r}$, where $\bar{r}$ is the mean down time of the primary supply. Then t_A , the length of time for which the storage unit should be able to support the critical load in the event of primary supply failure, is given by

$$\int_{t_A}^{\infty} r f_R(r) dr = \kappa \bar{r}. \quad (46)$$

It should be intuitively apparent that (37) and (46) would produce somewhat similar results; the proximity of these results will be clear from the following analysis. We know that

$$\bar{r} = \int_0^{\infty} r f_R(r) dr \Rightarrow \int_0^{\infty} (r - \bar{r}) f_R(r) dr = 0. \quad (47)$$

If one chose a value of t_A that satisfies (40), and this value were small, then from (47) it can be stated that

$$\begin{aligned} \int_{t_A}^{\infty} (r - \bar{r}) f_R(r) dr \approx 0 &\Rightarrow \int_{t_A}^{\infty} r f_R(r) dr \approx \bar{r} \int_{t_A}^{\infty} f_R(r) dr \\ &\Rightarrow \int_{t_A}^{\infty} f_R(r) dr \approx \kappa, \end{aligned} \quad (48)$$

i.e., the t_A that satisfies (46) exactly also satisfies (37), approximately. In other words, the more reliable the utility supply, the higher the equivalence between the two reliability targets.

Equation (46) can be further modified to the following form.

$$\int_0^{t_A} r f_R(r) dr = (1 - \kappa) \bar{r}, \quad (49)$$

since

$$\int_0^{t_A} r f_R(r) dr = \int_0^{\infty} r f_R(r) dr - \int_{t_A}^{\infty} r f_R(r) dr = \bar{r} - \kappa \bar{r}. \quad (50)$$

Equation (49) becomes useful when Monte Carlo simulation is used as shown in Fig. 3 to determine t_A from the interruption duration statistics collected—if the observed interruption times were sequentially stacked, then that time for which the mean of all equal and shorter interruptions is closest to $(1 - \kappa) \bar{r}$ gives the estimate of t_A . This has been illustrated in [12].

6 Concluding Remarks

This chapter described the role of emergency and standby systems in enhancing the reliability of power supply to a critical load, and showed an analytical approach to determine the size of a backup storage unit so as to meet a specified reliability target. The size of a storage unit is defined by its power capacity and its energy capacity. The analysis presented applies to electrical storage as well as fuel storage, and can be used by any entity, such as a hospital, a process plant, or a military base, that is considering acquisition of electrical storage to meet an increased reliability target. The method can also be used by a facility that already has some standby generation or storage, to further increase the reliability; in such a case, the combination of utility and existing backup is treated as the primary supply, and the proposed method is used to determine the required capacity of additional storage.

The proposed analysis is applicable directly to determine capacities of new backup schemes that comprise single storage units. Where a backup scheme consists of multiple storage components, such as several UPS units, or standby generators, or a combination thereof, and where other factors are considered, such as maintenance of components, dependent or common mode failures, failures of generators to start, or time-dependent factors like state of charge, more involved methods [6, 8, 12–14] are necessary to determine energy capacities of the storage components.

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