MATERIAL REMOVAL PROCESSES
Theory of Metal Machining

1. Overview
2. Theory of Chip Formation
3. Force Relationships
4. Power and Energy Relationship
5. Cutting Temperature

Introduction

• Everyday Experience: Scraping the ice from your windshield
  – Edge angle of the ice scraper
  – Force required depending on the characteristics of ice
• Incentives: Making a ceramic vase out of clay
  – Shaping
  – Removal of excess materials – ‘machining’
• Powder Metal or Cast
  – Exact dimension
  – Tolerance & Surface Finish

Classification

Material Removal Processes
A family of shaping operations through which undesired excess material is removed from a starting workpart so the remaining part become closer to the desired shape

• Categories:
  – Machining – material removal by a sharp cutting tool, e.g., turning, milling, drilling
  – Abrasive processes – material removal by hard, abrasive particles, e.g., grinding
  – Nontraditional processes - various energy forms other than sharp cutting tool to remove material

Machining

• A shearing process in which excess materials is removed by cutting tools.
  – A variety of work materials
  – ‘Repeatable’ regular geometries
  – Close tolerance (<0.025mm)
  – Smooth surface finish (0.4mm)
  – Waste, Expensive: Cost and Time
  – Other processes such as casting, forging, and bar drawing create the general shape
  – Machining provides the final shape, dimensions, finish, and special geometric details
Cutting condition

- Relative motion between tool and work
- Cutting conditions
  - Cutting speed, \( v \) (m/s) – Surface speed
  - Feed \( f \) (m): the lateral distance traveled by the tool during one revolution.
  - Depth of cut \( d \) (m)
- Material Removal Rate: \( MRR = v f d \)
  - Roughing - removes large amounts of material, at high feeds and depths, low speeds
  - Finishing - Achieves final dimensions, tolerances, and finish, Low feeds and depths, high cutting speeds

Machine Tools

- A power-driven machine that performs a machining operation
  - Holds workpart
  - Positions tool relative to work
  - Provides power and controls speed, feed, and depth.
  - Pumps a Cutting fluid

2. Theory of Chip Formation

- Orthogonal Cutting Model
  - Chip thickness ratio:
  \[ r = \frac{t_c}{t_l} = \frac{t_l \sin \phi}{t_l \cos(\phi - \alpha)} \]
  - By rearranging
  \[ \tan \phi = \frac{r \cos \alpha}{1 - r \sin \alpha} \]

Rake angle: \( \alpha \)
Shear angle: \( \phi \)
Chip thickness ratio:
\[ r = \frac{t_c}{t_l} = \frac{t_l \sin \phi}{t_l \cos(\phi - \alpha)} \]

As \( \phi \) (from 10° to 35°) increase, \( \gamma \) (from 5 to 2) decreases.

Shear Strain in chip

\[ \gamma = \frac{\sin(\alpha + \beta) - \sin \beta \cos \alpha \sin \beta}{\cos(\phi - \alpha) - \cos \alpha \cos \beta \sin \alpha \sin \beta} \]
\[ = \frac{\sin(\phi - \alpha)}{\cos \phi} \frac{\cos \alpha}{\sin \phi} \frac{\cos \alpha}{\sin \phi} \frac{\cos(\phi - \alpha)}{\sin \phi} \frac{\cos \alpha}{\sin \phi} \frac{\cos \alpha}{\sin \phi} \]

Shear Strain rate is around \( 10^3-10^5 \) sec\(^{-1} \)

Actual Chip Formation

(a) Discontinuous chip
  - Brittle materials at low cutting speed
  - High tool-chip friction and large feed and depth

(b) Continuous chip
  - Ductile materials with high speeds and small feed and depth of cut
  - Continuous chip with built-up edge
  - Ductile material at low to medium speeds

(c) Continuous chip with built-up edge
  - Brittle material at high cutting speeds
  - Ductile operating metals at high cutting speeds
The ‘Real’ Cutting Force

Cutting Forces are measured with Dynamometer.

- Area: \( A = bh \)
  - where \( b \) = chip width
  - \( h \) = chip thickness
- Temperature (500-1000°C)
- Pressure (1000-3000 MPa)

Force Diagram

\[
\begin{align*}
F &= F_c \sin \alpha + F_t \cos \alpha \\
N &= F_c \cos \alpha - F_t \sin \alpha \\
F_c &= F_t \sin \phi - F_c \cos \phi \\
F_t &= F_c \sin \phi + F_t \cos \phi \\
R &= \frac{F_t \sin \phi \cos(\beta - \alpha)}{\sin \phi \cos(\beta - \alpha)} \\
F_c &= \frac{F_t \cos(\beta - \alpha)}{-\sin \phi \cos(\beta - \alpha)} \\
F_t' &= \frac{F_c \sin(\beta - \alpha)}{-\sin \phi \cos(\beta - \alpha)}
\end{align*}
\]

Cutting Force

- Cutting Force: 
  \[ F_c = \frac{\sin(\beta - \alpha)}{\cos(\beta - \alpha)} \]
  \[ F_t = \frac{K_c}{K_t} \left( \frac{N}{\sin \mu} \right) \]
- Thrust Force: 
  \[ F_t = \frac{R \cos(\beta - \alpha)}{\sin \phi} \]

\( F_c \) and \( K_c \) must be calibrated through machining experiments.

The Merchant Equation

- Shear stress: \( \tau = \frac{F_c}{A_t} \)
- Shear Plane Area: \( A_t = \frac{t_c w}{\sin \phi} \)
- Shear stress: \( \tau = \frac{F_c \cos \phi - F_t \sin \phi}{t_c w / \sin \phi} \)
- Merchant’s Assumption: Shear plane angle will form to minimize energy
- After differentiating \( \tau \) w.r.t \( \phi \), Merchant’s Equation: 
  \[ \phi = 45 + \frac{\alpha + \beta}{2} \]

Implication of Merchant’s Eq.

- An increase in rake angle causes the shear plane angle to increase.
- A decrease in friction angle cause the shear plane angle to increase.
- The analysis from orthogonal cutting can be used in a typical turning if the feed is small relative to depth of cut.

Effect of shear plane angle \( \phi \):
  - (a) higher \( \phi \) with a resulting lower shear plane area,
  - (a) smaller \( \phi \) with a resulting larger shear plane area.
**Turning vs. Orthogonal**

- Feed \( f \)
- Uncut Chip thickness \( t_{uc} \)
- Depth \( d \)
- Width of cut \( w \)
- Cutting speed \( v \)
- Cutting force \( F_c \)
- Feed force \( F_f \)
- Thrust force \( F_t \)

**4. Power & Energy Relation**

- Power (energy per unit time) \( P = F v \)
- Horse power \( P_E = \frac{F v}{33,000} \) \( \text{in lb-min} \)
- Unit Power \( P_u = \frac{P}{MRR} \)
- Specific energy \( U = \frac{P}{\rho \cdot \pi \cdot r} \)

**Size Effect & Energy Distribution**

- Chip thickness before cut \( t \) vs Cutting speed \( v \)

**Specific Energy for various work materials \( (t_{uc}=0.25) \)**

<table>
<thead>
<tr>
<th>Materials</th>
<th>Brinell Hardness</th>
<th>Specific Energy ( U )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon Steel</td>
<td>150-200</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>200-250</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td>250-300</td>
<td>3.2</td>
</tr>
<tr>
<td></td>
<td>350-400</td>
<td>4.4</td>
</tr>
<tr>
<td></td>
<td>450-500</td>
<td>5.4</td>
</tr>
<tr>
<td></td>
<td>550-600</td>
<td>6.8</td>
</tr>
<tr>
<td></td>
<td>650-700</td>
<td>8.2</td>
</tr>
<tr>
<td></td>
<td>750-800</td>
<td>9.6</td>
</tr>
<tr>
<td>Aluminum</td>
<td>50-100</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>100-150</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>150-200</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>200-250</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>250-300</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>300-350</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>350-400</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>400-450</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>450-500</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>500-550</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>550-600</td>
<td>1.9</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>150-200</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>200-250</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>250-300</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>300-350</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>350-400</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>400-450</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td>450-500</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>500-550</td>
<td>2.6</td>
</tr>
<tr>
<td></td>
<td>550-600</td>
<td>2.8</td>
</tr>
<tr>
<td></td>
<td>600-650</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td>650-700</td>
<td>3.2</td>
</tr>
<tr>
<td></td>
<td>700-750</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>750-800</td>
<td>3.6</td>
</tr>
<tr>
<td></td>
<td>800-850</td>
<td>3.8</td>
</tr>
<tr>
<td></td>
<td>850-900</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>900-950</td>
<td>4.2</td>
</tr>
<tr>
<td></td>
<td>950-1000</td>
<td>4.4</td>
</tr>
<tr>
<td></td>
<td>1000-1050</td>
<td>4.6</td>
</tr>
<tr>
<td></td>
<td>1050-1100</td>
<td>4.8</td>
</tr>
<tr>
<td></td>
<td>1100-1150</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>1150-1200</td>
<td>5.2</td>
</tr>
<tr>
<td></td>
<td>1200-1250</td>
<td>5.4</td>
</tr>
<tr>
<td></td>
<td>1250-1300</td>
<td>5.6</td>
</tr>
<tr>
<td></td>
<td>1300-1350</td>
<td>5.8</td>
</tr>
<tr>
<td></td>
<td>1350-1400</td>
<td>6.0</td>
</tr>
<tr>
<td></td>
<td>1400-1450</td>
<td>6.2</td>
</tr>
<tr>
<td></td>
<td>1450-1500</td>
<td>6.4</td>
</tr>
<tr>
<td></td>
<td>1500-1550</td>
<td>6.6</td>
</tr>
<tr>
<td></td>
<td>1550-1600</td>
<td>6.8</td>
</tr>
<tr>
<td></td>
<td>1600-1650</td>
<td>7.0</td>
</tr>
<tr>
<td></td>
<td>1650-1700</td>
<td>7.2</td>
</tr>
<tr>
<td></td>
<td>1700-1750</td>
<td>7.4</td>
</tr>
<tr>
<td></td>
<td>1750-1800</td>
<td>7.6</td>
</tr>
<tr>
<td></td>
<td>1800-1850</td>
<td>7.8</td>
</tr>
<tr>
<td></td>
<td>1850-1900</td>
<td>8.0</td>
</tr>
<tr>
<td></td>
<td>1900-1950</td>
<td>8.2</td>
</tr>
<tr>
<td></td>
<td>1950-2000</td>
<td>8.4</td>
</tr>
</tbody>
</table>

**5. Cutting Temperature**

- **Cook’s dimensional analysis** \( T = 0.4 \frac{U}{\rho C} \left( \frac{\nu}{K} \right)^{0.333} \)

**Experimental Measurement**

- Tool-chip thermocouple
- Trigger’s results \( T = K \nu^{0.8} \)
- RC-1308 Ti \( (T=479\nu^{0.8}) \)
- 18-8 Stainless steel \( (T=135\nu^{0.361}) \)
- B113 Free machining steel \( (T=86.2\nu^{0.348}) \)
Chatter Analysis

- Mechanical vibration
  
  Free Vibration: \( mx'' + cx + kx = 0 \)
  
  Forced Vibration: \( mx'' + cx + kx = F_0 \sin \omega t \)

  Assume \( x(t) = X \sin(\omega t + \phi) \)

  Or using complex harmonic functions

  \( s^2 \Rightarrow c + j \omega \)

  Assume \( x(t) = X e^{j \omega t} \)

  \( (k - j \omega^2 m + j \zeta \omega m) X e^{j \omega t} = F_0 e^{j \omega t} \)

  Magnitude ratio:

  \[ \left| \frac{X'}{X} \right| = \frac{1}{\sqrt{\left( k - \omega^2 m + j \zeta \omega m \right)^2 + \left( \omega m \right)^2}} \]

  Phase:

  \( \phi = \tan^{-1} \frac{-\omega \zeta}{\omega^2 - \omega_n^2} \)

  where \( \omega_n = \sqrt{\frac{k}{m}} \)

  and \( \zeta = \frac{c}{2m \sqrt{k/m}} \)