1. Consider the system
\[ \dot{x}_1 = x_2, \quad \dot{x}_2 = -x_1 + \mu x_2 (1 - 3x_1^2 - 2x_2^2) \]
where \( \mu \) is a constant that satisfies \( |\mu| < 2 \).

(a) Show that the origin \( x = 0 \) is the unique equilibrium point.

(b) Determine the type of the equilibrium point when \( \mu < 0 \) and \( \mu > 0 \).

(c) Using the Poincaré-Bendixson criterion, show that when \( \mu > 0 \) there is a periodic orbit in the set \( \{x_1^2 + x_2^2 \leq \frac{1}{2}\} \).

(d) The phase portraits for \( \mu = -0.2 \) and \( \mu = 0.2 \) are shown in Figure 1. Mark the arrowheads and discuss the qualitative behavior in each case.

(e) Find bifurcations that occur as \( \mu \) varies over the interval \((−2, 2)\). Sketch the bifurcation diagram.

Figure 1: Exercise 1.