1. **Multiple Choice**: Please read every question carefully; do not rush to judgement

(a) At the interface between two different perfect dielectric media
   i. The tangential component of the flux density in continuous
   ii. The normal component of the polarization is continuous
   iii. The normal component of the electric field is continuous
   iv. The tangential component of the electric field is continuous
   v. All of the above

(b) Poisson’s equation for the electrostatic problem is given by
   i. $\nabla^2 V(r) = V(r)$
   ii. $\nabla^2 V(r,t) = \partial_t \rho(r,t)$
   iii. $\nabla \times \mathbf{E} = 0$
   iv. $\nabla^2 V(r) = \rho(r)/\varepsilon_0$
   v. Items 1 and 3

(c) The solution to either the Laplace or Poisson for electrostatic is unique provided
   i. The potential on the boundary is specified
   ii. The normal derivative of the potential is specified
   iii. On a part of the body, the potential is specified and the normal derivative of the potential is specified.
   iv. The normal component of the electric field is specified
   v. All of the above

(d) The energy density that is stored in an electric field is
   i. Proportional to the square of the magnitude of the electric field.
   ii. Proportional to the square of the magnitude of the electric flux density
   iii. Proportional to the product of the electric field and the electric flux density
   iv. All of the above
   v. None of the above

(e) The polarization is defined by
   i. $\mathbf{P} = \varepsilon_0 \chi \mathbf{E}$
   ii. $\mathbf{P} = \mathbf{D} - \varepsilon_0 \mathbf{E}$
   iii. $\mathbf{P} = \mathbf{D}/\varepsilon_0$
   iv. All of the above
   v. Items 1 & 2
(f) The capacitance is
   i. \( Q/V \)
   ii. \( -\varepsilon_0 \int dS \hat{n} \cdot \mathbf{E} / \int dl \cdot \mathbf{E} \)
   iii. \( -\int dS \hat{n} \cdot \mathbf{E} / \int dl \cdot \mathbf{E} \)
   iv. All of the above
   \( \checkmark \) Items 1 & 2

(g) Consider the interface between two perfect dielectric materials \( \varepsilon_1 \) and \( \varepsilon_2 \). The potential in each region is denoted by \( V_1 \) and \( V_2 \). At the interface between the two dielectrics
   i. \( V_1 = V_2 \)
   ii. \( \partial_n V_1 = \partial_n V_2 \)
   iii. \( \varepsilon_1 \partial_n V_1 = \varepsilon_2 \partial_n V_2 \)
   iv. Items 1 & 2
   v. All of the above

(h) Consider two parallel plate capacitors; plates of area \( d^2 \) and separated by a distance \( d \). In the first, the space between the two plates is filled with a material with permittivity \( \varepsilon_1 \), and the second is half full of material with permittivity \( \varepsilon_1 \).
   i. The capacitance of both capacitors are the same
   \( \checkmark \) The capacitance of the first is larger
   iii. The capacitance of the second is larger

(i) A dielectric is subjected to an electric field
   i. The volume polarization charge density is \( \nabla \cdot \mathbf{P} \)
   ii. The surface polarization charge density is given by \( \hat{n} \cdot \mathbf{P} \)
   iii. The total polarization charge is zero by the virtue of fact that the body is electrically neutral prior to being subjected to the electric field.
   iv. 1, 2, & 3
   \( \checkmark \) 2 & 3

(j) At the interface between a dielectric and a conductor
   i. The tangential component of the electric field is continuous
   ii. The normal component of the electric field is continuous
   iii. The normal component of the electric field is proportional to the surface charge density
   iv. All of the above
   \( \checkmark \) 1 & 3
2. Consider a square box of dimension $a$. A charge of $1 \mu C$ is placed on the bottom, and $2 \mu C$ on the top. The charge on the sides is zero. Find the potential distribution inside the square. (Hint: We have done a very similar problem in class; the charge on a surface is related to $\frac{\partial V}{\partial n}$.)

$$\nabla^2 V = 0$$

$$V = X(x)Y(y)$$

**Periodic in x direction.**

$$\Rightarrow \frac{d^2}{dx^2} X + \lambda^2 X = 0$$

$$\Rightarrow X(x) = A \sin \lambda x + B \cos \lambda x$$

$$V(x,y) = X(x) Y(y) [A \sin \lambda x + B \cos \lambda x]$$

$$\left. \frac{\partial V}{\partial x} \right|_{x=0} = 0 \Rightarrow A = 0 \Rightarrow \left. \frac{\partial V}{\partial x} \right|_{x=0} = 0 \Rightarrow \sin \lambda a = 0 \Rightarrow \lambda = \frac{n \pi}{a}, n = 0, 1, ...$$

$$Y(y) = (Ce^{-\frac{\pi y}{a}} + De^{\frac{\pi y}{a}})$$

$$V(x,y) = \sum_{n=0}^{\infty} (C_n e^{-\frac{n \pi y}{a}} + D_n e^{\frac{n \pi y}{a}}) \cos \frac{n \pi x}{a}$$

**Boundary condition on top:**

$$\left. \frac{\partial V}{\partial y} \right|_{y=0} = \frac{E_0}{\varepsilon_0} = \sum_{n=0}^{\infty} \frac{n \pi}{a} (-C_n + D_n) \sin \frac{n \pi x}{a}$$

$$\left. \frac{\partial V}{\partial y} \right|_{y=\frac{a}{2}} = \frac{E_0}{\varepsilon_0} = \sum_{n=0}^{\infty} \frac{n \pi}{a} (-C_n e^{-\frac{n \pi}{2 a}} + D_n e^{\frac{n \pi}{2 a}}) \cos \frac{n \pi x}{a}$$

Solve for $C_n$ and $D_n$ using orthogonality.
3. Consider a coaxial cable. The inner and outer radii of the conductors are 5 and 7 mm, respectively. The two conductors are separated by a cladding that comprises of two dielectric materials with relative permittivities 2 and 3, respectively, and each of thickness 1mm. Find

(a) Find an expression for the electric field between the conductors? (Hint: Gauss’s law)

(b) Find the capacitance per unit length?

\[
\begin{align*}
\mathbf{E} &= \mathbf{E}_1 \mathbf{n} + \mathbf{E}_2 \mathbf{n} \\
E_1 &= \frac{Q}{2\pi\varepsilon_0 l} \\
E_2 &= \frac{Q}{2\pi\varepsilon_0 l} \left[ \ln \frac{5}{6} \frac{1}{\varepsilon_1} + \ln \frac{7}{6} \frac{1}{\varepsilon_2} \right] \\
\end{align*}
\]