Time allotted: 50 mins. Answer All
State your assumptions clearly and show all steps

BEST OF LUCK
1. *Multiple Choice*

(a) If the cross product between two vectors vanishes, then the vectors are
   
i. The vectors are perpendicular
   
ii. The vectors are parallel
   
iii. The vectors are co-planar
   
iv. All of the above
   
v. None of the above

(b) The gradient of a scalar field is
   
i. Conservative
   
ii. Irr-rotational
   
iii. A vector
   
iv. All of the above

(c) Which of the following unit vectors do NOT vary with position
   
i. \( \hat{R}, \hat{\theta}, \hat{\phi} \)
   
ii. \( \hat{r}, \hat{\phi}, \hat{z} \)
   
iii. \( \hat{r} \cos \phi - \hat{\theta} \sin \phi, \hat{r} \sin \phi + \hat{\phi} \cos \phi, \hat{z} \)
   
iv. All of the above
   
v. None of the above

(d) The electric field is
   
i. A measure of the force between two charges
   
ii. Is related to the gradient of the potential
   
iii. Is proportional to the charge
   
iv. All of the above
   
v. None of the above

(e) Gauss law states that
   
i. The flux leaving a closed surface is proportional to the charge that is contained within it.

ii. A volume integral of the charge density is equal to an integral of the electric field on the surface that bounds the volume.
iii. The surface integral of the electric field over a surface is a constant.
iv. All of the above
\( \square \) Only 1 and 2

2. Answer True or False and justify your answer.

(a) The divergence of the gradient of a scalar field is equal to zero; i.e., \( \nabla \cdot \nabla \Phi(x, y, z) = 0 \). \( \underline{\text{F}} \)

(b) If \( \hat{n} \) is the unit normal to the surface, the vector \( \mathbf{C} = \hat{n} \times \hat{n} \times \mathbf{A} \) is perpendicular to \( \hat{n} \). \( \underline{T} \)

(c) Two different equipotential surfaces can intersect. \( \underline{F} \)

(d) A charge exist between two infinite parallel plates. The plates carry equal and opposite charge densities. Then the work done in moving this charge in a direction parallel to the plate is a non-zero constant. \( \underline{F} \)

(e) In the last example, assume that the plates are parallel to the \( x-y \) plane. Then the work done in moving a charge along the \( z \) axis is zero. \( \underline{F} \)

\[ \begin{align*}
(\text{a}) \quad \nabla \cdot \mathbf{V} &= \nabla \cdot \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \\
(\text{b}) \quad \mathbf{n} \times (\mathbf{n} \times \mathbf{A}) &= (\hat{n} \cdot \mathbf{A}) \hat{n} - (\hat{n} \cdot \mathbf{A}) \mathbf{n} \\
&= \mathbf{n} \cdot (\mathbf{n} \times (\mathbf{n} \times \mathbf{A})) = (\hat{n} \cdot \mathbf{A}) (\hat{n} \cdot \mathbf{A}) - (\hat{n} \cdot \mathbf{A}) \hat{n} \cdot \mathbf{A} = 0
\end{align*} \]

\( \text{c) False by the definition} \)

\( \text{d) Force to motion} = \mathbf{F} \cdot \mathbf{d} = 0 \)

\( \text{e) No change in potential} \equiv \text{Work done} \)

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3. Two charges \(-2\mu C\) and \(2\mu C\) exist at \((-1,0,0)\) and \((1,0,0)\), respectively. Sketch the potential and the magnitude of the electric field in the range \((-5,0,0)\) to \((5,0,0)\).

Steps asked for:  
1) Derive formula:  
2) Evaluate at a set of points:  

\[
\mathbf{E}(\mathbf{r}) = \sum_{n=1}^{2} \mathbf{E}_n(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \left[ \frac{\mathbf{q}}{|\mathbf{r} - \mathbf{r}_n|^3} - \frac{\mathbf{q}}{|\mathbf{r} - \mathbf{r}_{n+1}|^3} \right]
\]

\[
V(\mathbf{r}) = \sum_{n=1}^{2} V_n(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \left[ \frac{\mathbf{q}}{|\mathbf{r} - \mathbf{r}_n|} - \frac{\mathbf{q}}{|\mathbf{r} - \mathbf{r}_{n+1}|} \right]
\]

Use these formulae to put values.
4. Two rings of radius $a$ carrying a charge density $\rho_1$. These rings are parallel to the $x$-$y$ plane, and their centers are located at $(0,0,-a)$ and $(0,0,a)$.

(a) Find the field at a point $\mathbf{r} = x\hat{x} + y\hat{y} + z\hat{z}$. Please do not evaluate any integrals, but set up the problem completely.

(b) Find the field at the origin. Evaluate all integrals.

$$
\mathbf{E} = \sum_{i=1}^{2} E_i(\mathbf{r})
$$

$$
\mathbf{E}_1(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int dl \frac{\rho (\mathbf{r} - \mathbf{r}_i)}{|\mathbf{r} - \mathbf{r}_i|^{3/2}} = \frac{\rho}{4\pi\varepsilon_0} \int d\phi \frac{(\mathbf{r} - \mathbf{a})^\hat{x} + (\mathbf{r} + \mathbf{a})^\hat{z}}{(r^2 + (r - a)^2)^{3/2}}
$$

$$
\mathbf{E}_2(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int dl \frac{\rho (\mathbf{r} - \mathbf{r}_2)}{|\mathbf{r} - \mathbf{r}_2|^{3/2}} = \frac{\rho}{4\pi\varepsilon_0} \int d\phi \frac{(\mathbf{r} + \mathbf{a})^\hat{z} + (\mathbf{r} - \mathbf{a})^\hat{x}}{(r^2 + (r + a)^2)^{3/2}}
$$

6) Let $\mathbf{r} = 0$ in the above equations.

We know $\int dq \phi = 0 \Rightarrow$ By inspection 2 equations with negative signs $\Rightarrow \mathbf{E} = 0$ at the origin.