0. Re-do the proof of the Lyapunov’s theorem in Lecture 8.

1. **(Gradient systems)** A dynamical system $\dot{x} = F(x)$, where $F(x) = -\text{grad}V(x) = -\left(\frac{\partial V(x)}{\partial x}\right)^T$ (minus gradient of a smooth function $V$), is called a gradient system.

   a) If $x(t)$ is a trajectory of a gradient system $\dot{x} = -\text{grad}V(x)$, show that
   
   $$\frac{d}{dt}V(x(t)) = -\|\text{grad}V(x(t))\|^2.$$ 

   b) If the Hessian
   
   $$D^2V = \begin{pmatrix}
   \frac{\partial^2 V}{\partial x_1^2} & \cdots & \frac{\partial^2 V}{\partial x_1 \partial x_n} \\
   \vdots & \ddots & \vdots \\
   \frac{\partial^2 V}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 V}{\partial x_n^2}
   \end{pmatrix}$$

   of $V$ (i.e., the matrix of second partial derivatives of $V$) is nonsingular at the equilibria of $\dot{x} = -\text{grad}V(x)$, show that the equilibria cannot be spirals or centers.

2. Exercise 4.3-part (2) of the textbook.

3. Exercise 4.4 of the textbook.