Hanging Sign problem: study of EP at \([ vx=0, vy=0, xm= 1.0, ym= 0 \).]

Design condition: \( L1f= 1.2756, L2f= 1.098 \). (spring free lengths)

Run 1. Verify that EP is where expected for design condition. \((b= 0.1)\)

Verification that we did find an equilibrium point. \(X0= [0; 0; 1.0; 0]\)
Numerical linearization at the EP.

This code will generate an A matrix (Alin) at the EP numerically.

```matlab
% This file linearizes the HSeqns function.
% t= 0.0;
xep= [ 0.0; 0.0; 1.0; 0.0 ];
% xdot= [ 0.0; 0.0; 0.0; 0.0 ];
% xdot= HSeqns(t,xep);
xdot;
Alin= [ 0 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0];
% delta= 0.0001;
for j= 1:4
  x= xep;
  x(j)= x(j)+delta;
  xdotj= HSeqns(t,x);
  Alin(:,j)= (xdotj-xdot)/delta;
end
% 
Alin;
```

Here is the diary report plus the eigenvalues.

```
Alin

Alin =

    -0.0100   0  -154.9041   45.0949
    0   -0.0100   45.0981  -45.0973
    1.0000   0   0   0
    0    1.0000   0   0

Evals =eig(Alin)

Evals =

    -0.0050+13.0786i  
    -0.0050-13.0786i
    -0.0050+ 5.3806i
    -0.0050- 5.3806i

diary off
```

These eigenvalues tell us that the EP is stable but response near the EP is very oscillatory.
Hseqns.m: This file defines the system equations.

function xdot= HSeqns(t,x)
% This file defines a set of system equations for the hanging sign problem.
% In vector terms:  xdot= f(x,U)
% x= [ vx, vy, xm, ym ]
% global A, global B, global g, global k1, global Lf1, global k2, global Lf2
% global m, global b
% % Set up local variables for convenience in coding
% vx= x(1);  % x-velocity of mass
% vy= x(2);
% xm= x(3);  % x-position of mass
% ym= x(4);
%  % Calculate spring lengths from [ xm,ym ]
L1= sqrt(xm^2+(A-ym)^2);  % spring 1 length
L2= sqrt(xm^2+(B+ym)^2);  % spring 2 length
%  % Calculate spring forces from [ L1,L2 ]
F1= k1*(L1-Lf1);
F2= k2*(L2-Lf2);
%  % Calculate transformation array between [F1,F2] and [Fxp,Fyp]
t1x= xm/L1;
t1y= (ym-A)/L1;
t2x= xm/L2;
t2y= (B+ym)/L2;
Fxp= t1x*F1 +t2x*F2;  % force on joint P in x-dir from springs
Fyp= t1y*F1 +t2y*F2;  % force on joint P in y-dir from springs
%  % Set up the input forces
Fx= 0.0;  % no external x force
Fy= -m*g;  % gravity force
%  xdot(1)= (1/m)*(-b*vx -Fxp +Fx);
xdot(2)= (1/m)*(-b*vy -Fyp +Fy);
xdot(3)= vx;
xdot(4)= vy;
%  xdot= xdot';  % return xdot as a column vector
%
**HSparams.m: This file defines the global parameters.**

```
% HSparams.m setup file for hanging sign global parameters
% RCR: 02/27/02
%
% x=[vx, vy, xm, ym] state vector: mass velocity components, mass position
%
% Set global variables for convenience and efficiency of calculation
global A, global B, global g, global k1, global Lf1, global k2
global Lf2, global m, global b
A= 1.0; % y offset (above 0) of spring 1 attachment point
B= 0.0; % y offset (below 0) of spring 2 attachment point
g= 9.81; % gravity acceleration
k1= 100.0; % spring 1 stiffness
%Lf1= A*sqrt(2); % spring 1 free length
Lf1= 1.2756;
k2= 100.0;
%Lf2= A;
Lf2= 1.098;
m= 1.0; % mass
b= 1.0; % damping introduced for computational stability
%
These data show eigenvectors - the modes of response at the EP.

EDU» E (eigenvalues)

E =

```
  -0.5000 +10.0070i     0         0         0
   0    -0.5000 -10.0070i    0         0
   0         0     -0.5000 +19.3174i    0
   0         0         0      -0.5000 -19.3174i
```

EDU» V (eigenvectors)

V =

```
  0.9501 + 0.0475i  0.9501 - 0.0475i  0.2930 + 0.0021i  0.2930 - 0.0021i
  0.2916 + 0.0146i  0.2916 - 0.0146i -0.9547 - 0.0067i -0.9547 + 0.0067i
  0.0000 - 0.0949i  0.0000 + 0.0949i -0.003 - 0.0152i -0.003 + 0.0152i
  0.0000 - 0.0291i  0.0000 + 0.0291i  0.0009 + 0.0494i  0.0009 - 0.0494i
```
Behavior in vicinity of EP. $X_0 = [0, 0, 1.0, 0.01]$

Run near EP, $b = 0.1$