HW #2  Key

1) \[ V_o(s) = Z_f(s) I_f(s) \]
\[ V_i(s) = Z_i(s) I(s) \]
\[ I_f(s) = -I(s) \]

\[ G_i(s) = \frac{Z_f(s) I_f(s)}{Z_i(s) I(s)} = \frac{-Z_f(s) I(s)}{Z_i(s) I(s)} = \frac{-Z_f(s)}{Z_i(s)} \]

2) \[ Z_f(s) = \frac{1}{R_2 s C_2} \]
\[ Z_i(s) = \frac{1}{R_1 s C_1} \]

\[ G_i(s) = -\frac{Z_f(s)}{Z_i(s)} \]

\[ G_i(s) = -\left( \frac{R_2}{s C_2} \right) \left( \frac{1}{s C_1} \right) = -\frac{R_2 R_1}{s C_2} + \frac{R_2}{s^2 C_1 C_2} = -\frac{R_2}{s} \left( \frac{R_1}{C_2} + \frac{1}{s C_1 C_2} \right) \]

\[ = -\frac{R_2}{s^2 C_1 C_2} \left( R_1 C_1 s + 1 \right) \]
\[
\frac{v_1(t) - v_2(t)}{R_1} + C \frac{d}{dt}(v_1(t) - v_2(t)) = \frac{v_2(t)}{R_2}
\]

\[
\frac{v_1(s) - v_2(s)}{R_1} + Cs(v_1(s) - v_2(s)) = \frac{v_2(s)}{R_2}
\]

\[
v_1(s)\left(\frac{1 + R_1Cs}{R_1}\right) = v_2(s)\left(\frac{R_1 + R_2 + R_1R_2Cs}{R_1R_2}\right)
\]

\[
\frac{v_2(s)}{v_1(s)} = \frac{R_2(1 + R_1Cs)}{R_1 + R_2 + R_1R_2Cs}
\]

\[
\frac{v_1(t) - v_x(t)}{R_1} = \frac{v_x(t)}{R_2} + \frac{1}{L} \int_0^t (v_x(t) - v_2(t)) \, dt
\]

\[
\frac{v_1(s) - v_x(s)}{R_1} = \frac{v_x(s)}{R_2} + \frac{v_x(s) - v_2(s)}{sL}
\]

\[
\frac{v_1(s)}{R_1} = \left(\frac{R_1 + R_2}{R_1R_2}\right) v_x(s) + \frac{v_x(s) - v_2(s)}{sL}
\]  \hspace{1cm} (1)

\[
\frac{1}{L} \int_0^t (v_x(t) - v_2(t)) \, dt = \frac{v_2(t)}{R_3}
\]

\[
\frac{v_x(s) - v_2(s)}{sL} = \frac{v_2(s)}{R_3}
\]  \hspace{1cm} (2)

\[
v_x(s) = v_2(s)\left(\frac{sL + R_3}{R_3}\right)
\]  \hspace{1cm} (3)

Combining equation 1, 2 and 3 and simplifying

\[
\frac{v_1(s)}{R_1} = \left(\frac{R_1 + R_2}{R_1R_2}\right)\left(\frac{sL + R_3}{R_3}\right) v_2(s) + \frac{v_2(s)}{R_3}
\]

\[
\frac{v_2(s)}{v_1(s)} = \frac{R_2R_3}{(R_1 + R_2)(sL + R_3) + R_1R_2}
\]
\[ v_i(t) = v_0(t) \]
\[ v_i(s) = v_0(s) \]
\[ \frac{v_0(s)}{v_i(s)} = 1 \]

\[ \frac{0 - v_i(t)}{R_i} = -\frac{v_0(t) - v_i(t)}{R_f} \]
\[ \frac{v_i(s)}{R_i} = \frac{v_0(s) - v_i(s)}{R_f} \]
\[ \frac{v_0(s)}{v_i(s)} = \frac{R_f + R_i}{R_i} \]

\[ C \frac{d}{dt} v_i(t) = -\frac{v_0(t)}{R} \]
\[ Cs v_i(s) = -\frac{v_0(s)}{R} \]
\[ \frac{v_0(s)}{v_i(s)} = -RCs \]
4)\[ V_1(s) = R_1 + R_2 + \frac{1}{sC} \]
\[ V_2(s) = \frac{1}{sC} \]
\[ \frac{V_2(s)}{V_1(s)} = \frac{\frac{1}{sC}}{R_1 + R_2 + \frac{1}{sC}} = \frac{1}{(R_1 + R_2)sC + 1} \]

5) \[ R_1 = 1 \quad R_2 = 1 \quad C = 2 \]
\[ G(s) = \frac{1}{4s + 1} \]

\[ G = tf(1,[4 1]); \]
\[ impulse(G); \]
\[ step(G); \]