1 Problem

\[ G(s) = \frac{L[y(t)]}{L[u(t)]} \]

\[ 1 = \text{Laplace TF of output signal.} \]

\[ \text{Laplace transform of the impulse response.} \]
b. (5 points \times 4) From Fig. 2, find the following transfer functions

- from \( u(t) \) to \( y(t) \), i.e., \( T_{u\rightarrow y}(s) \),
  \[ \frac{K}{(1 - KG)} \]

- from \( u(t) \) to \( \ell(t) \), i.e., \( T_{u\rightarrow \ell}(s) \),
  \[ \frac{KG}{1 - KG} \]

- from \( u(t) \) to \( m(t) \), i.e., \( T_{u\rightarrow m}(s) \),
  \[ \frac{G(s)}{1 - KG} \]

- from \( y(t) \) to \( m(t) \), i.e., \( T_{y\rightarrow m}(s) \),
  \[ \frac{G(s)}{1 - KG} \]

in terms of \( K \) and \( G(s) \).
c. (10 points) Consider a case with

\[ G(s) = \frac{(s - 1)}{s^2 + s + 1} \]

in Fig. 2. Determine the stability of this (OPEN-LOOP) transfer function \( G(s) \) with a justification, (e.g., stable, marginally stable, or unstable?).

Stable since all poles are in the open left half plane.

\[ CE. \quad (-1) = 0 \Rightarrow 1 - \frac{k(s-1)}{s^2 + s + 1} = 0 \]

\[ \Rightarrow s^2 + s + 1 - ks + k = 0 \]

\[ = 1.s^2 + (-k) s + k+1 = 0 \]

Answer \( \cap \) \( 1 \)

\[ \boxed{-1 < k < 1} \]

d. (20 points) For the given \( G(s) = \frac{(s-1)}{s^2 + s + 1} \), determine the range of the constant gain \( K \) for which the CLOSED-SYSTEM in Fig. 2 is stable by using the Routh-Hurwitz stability criterion.
2 Problem

Given the differential equation $\dot{x} = f(x, u)$, linearize the equation about $x_0 = 2$ if the function $f(x, u)$ is given by

(20 points) $f(x, u) = 2^3 - x^3 + xu.$

Step 1  Find $u_0$

$\dot{x}_0 = 0 = f(x_0, u_0) = 3^3 - 2^3 + 2 \cdot u_0 = 0$

$\Rightarrow u_0 = 0$

Step 2  New coordinate system

$\delta x = x - 2$

$\delta u = u - 0$

Step 3  $\frac{\partial f}{\partial x} = -3x^2 + u \Big|_{x_0 = 2, u_0 = 0} = -3 \cdot 2^2 + 0 = -12$

Step 4  $\frac{\partial f}{\partial u} = x \Big|_{x_0 = 2} = 2$

$\delta x = -12 \cdot \delta x + 2 \cdot 2 \delta u$
3 Problem

Figure 2: OP amp.

(10 points) Determine the transfer function from $V_i(s)$ to $V_o(s)$. (Hint: $i^- = 0$ and $v_d = 0$.)

(10 points) Determine the nature of the stability (stable, marginally stable, or unstable). Also give a justification to your answer.

\[
\frac{V_o(s)}{V_i(s)} = -\frac{Z_f}{Z_i} = -\frac{R + \frac{1}{sC}}{sL}
\]

where

$Z_f = R + \frac{1}{sC}$  \hspace{1cm}  $Z_i = sL$

(2) unstable

since it has multiple poles at $j\omega$ axis.