1 Problem

Problem: 1 2 3 Total
Max. Grade: 30 20 40
Grade:

1. (5 points × 2) State two definitions for “transfer function”.

2. (5 points × 2) From Fig. 1, find the following transfer functions
   - from \( u(t) \) to \( y(t) \), i.e., \( T_{u \rightarrow y}(s) \),
   - from \( u(t) \) to \( e(t) \), i.e., \( T_{u \rightarrow e}(s) \).

3. (10 points) Consider Fig. 1 with \( K = 1 \), \( G(s) = \frac{1}{s} \), and a unit impulse input \( u(t) = \delta(t) \), i.e., \( U(s) = L(\delta(t)) = \int_0^\infty \delta(t)e^{-st}dt = 1 \). Find \( y(t) \) when all initial conditions are zeros. Hint: \( L[e^{-at}] = \int_0^\infty [e^{-at}]e^{-st}dt = \frac{1}{s+a} \).

Figure 1: A block diagram.

\[ L\left[\frac{1}{s+a}\right] = \int_0^\infty [e^{-at}]e^{-st}dt = \frac{1}{s+a} \]
\[ \frac{Y(s)}{U(s)} = \frac{1 \cdot \frac{1}{s}}{1 + 1 \cdot \frac{1}{s}} = \frac{1}{s+1} \]

\[ U(s) = 1 \iff L[\delta(t)] = 1 \]

\[ Y(s) = \frac{1}{s+1} \times 1 \]

\[ \mathcal{L}^{-1}[Y(s)] = e^{-t} \]

\[ y(t) = e^{-t} \]
2 Problem

(20 points) Consider the OP amp circuit in Fig. 2. Find the transfer function from the input voltage $v_i(t)$ to the output voltage $v_o(t)$.

\[
\frac{1}{Z_f} = \frac{1}{R + sC} = \frac{1 + RCs}{R}
\]

\[ Z_f = \frac{R}{1 + RCs} \]

\[ Z_i = R + sL \]

\[
T(s) = \frac{V_o(s)}{V_i(s)} = -\frac{Z_f}{Z_i} = -\frac{R}{(sL + R)(1 + RCs)}
\]
3 Problem

Figure 3: A simplified quarter car model.

Figure 3 shows a simplified quarter car model. Assume no gravity and zero initial conditions. Notations are as follows.

- \( u(t) \) : the end position of the system with the coordinate shown in Figure 3.
- \( x(t) \) : the position of the mass with the coordinate shown in Figure 3.
- \( m \) : the mass of the car.
- \( k \) : the spring constant of a linear spring.
- \( b \) : the damping coefficient of a linear damper.

a. (20 points) Draw the complete free body diagram of the quarter car model.

b. (10 points) Determine the equation of motion.

c. (10 points) Determine the transfer function from the input \( u(t) \) to the output \( x(t) \), i.e.,

\[
G(s) := \frac{X(s)}{U(s)} = \frac{\mathcal{L}(x(t))}{\mathcal{L}(u(t))}.
\]
\[ F = ma \]
\[ m \ddot{x} = -b(x - \dot{u}) - k(x - u) \]
\[ (m \ddot{x} + b\dot{x} + kx) = b\dot{u} + ku \]

Thus,
\[ X(s) \left[ ms^2 + bs + k \right] = \frac{X(s)}{U(s)} = \frac{bs + k}{ms^2 + bs + k} \]