1 Problem

Answer the following questions briefly.

a. (10 points) Consider a transfer function $G(s)$. What is the condition for which $G(s)$ is stable?

b. (5 points × 4) Determine if $G(s)$ in the table is stable, marginally stable, or unstable.

<table>
<thead>
<tr>
<th>$G(s)$</th>
<th>stable/ marginally stable/ unstable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{(s+1)}{(s-1)(s+2)}$</td>
<td>poles, $1 - 2 \rightarrow$ unstable</td>
</tr>
<tr>
<td>$\frac{(s^2-1)}{(s+1)^2}$</td>
<td>poles, $-1 - 1 \rightarrow$ stable $\rightarrow$ zeros, $1,1 \rightarrow$ closed loop unstable beyond certain range.</td>
</tr>
<tr>
<td>$\frac{s-1}{s^2+1}$</td>
<td>poles, $\pm i \rightarrow$ marginally stable.</td>
</tr>
<tr>
<td>$\frac{1}{(s^4+100)^2}$</td>
<td>poles, $\pm 10i, \pm 10i \rightarrow$ unstable</td>
</tr>
</tbody>
</table>

a) For $G(s)$ to be stable, the real part of the roots, i.e. poles and zeroes, should lie on the negative part of the real axis.
2 Problem

Consider a PI controller
\[ C(s) = \frac{k_1 s + k_2}{s} \]
for an unstable transfer function
\[ G(s) = \frac{1}{(s^2 + s - 1)} \]
in the feedback system in Fig. 1.

a. (10 points) Find the range of \((k_1, k_2)\) for which the closed-loop system in Fig. 1 is stable. In particular, draw the range in the \(k_1\) (x axis) and \(k_2\) (y axis) plot.

b. (10 points) For values of \((k_1 = 3, k_2 = 1)\) and a unit step input \(U(s) = \frac{1}{s}\), compute the steady state error \(e_{ss} := \lim_{t \to \infty} [e(t) := u(t) - y(t)]\).

**Open loop TF** \[ \rightarrow \quad \frac{k_1 s + k_2}{s (s^2 + s - 1)} \]

\[ \text{Zero \(s = \frac{-k_2}{k_1} \)} \]

\[ \text{Poles} \rightarrow s = 0, \quad -1 \pm \frac{\sqrt{5}}{2} \]

**Closed loop transfer function** \[ \rightarrow \quad \frac{k_1 s + k_2}{s^2 + s - s + k_1 s + k_2} \]

3
Using Routh Array, for \( s^3 s^2 + (k_1 - 1) s + k_2 \)

\[
\begin{array}{cccc}
  s^3 & 1 & (k_1 - 1) & 0 \\
  s^2 & 1 & k_2 & 0 \\
  s^1 & k_1 - k_2 > 0 \Rightarrow k_1 > k_2 + 1 \\
  s^0 & k_2 \Rightarrow k_2 > 0 \\
\end{array}
\]

\[ k_1 - k_2 = 1 \]

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b) Step Error:

\[
C_{ss} = \frac{s(1)}{s \to u.1 + G(s)} = \frac{1}{1 + \frac{k_2}{0}} = 0
\]

\[ G(s) = \frac{k_1 s + k_2}{s(s + k_1 - 1)} \]
3 Problem

(10 points) Find the feasible pole locations of the second order system

\[ G(s) = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \]

in the s-domain that satisfy the following specifications in the time domain.

1. The settling time \( t_s = 5\% \leq 3 \text{ sec} \). Hint: \( t_s = \frac{3}{\zeta \omega_n} \).

2. The percent overshoot \( PO \geq 4.3\% \). Hint: \( PO = 100 \exp\left(-\frac{\pi}{\tan \theta}\right) \).

3. The transfer function \( G(s) \) is stable.

1. \( t_s \leq 3 \text{ sec} \Rightarrow \omega_n \geq 1 \Rightarrow \boxed{-\omega_n \leq -1} \)

2. \( PO \geq 4.3\% \Rightarrow 100 \exp\left(-\frac{\pi}{\tan \theta}\right) \geq 4.3 \)

   \[ \theta_1, \quad \tan \theta = \frac{-\pi}{\omega_n \left(\frac{4.3}{100}\right)} \approx 1 \]

   \[ \theta_1, \quad \theta \approx 45^\circ \]

3. \( G(s) \) in stable \( \Rightarrow \) lies entirely in left half \( s \) complex plane.

Feasible pole region for given condition