Problem 1. (25 pts.) In the system of fig. 1, \( x(t) \) is the input displacement and \( y(t) \) is the output displacement. The initial conditions are \( x(0) = -1 \) and \( y(0) = 0 \).

a) Find the transfer function between \( Y(s) \) and \( X(s) \).
b) From the Transfer function, obtain the response \( y(t) \) when \( x(t) \) is a unit step input.

![Figure 1: Mechanical System](image-url)
Problem 2. (25 pts.) When the system shown in Fig. 2(a) is subjected to a unit-step input, the system output responds as shown in Fig. 2(b).

a) Show that the closed loop transfer function is given by

\[ \frac{C(s)}{R(s)} = \frac{K}{Ts^2 + s + K} \]

b) Determine the values of K and T from the response curve.

Figure 2: a) Closed loop system b) Unit step response curve
Problem 3. (25 pts.) Given the non-linear differential equation for cutter displacement

\[ \dot{x} = 1.5u^2 - 3.5 - 3x - x^2 \]

where \( x = x(t) \) is the cutter displacement in centimeters and \( u = u(t) \) is the drive voltage,
a) Find an appropriate operating value \( u = u_0 \) to linearize the above differential equation for the cutter at the operating point \( x(t) = x_0 = 2 \text{cm} \).
b) Linearize the non-linear differential equation at the \( x(t) = x_0 = 2 \text{ cm} \) operating point.
Problem 4. (25 pts) Write short answers to the following

(a) (12 pts.) If the Laplace transform of a system is \( X(s) = \frac{3}{s(s+1)(s-3)(s+2)^2+4} \), predict the final value for \( x(\infty) = \lim_{t \to \infty} x(t) \) if it exists. If not why not?

(b) (13 pts.) Find the inverse Laplace Transform of

\[
Y(s) = 2 + \frac{2s + 5}{s(s + 1)}
\]