ME 451 HW 5 Solutions

1)

\[
T(s) = \frac{Y(s)}{R(s)} = \frac{K K_1}{s + k_1(k + k_2)}
\]
b. \[ S_{k_i} = \frac{\partial T}{\partial k_i} \frac{k_i}{T} = \frac{[S + k_1(k+k_2)]k_i - k_1k_i}{[S + k_1(k+k_2)]^2} \cdot \frac{k_1}{k_1} \cdot \frac{[S + k_1(k+k_2)]}{k_1k_1} \]

\[ = \frac{S}{S + k_1(k+k_2)} \]

\[ S_{k_i} = \frac{\partial T}{\partial k_i} \frac{k}{T} = \frac{[S + k_1(k+k_2)]k_i - k_1k_i}{[S + k_1(k+k_2)]^2} \cdot \frac{k}{k_1} \cdot \frac{[S + k_1(k+k_2)]}{k_1k_1} \]

\[ = \frac{S + k_1(k+k_2) - k_1k}{S + k_1(k+k_2)} \]

\[ = \frac{S + k_1k_2}{S + k_1(k+k_2)} \]

\[ \text{Re-arrange} \]

\[ Y(s) \]

\[ T_d(s) \]
\[ \frac{Y}{T_d} = \frac{-\frac{1}{s}}{1 + \frac{1}{2}k_1(k+k_2)} \]

\[ = \frac{-1}{s + k_1(k+k_2)} \]

\[ E = n - u \quad n = 0 \]

\[ E = -u \]

\[ = \frac{k}{s + k_1(k+k_2)} \cdot \frac{1}{s} \]

\[ e_{ss} = \lim_{s \to 0} \frac{k}{s + k_1(k+k_2)} \cdot \frac{1}{s} \]

\[ = \frac{k}{k_1(k+k_2)} \]
\[ Y(\infty) = \frac{K \cdot 1}{S + K_1 (k + K_2)} \quad \text{for} \quad k_1 = k_2 = 1, \quad R = \frac{1}{3} \]

\[ Y(s) = \frac{K}{S + (k+1)} \cdot \frac{1}{S} \]

\[ = \frac{A}{S + (k+1)} + \frac{B}{S} \]

\[ = \frac{A S + B [S + (k+1)]}{[S + (k+1)] S} \]

\[ S' : \quad A + B = 0 \]

\[ S'' : \quad (k+1) B = K \quad \Rightarrow \quad B = \frac{k}{k+1}, \quad A = -\frac{K}{k+1} \]

\[ Y(s) = \frac{k}{k+1} \left[ \frac{1}{S} - \frac{1}{S + (k+1)} \right] \]

\[ y(t) = \frac{k}{k+1} \left[ 1 - e^{-(k+1)t} \right] \]

*Fastest response when \( k = 10 \)*
\[ E(s) = I_2 - G_3 \cdot Y(s) \]

\[ Y(s) = E(s) \cdot G_2 \]

\[ E(s) = I_2 - E(s) \cdot G_1 \cdot G_2 \cdot G_3 \]

\[ E(s) = \frac{I_2}{1 + G_1 G_2 G_3} \]

\[ E_{ss} = \lim_{s \to 0} \frac{s \cdot \frac{1}{s}}{s + 0} = \frac{1}{1 + \frac{k(s+50)}{s+200} + \frac{46.24}{s^2+16.75+72.9} + \frac{42.5}{s+425}} \]

\[ = \frac{1}{1 + \frac{k}{4} \cdot \frac{46.24}{72.9} \cdot 1} \]

\[ = \frac{1}{1 + \frac{k}{6.3}} \]

\[ = \frac{6.3}{6.3 + k} \]
3) $s_{15} \rightarrow e$

Unity feedback model

\[ \frac{200}{(s+1)(s+3)} \]

\[ R(s) \leftrightarrow \frac{(0.02)(200)}{(s+1)(s+3)} G(s) \]

\[ C(s) \]

a. i) \( k_p = \lim_{s \to 0} \frac{0.02(200)}{(s+1)(s+3)} \cdot \frac{1}{1} \)

\[ = \frac{4}{3} \]

\[ e_{ss} = \frac{1}{1 + \frac{4}{3}} = \frac{3}{7} = 0.42857 \text{ V} \]

ii) \( k_v = \lim_{s \to 0} \frac{5 \cdot 0.02(200)}{(s+1)(s+3)} \)

\[ = 0 \]

\[ e_{ss} = \frac{1}{0} \to \infty \]
\[ G_{\text{eq}} = 1 + \frac{s}{5} \]

\[ \frac{0.12 \cdot (200)}{(5 + 0.5)} \cdot \left(1 + \frac{s}{5}\right) \]

\[ \frac{4.8 + 0.4}{s(s+1)(s+3)} \]

i) \[ k_p = \lim_{s \to 0} \frac{4s + 4}{s(s+1)(s+3)} \]

\[ k_p \to \infty \]

\[ e_{ss} = \frac{1}{1 + \infty} \Rightarrow e_{ss} = 0 \text{ V} \]

ii) \[ k_v = \lim_{s \to 0} \frac{s \cdot (4s + 4)}{s(s+1)(s+3)} \]

\[ = \frac{0.4}{3} \]

\[ e_{ss} = \frac{3}{0.4} = 7.5 \text{ V} \]
$G_2 = 1 + 0.3 \frac{s}{1 + 0.3 \frac{s}{(s+1)(s+3)}}$

\[ = \frac{4 + 1.2s}{(s+1)(s+3)} \]

i) \[ k_p = \lim_{s \to \infty} \frac{4 + 1.2s}{(s+1)(s+3)} = \frac{4}{3} \]

\[ \varepsilon_{ss} = \frac{3}{4} = 0.75 \]

ii) \[ k_v = \lim_{s \to \infty} 5 \frac{4 + 1.2s}{(s+1)(s+3)} = 0 \]

\[ \varepsilon_{ss} = \infty \]

d. The integral term reduces the steady-state error by increasing the system type by 1.

e. PD will have no effect on ss error (doesn't change system type.)
4)

\[ \frac{4(s+3)}{s^2 - 2s + 10} \]

\[ s = \frac{2 \pm \sqrt{4 - 40}}{2} \]

\[ s = 1 \pm 3i \]

Unstable

6.

\[ \frac{4(s+3)}{s^2 - 2s + 10} \]

\[ \Rightarrow \frac{4(s+3)}{s^2 - 2s + 10 + 4s + 12} \]

Char. eqn \[ s^2 + 2s + 22 = 0 \]

\[ s = \frac{-2 \pm \sqrt{4 - 4 \cdot 22}}{2} \]

\[ s = \frac{-2 \pm 8\sqrt{11}}{2} \]

\[ x \]

Stable
- Open loop

\[ s^2 + 2s + 10 = 0 \]

\[ s = \frac{-2 \pm \sqrt{-36}}{2} \]

\[ s = -1 \pm 3i \]

Stable

- Closed loop

\[ s^2 + 2s + 10 + 4s - 12 = 0 \]

\[ s^2 + 6s - 2 = 0 \]

\[ s = \frac{-6 \pm \sqrt{36 + 8}}{2} \]

\[ s = -3 \pm 3.3 \]

Unstable