1. (20 points) Write the differential equations of motion for the system shown below. Put the variables on the left \((x_1, x_2, x_3)\) and the forces \((F(t))\) on the right. Do not solve the equations.

\[
m_1 \ddot{x}_1 + c \dot{x}_1 + k_1 x_1 - c \dot{x}_3 = 0
\]

Answer: 
\[
m_2 \ddot{x}_2 + (k_2 + k_3) x_2 - k_3 x_3 = 0
\]
\[
m_3 \ddot{x}_3 + c \dot{x}_3 + (k_4 + k_5) x_3 - c \dot{x}_1 - k_3 x_2 = -F(t)
\]

2. (15 points) Given the equations below, find the transfer function.

\[
\ddot{x}_1 + \dot{x}_1 + x_2 = 0, \quad \ddot{x}_2 = F(t), \quad \frac{X_1(s)}{F(s)}
\]

Answer: 
\[
\frac{X_1(s)}{F(s)} = \frac{-1}{s^2(s+1)}
\]

3. (25 points) Given the block diagram below, write the state equations for this system \((\dot{x} = Ax + Bu(t) \text{ and } y = Cx)\) using the states defined as shown.

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 6 \\
-1 & -3 & -4
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} r(t)
\]

Answer: 
\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 6 \\
-1 & -3 & -4
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} r(t)
\]

\[
y =
\begin{bmatrix}
1 & 2 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
\]
4. (15 points) In heat transfer, at the surface of a solid material, the radiative boundary condition is \( -k \frac{\partial T}{\partial x} = \sigma \varepsilon (T^4 - T_o^4) \). This non-linear equation will not allow us to solve the conduction problem exactly, but if we have a convective boundary condition like \( -k \frac{\partial T}{\partial x} = h(T - T_o) \) we can solve the equation. Find a value of \( h \) which will allow us to approximate the non-linear radiative boundary condition as a linear convective condition.

Answer: \( -k \frac{\partial T}{\partial x} = 4\sigma \varepsilon T^3 (T - T_o) \)

5. (25 points) Write the transfer function \( \frac{Y(s)}{R(s)} \) for the system shown below:

Answer: \( \frac{Y(s)}{R(s)} = \frac{6}{s^3 + 4s^2 + 18s + 6} \)