Density Change of Air

Air can be assumed to be an ideal gas and therefore obeys the following relation
\[ P = \rho RT \]
where \( P \) = absolute pressure, \( \rho \) the density, \( R \) the gas constant \((R_u = 8.31 \text{ kPa m}^3/\text{kmol K}, R_{\text{air}} = R_u/M = 0.287 \text{ kJ/kgK})\), and \( T \) the absolute temperature.

Example: Air at 323K, \( \rho = P/(RT) = 101.3/0.287/323 = 1.093 \text{ kg/m}^3 \)

Net Buoyancy Force

The net buoyancy force acting on a body is the difference between the weight of the body and the weight of the fluid displaced by the body, i.e.
\[ F_{\text{net}} = \Delta \rho g V_{\text{body}} \] (for a + direction upward)
where \( g = -9.81 \text{ m/s}^2 \)
Example: For air heated to 50°C and a 1m³ container in a room at 20°C,
\[ F_{\text{net}} = (1.093 - 1.20465)\times(-9.81)\times1 = 1.1 \text{ kg m/s}^2 = 1.1 \text{ N} \text{ or about } 110 \text{ grams.} \]

Other more complicated approach

The volume expansion coefficient is defined as
\[ \beta = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_p \] and is a property of a substance. This property can be looked up in tables for a variety of substances. However, for gases, one can show with \( \rho = P/(RT) \) that
\[ \beta_{\text{ideal gas}} = 1/T \]
where the temperature is expressed in Kelvins.

At constant pressure,
\[ \beta \approx -\frac{1}{\rho} \frac{\Delta \rho}{\Delta T} = -\frac{1}{\rho} \frac{\rho_x - \rho}{T_x - T} \]
or
\[ \rho = \frac{\rho_x}{1 + \beta(T - T_x)} \]