velocity potential for rectangular cavity with rigid boundary

\[ \Phi = A e^{j \omega t} \cos(k_x x) \cos(k_y y) \cos(k_z z), \] 

with \( k_x = \frac{\pi}{L_x} l \) \( l = 0, 1, 2, \ldots \)

\[ k_y = \frac{\pi}{L_y} m \] \( m = 0, 1, 2, \ldots \)

\[ k_z = \frac{\pi}{L_z} n \] \( n = 0, 1, 2, \ldots \)

\[ \rho = -\rho \frac{\partial \Phi}{\partial t} = -\rho j \omega A e^{j \omega t} \cos(k_x x) \cos(k_y y) \cos(k_z z) \]

\( |\Phi| \) has peaks at \( x = 0, x = L_x, y = 0, y = L_y, \)

\( z = 0, z = L_z \)

i.e., \( |\cos(k_x x)| \big|_{x=0} = 1, \big|\cos(k_x L_x)\big| = 1, \) etc.
\[ \mathbf{v} = \nabla \mathbf{u} = A e^{j \omega t} \left[ -k_x \sin(k_x x) \cos(k_y y) \cos(k_z z) \mathbf{\hat{x}} \\
- k_y \sin(k_y y) \cos(k_x x) \cos(k_z z) \mathbf{\hat{y}} \\
- k_z \sin(k_z z) \cos(k_x x) \cos(k_y y) \mathbf{\hat{z}} \right] \]

Normal component of \( \mathbf{v} = u_x \mathbf{\hat{x}} + u_y \mathbf{\hat{y}} + u_z \mathbf{\hat{z}} \)

is \( u_x \) in the \( x \)-dir,

\( u_y \) in the \( y \)-dir, and

\( u_z \) in the \( z \)-dir.

**Key:** Need normal component of \( \mathbf{v} \) to equal zero at each boundary.

To show this, factor out (i.e., ignore) the common \( A e^{j \omega t} \) term, so

\[ u_x = -k_x \sin(k_x x) \cos(k_y y) \cos(k_z z) \]

\[ u_y = -k_y \sin(k_y y) \cos(k_x x) \cos(k_z z) \]

\[ u_z = -k_z \sin(k_z z) \cos(k_x x) \cos(k_y y) \]
need \( u_x(x=0) = 0 \) and \( u_x(x=L_x) = 0 \)

for all values of \( y+z \).

This is clearly true, as \( \sin(k_x 0) = 0 \)

and \( \sin(\frac{\pi L_y}{L_x}) = 0 \)

so these terms vanish (as they should)

across the rigid boundary.

Can show the same goes for \( u_y(y=0) = 0, u_y(y=L_y) = 0, u_2(z=0) = 0, \) and \( u_2(z=L_z) = 0. \)

So, the normal component of \( \vec{v} \) is indeed equal to zero on the face of every rigid boundary, and the necessary B.C.'s are satisfied.