Additional Exercises for Chapter 6

1. Consider the system

\[ \dot{x}_1 = -x_1 + x_2, \quad \dot{x}_2 = -h(x_1) - x_2 + u, \quad y = x_2 \]

where \( h \in [k, \infty) \) for some \( k > 0 \).

(a) Let \( u = 0 \). Using

\[ V(x) = \int_0^{x_1} h(z) \, dz + \frac{1}{2}x_2^2 \]

show that the origin is globally asymptotically stable.

(b) Show that the system is strictly passive and output strictly passive.

(c) Show that the system is finite-gain \( L_2 \) stable and find an upper bound on the \( L_2 \) gain.

(d) Consider the feedback connection of the dynamical system given above with a time-varying memoryless nonlinearity that belongs to the sector \( [-\frac{1}{2}, \frac{1}{2}] \). Show that the origin of the unforced closed-loop system is globally uniformly asymptotically stable.