Mutation Analysis Models for Visual Servoing in Nanomanipulations

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Abstract—This paper has two purposes: investigating a featureless visual servoing approach based on mutation analysis and proposing a visual servo control method for nanomanipulations. For the first purpose, the featureless visual servoing method is needed because traditional visual servoing relies heavily on robust feature extraction and tracking, which are very difficult in natural environment. The mutation analysis based approach in this paper considers the image as a set, and designs a controller to make the distance between the initial and goal image sets converge to zero, thereby steering the initial image to the goal image. For the second purpose, atomic force microscopic (AFM) based nanomanipulations with subnanometer accuracy are very difficult because the position sensor cannot provide valuable feedback due to large noises at this precision level. We propose to use the images obtained by AFM and perform a visual servo control. This method, independent of external sensors, can directly perform control on the AFM end tip’s position. The featureless controller is successfully validated on AFM images and the results suggest a potential precision enhancement for nanomanipulations.

I. INTRODUCTION

Visual servoing uses vision information to control the motion of a mobile robot or a manipulator for various tasks such as navigation or manipulation. Consider the eye-in-hand configuration when the camera is rigidly attached to a robot or a manipulator. The goal for visual servoing is to design a control law, under which the mobile robot or manipulator can move, so that the current image will eventually converge to a specified goal image [1]. This problem is usually solved in vector space by minimizing the error \( e(t) \) defined as \( e(t) = s(t) - s^* \), where \( s(t) \) and \( s^* \) are the current and desired position vectors of features from the image, respectively [1]. This approach, however, relies heavily on robust feature extraction and tracking. In fact, good features may be difficult to find and they may also become occluded during the movement in natural environment [2]. Furthermore, reliable feature tracking for visual servo in complex real environments is quite challenging [3]. To address these problems, artificial fiducial markers are employed for the experiments in most of visual servoing literature.

Instead of using fiducial markers, there is a trend to come up with featureless visual servoing techniques in recent years. In these techniques, the servo control is directly performed based on pixel intensity information in the image, and no explicit feature extraction and tracking is required. Several different featureless visual servoing methods are proposed in literature. In general, they can be classified into four categories based on how the error \( e(t) \) is defined.

The first method [4], considering the whole image as a feature, performs principle component analysis on the original image to obtain an eigen space with reduced dimension. Then the control is performed to make the error in this eigen space converge to zero. This method, although not depending on explicit feature extraction, requires processing on the entire image; moreover, formal proof of convergence is not provided. The second method [5], similar to the first one, processes the original image with a spatial sampling function to derive a kernel measurement, and a controller is designed to make the measurement error converge to zero. But the controller is only obtained for a subset of rigid body motions instead of a general six degree-of-freedom motion. The third method tries to make the sum-of-squared-difference of intensities between two images converge to zero, and the matrix relating the camera motion to the motion of individual pixels is obtained through the optical flow equation. Two different implementations exist for this method. In [6], the optimization based control is performed without formal convergence proof, while in [7], [8], similar gradient control is designed to stabilize a helicopter but with a formal stability proof. It’s also worth mentioning that the latter considers a dynamic model with force/torque input instead of a kinematic model with only velocity input. The last method employs the entropy from information theory to obtain the mutual information between two images and uses an optimization approach to maximize it [9]. This method is robust with respect to different lighting conditions.

The mutation analysis based visual servoing approach is another featureless method proposed earlier than all above methods [10]. The basic idea for this approach is shown in Fig. 1. Both the current and goal images are considered as sets, and the error between two images is defined as the distance between two sets. A controller is then designed to make the image set distance converge to zero. Nevertheless, since the space of sets does not have the linear structure of vector space, the traditional dynamics model in vector space

![Fig. 1. The scheme for mutation analysis based visual servoing](image-url)
cannot be used; instead, the non-vector mutation analysis for set evolution should be employed \([11],[12]\). Therefore, this method provides a fundamentally different way to solve the featureless serving problem.

The purpose of this paper is twofold. On one hand, we want to bring this mutation analysis based visual serving method into the community’s attention. Essentials of mutation analysis will be explained, and the controller for binary images in \([10]\) will be extended to gray scale images. On the other hand, we propose a potential application for visual serving: atomic force microscope (AFM) based nanomanipulation, and validate the mutation analysis based method with simulations on AFM images. Note that a literature review for AFM based nanomanipulations will be given in section IV.

The rest of paper is organized as follows. First of all, essential concepts in mutation analysis will be presented in section II. After that, the mutation equation model for visual serving and the controller design are given in section III. Based on the controller, the potential application for nanomanipulation with visual serving is presented in section IV, and the simulation results with AFM images are demonstrated in section V. Finally, the conclusions are given.

II. Mutation Analysis for Set Evolutions

Mutation analysis deals with the dynamics of sets instead of vectors. In this case, the linear structure of vector space does not exist because the addition and scalar multiplication of vectors. In this case, the linear structure of vector space based on transitions. For a function \( f : \mathbb{R}^n \to \mathbb{R}^n \), denote the set of all such functions as \( BL(E, \mathbb{R}^n) \). For ordinary differential equation (ODE) \( \dot{x} = \varphi(x) \) with initial condition \( x(0) = x_0 \). The transition for \( \varphi \) is defined as:

\[
T_\varphi(t, x_0) = \{ x(t) : \dot{x} = \varphi(x), x(0) = x_0 \}
\]

(4)

where \( x(t) \) is the solution to above ODE. In other words, the transition is nothing but the solution to a given ODE at time \( t \). Note that a rigorous definition for transitions requires it to satisfy four conditions \([12]\), but for simplicity, we use above definition which can be shown to satisfy those four conditions (Ch. 1, Example 3, \([12]\)). The above definition can be extended when the initial condition is a set instead of a point:

\[
T_\varphi(t, K_0) = \{ x(t) : \dot{x} = \varphi(x), x(0) \in K_0 \}
\]

(5)

where \( K_0 \) is a set containing all possible initial conditions which implies \( T_\varphi(t, K_0) \) is also a set. In fact, \( T_\varphi(t, K_0) \) can be considered as a reachable set of points at time \( t \) generated by \( \varphi \) with the initial points in \( K_0 \). It can also be considered as a tube evolving from \( K_0 \) under the rule given by \( \varphi \). Note that this definition can also be shown to satisfy the four conditions for transitions (Ch. 1, Example 4, \([12]\)).

C. Mutation Equations

To describe the set dynamics, the evolution of a set, called tubes, should be defined at first. A tube \( K(t) \subset \mathbb{R}^n \) is defined as \([10]\):

\[
K(\cdot) : \mathbb{R}^+ \to 2^{\mathbb{R}^n}
\]

(3)

where \( 2^{\mathbb{R}^n} \) is the power set of \( \mathbb{R}^n \) which is the collection of all subsets of \( \mathbb{R}^n \). Let \( \varphi : E \to \mathbb{R}^n \) with \( E \subset \mathbb{R}^n \) be a bounded Lipschitz function. Denote the set of all such functions as \( BL(E, \mathbb{R}^n) \). Note that this definition can also be shown to satisfy the four conditions (Ch. 1, Example 3, \([12]\)).
where $K(t + \Delta t)$ is the set at time $t + \Delta t$ according to the tube $K(t) : \mathbb{R}^+ \rightarrow 2^{\mathbb{R}^n}$. Note that for a given tube, there may be none or multiple $\varphi$ satisfying Eq. (7); therefore, the mutation of a tube $K(t)$ is defined as the set, possibly empty, of all $\varphi$:

$$\hat{K}(t) = \{ \varphi(x) \in \mathcal{B}(E, \mathbb{R}^n) : \text{Eq. (7) is satisfied} \}$$

(8)

Based on the mutation of a tube, the mutation equation describing set dynamics is defined as:

$$\varphi(x) \in \hat{K}(t)$$

(9)

The mutation equation defined above has only $x$ as its parameter. To add the control input to the equation, consider a map $\varphi : E \times U \mapsto \mathcal{B}(E, \mathbb{R}^n)$ where $U$ is the set of all possible controls $u$. Then the controlled mutation equation can be defined as:

$$\varphi(x(t), u(t)) \in \hat{K}(t) \quad \text{with} \quad u(t) = \gamma(K(t))$$

(10)

where $\gamma : 2^{\mathbb{R}^n} \mapsto U$ is the feedback map from current set $K(t)$ to the control input.

**III. Mutation Analysis Based Visual Servoing**

Based on mutation analysis, the visual servoing problem can be formulated as a stabilization problem: Given a goal image set $\hat{K}$ and an initial image set $K(0)$, design a feedback controller $u(t) = \gamma(K(t))$ based on current image set $K(t)$ such that $dh(K(t), \hat{K}) \rightarrow 0$ as $t \rightarrow \infty$.

To solve this problem, the mutation equation should be first derived, which requires to determine $\varphi(x(t), u(t))$. In the visual servoing case, $x(t)$ will be the trajectory for individual pixel in the image. Therefore, we can denote $x = [x_1, x_2, x_3]^T$ where $x_1$ and $x_2$ represent the pixel position and $x_3$ is the pixel intensity. The control input $u(t)$, if a kinematic model is considered, will be the camera’s spatial velocity which consists of three linear velocity components and three angular velocity components; let $u(t) = [v_x, v_y, v_z, \omega_x, \omega_y, \omega_z]^T$. Based on the definition of $\varphi(x(t), u(t))$, all we need to do is to find the relation $\dot{x}(t) = \varphi(x(t), u(t))$. In fact, $x_1(t)$ and $x_2(t)$ are related to $u(t)$ by the interaction matrix [11]. In what follows, $\varphi(x(t), u(t))$ will be obtained based on the interaction matrix.

Under invariant lighting condition, the projection of a 3D point in the environment onto the image plane will have a constant intensity; therefore, we have $\dot{x}_3(t) = 0$. Suppose the camera is calibrated, i.e., the intrinsic parameters such as skew factor or pixel aspect ratio are known. Then with the perspective projection model, any point in the image plane with coordinates $x_1$ and $x_2$ is related to the corresponding 3D point $P$ in the environment with coordinates $P = [p_x, p_y, p_z]^T$ by:

$$x_1 = \lambda p_x / p_z, \quad x_2 = \lambda p_y / p_z,$$

(11)

where $\lambda$ is the focal length. Without loss of generality, we can assume $\lambda = 1$. By direct differentiation Eq. (11) and considering $\dot{x}_3 = 0$, we have:

$$\dot{x}(t) = L u(t)$$

(12)

where

$$L = \begin{bmatrix}
-1/p_z & 0 & x_1/p_z & x_1 x_2 & (1 + x_2^2) & x_2 \\
0 & -1/p_z & x_2/p_z & 1 + x_2^2 & -x_1 x_2 & -x_1 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Note that $L$ is obtained by adding a third row to the traditional interaction matrix to consider an extra intensity element $x_3$ in $x$. Similar to the usual visual servoing, the depth $p_z$ should be determined, whose value can be directly estimated from the feature variations in different images [15]. In this paper, however, we consider $p_z$ to be a constant in order to simplify the analysis. In this case, $L$ only depends on $x$. Based on Eq. (12), we have $\varphi(x(t), u(t)) = L(x(t)) u(t)$.

Then the tube $K(t)$ for image set evolution induced from $\varphi(x(t), u(t))$ is:

$$K(t) = T_{\varphi}(t, K_0) = \{ x(t) : x = \varphi(x(t), u(t)), x(0) \in K_0 \}$$

(13)

where $K_0$ is the initial image set. With this $K(t)$, Eq. (7) is satisfied; therefore, the controlled mutation equation is:

$$L(x(t)) u(t) \in \hat{K}(t)$$

(14)

To simplify the notation, $t$ will be omitted in the following discussion, and above equation is denoted as $L(x) u \in \hat{K}$.

Because $\varphi(x, u) = L(x) u$ is linear in $u$, the same controller derived in [10] can be used. To obtain the controller, the following Lyapunov function candidate is defined [10]:

$$V(K) = \int_K d^2_K(x) dx + \int_K d^2_K(x) dx$$

(15)

Note that if we define the error as $e = dh(K, \hat{K})$, then the Lyapunov function candidate is usually defined as $e^2 / 2$. With this candidate, however, the possibility of differentiation is unknown. For the Lyapunov function candidate in Eq. (15), we can verify that if $V(K) \rightarrow 0$, then $e \rightarrow 0$. In fact, if $V(K) \rightarrow 0$, we have both $d^2_k(x) \rightarrow 0$ with $x \in K$ and $d^2_k(x) \rightarrow 0$ with $x \in \hat{K}$ because the distance is nonnegative. Then $d(K, \hat{K}) \rightarrow 0$ and $d(K, \hat{K}) \rightarrow 0$. The definition of Hausdorff distance implies $e \rightarrow 0$.

Note that the Lyapunov function candidate in Eq. (15) is different from traditional Lyapunov function in vector space because it’s a function of a time varying set. In fact, it is called shape Lyapunov function [16]. Based on the Lyapunov function candidate in Eq. (15), we can have the following theorem, whose proof is given in [10].

**Theorem 1** [10]: For the system $L(x) u \in K(t)$ with $x \in \mathbb{R}^n$, $L(x) \in \mathbb{R}^{n \times n}$, $u \in \mathbb{R}^n$, and $K(t) \subset \mathbb{R}^n$, the following controller can locally exponentially stabilize it at $\hat{K}$:

$$u(t) = \gamma(K) = -\alpha A(K)^+ V(K)$$

(16)

where $\alpha > 0$ is a gain factor and $A(K)^+$ is the Moore-Penrose pseudoinverse of $A(K) \in \mathbb{R}^{1 \times n}$ defined by:

$$A(K) = \int_K d^2_k(x) \sum_{i=1}^m OL_i / \partial x_i + \int_K [x - \Pi_K(x)]^T L(x) dx$$

$$-2 \int_K [x - \Pi_K(x)]^T L(\Pi_K(x)) dx$$

where $L_i (i = 1, 2, \ldots, m)$ are the $m$ row vectors in matrix $L$, and $\partial L_i / \partial x_i$ is also a row vector with the same dimension.
IV. Application to Nanomanipulations

Atomic Force Microscopy (AFM) is one of the primary tools for imaging and manipulating matters at nanoscale. As shown in Fig. 2, the AFM consists of a piezo-actuated cantilever with a sharp tip at its end. Once the tip interacts with the sample surface, the atomic force will cause a deflection of the cantilever which will be recorded by a position sensitive device (PSD). The amplitude of deflection represents the height of sample point. In this way, the topography of the sample surface can be obtained [17].

![Fig. 2. Principle of Atomic Force Microscopy](image)

Traditionally, AFM is used as a high resolution imaging tool for sample topography interrogating. In recent years, by considering the cantilever as a micro robotic arm, the AFM is used for manipulation to mechanically push, pull, or cut structures at nanoscale [18]. Due to the invisibility of the nano world, the manipulation is usually achieved in a scan-manipulation-scan fashion. First of all, an area of interest is scanned and the resulting image is used to identify the objects intended to be manipulated. Based on the location of the object in the image, a control signal is designed and applied to the AFM, leading to an open loop movement for the cantilever. Finally, the area of interest is scanned again to verify the manipulation results.

According to above process, the fundamental requirement for successful nanomanipulation is precise point to point motion, i.e., the horizontal position for the AFM tip should be controlled precisely enough. Nevertheless, the movement induced vibration, low frequency creep effect, thermal drift, and inherent hysteresis for piezoactuator make the desired position accuracy difficult to achieve [19]. Although position sensors can be added to provide state feedback, they can only measure pizeotube’s position instead of tip’s position; the large sensor noise at such precision level also makes the sensor incapable of providing valuable feedback. Therefore, sensor deficiencies exclude the use of standard feedback control methods.

To address these problems, the model inversion based feedforward compensation is always used [20]. Although this method provides satisfactory results in some cases, the identification process is quite complicated, and the performance relies on the accuracy of the forward model [21]. Several other methods such as combined feedforward/feedback and $H_{\infty}$ can also be employed to enhance the performance [22].

Visual servoing provides another approach for manipulation in the nano world. In fact, the AFM can be considered as a single pixel image sensor with two translational degree of freedoms, and an image is obtained by sequentially moving the AFM tip on the sample surface. Based on this idea, the visual servoing based nanomanipulation can be implemented in two cases. For both cases, suppose an area of interest is scanned and an image $I$ is obtained, from which the desired image set can be specified.

The first case is when the desired AFM tip position is sufficiently close to the current tip position. The goal image $\hat{K}$, containing the desired tip position in the image, can be specified in image $I$. The current image $K$ is obtained by using the AFM for a local scan around its current tip position [23]. Once $K$ is obtained, the current tip position can be obtained by comparing $K$ with $I$ using image registration methods such as template matching. This case is shown in Fig. 3(a), where the current and desired positions are shown as solid circles and the corresponding images are shown as squares. Then the controller in Eq. (16) can be used to steer the AFM tip to the desired position. Note that during the implementation, when the tip arrives an updated position, a local scan should be performed to obtain an updated image.

The second case corresponds to when the desired AFM tip position is far away from the current position. In this case, a series of intermediate images, with their image centers along a planned trajectory between the current and desired positions, can be used as the intermediate goal images. The consecutive intermediate images should be sufficiently close so that the same approach for the first case can be employed. This case is shown in Fig. 3(b), where four intermediate images along a straight line trajectory are shown.

The nanomanipulation discussed above can be considered as a special case with only two translational velocity components $v_x$ and $v_y$ as the inputs. Without loss of generality, assume $p_z = 1$, then we can obtain $\phi(x, u)$ from Eq. (12) as:

$$\phi(x, u) = Lu$$

where

$$L = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}$$

is a constant matrix. In this case, $A(K)$ in the controller given

![Fig. 3. Illustration of two cases for visual servoing based nanomanipulation](image)
by Eq. (16) is simplified to:

$$A(K) = 2\left\{ \int_{K} [x - \Pi(x)]^T dx - \int_{K} [x - \Pi(x)]^T dx \right\} L$$

It should be noted that the scanning images have been used to obtain the parameters for the forward model [20] and compensate the dynamic effects [24]. However, to control directly with the image information is, to the best of our knowledge, never considered before. On the other hand, our approach is very similar to the vision guided robot navigation based on image memory [25]; however, our approach is different because no feature is used.

V. SIMULATION RESULTS

In this section, we validate our controller using AFM images for the above two cases. For the first case, the simulation flow chart is shown in Fig. 4. First of all, a goal image \( \hat{K} \) and a current image \( K \), sufficiently close to each other, will be chosen from a given whole image \( I \). Note that in real AFM implementation, both \( \hat{K} \) and \( K \) and its position in \( I \) are specified, but \( K \) is obtained by a local scan and its position is derived by a image registration using \( K \) and \( I \). In our simulation, it’s assumed that both \( K \) and its position are known in advance. Based on \( \hat{K} \) and \( K \), the control input \( u \) is calculated, which is used to update the position for \( K \). Although the control signal is the translational velocity \( v_x \) and \( v_y \), it can be considered as displacements if the time intervals for each iteration are the same. Therefore, an updated \( K \) is obtained by moving \( K \) in \( I \) with control input as the displacements. Note that in AFM implementation, we should move the AFM tip to the updated position and perform a local scan to update \( K \). Using the updated \( K \), the same process is performed again to obtain a new control signal. The program terminates until \( dh(\hat{K}, \hat{K}) = 0 \).

The simulation is performed based on two gray scale images with size 256 \( \times \) 256 pixels as shown in Fig. 5. The image in Fig. 5(a) has a scan size 25 \( \mu \)m and is scanned for nanowire manipulation, while the image in Fig. 5(b) has a scan size 32 \( \mu \)m and is scanned for hacat cell manipulation. The initial and goal images, with size 30 \( \times \) 30 pixels, are labeled in original image and also enlarged on the right.

For the nanowire image, the pixel distance between initial and goal images is ten pixels horizontally and seven pixels vertically. The simulation results for this image are shown in Fig. 6(a), where both values for Lyapunov function and Hausdorff distance for all the iterations are shown. From the figure, both values decrease monotonically, and after five iterations, the current image coincides with the goal image.

For the hacat cell image, the pixel distance between initial and goal images is ten pixels horizontally and fifteen pixels vertically. The simulation results shown in Fig. 6(b) demonstrate that the control goal can be achieved. Although the Lyapunov function value decreases, the Hausdorff distance increases at the third iteration. This is because the decrease of Lyapunov function cannot guarantee the decrease of Hausdorff distance, but if the Lyapunov function approaches zero, the Hausdorff distance must also approach zero as discussed in section III.

For the second case when the initial image and goal image are far away from each other, a straight line trajectory is used to generate the intermediate images. To guarantee the convergence at those intermediate goal images, the pixel distance between two consecutive images is chosen to be four pixels vertically, while the horizontal pixel distance is obtained from the equation of the line trajectory. The same images, as shown in Fig. 7, are used for the simulation, where the initial and goals images are also shown. For both simulations, the pixel distance between the initial and goal images is 40 pixels horizontally and 60 pixels vertically. Therefore, the number of intermediate goal images is 14.

The simulation results for both images are shown in Fig. 8, where the center positions in pixels for the desired trajectory and the true trajectory are plotted. The end points for the black solid lines are the center positions for the intermediate
goal images, while the end points, shown as small circles in the figure, for the red dashed lines are the center positions for the images obtained by servo control. From the figure, we can see that although the true positions may deviate from the desired positions sometimes, the final goal position can be achieved. For the nanowire image, a total number of 38 iterations is performed to arrive the goal image, which means an average of 2.5 iterations from one intermediate goal image to the next one, while for the cell image, the total number is 29 and the average number is thus 1.9.

![Fig. 7. Simulation images for the second case](image1)

(a) Nanowire AFM images  
(b) Hacat cell AFM images

![Fig. 8. Simulation results for the second case](image2)

(a) Results for nanowire images  
(b) Results for hacat cell images

VI. CONCLUSIONS

In this paper, the framework of using mutational analysis for featureless visual servoing is reinvestigated. Different from existing featureless methods, this set evolution based approach considers the image as a set, and the controller is designed based on the distance between two image sets. The simplified controller with two translational degree of freedoms is validated using AFM images. If the initial image and goal image are close enough, the controller can successfully steer the initial image to the goal image. On the other hand, if they are far apart, a set of intermediate goal images will be obtained with consecutive images sufficiently close to each other. Then the initial image can also be steered to the goal image successfully. The method presented in this paper can be considered as a new approach for precise nanomanipulations, and the implementation on an AFM system will be performed in the future.

REFERENCES