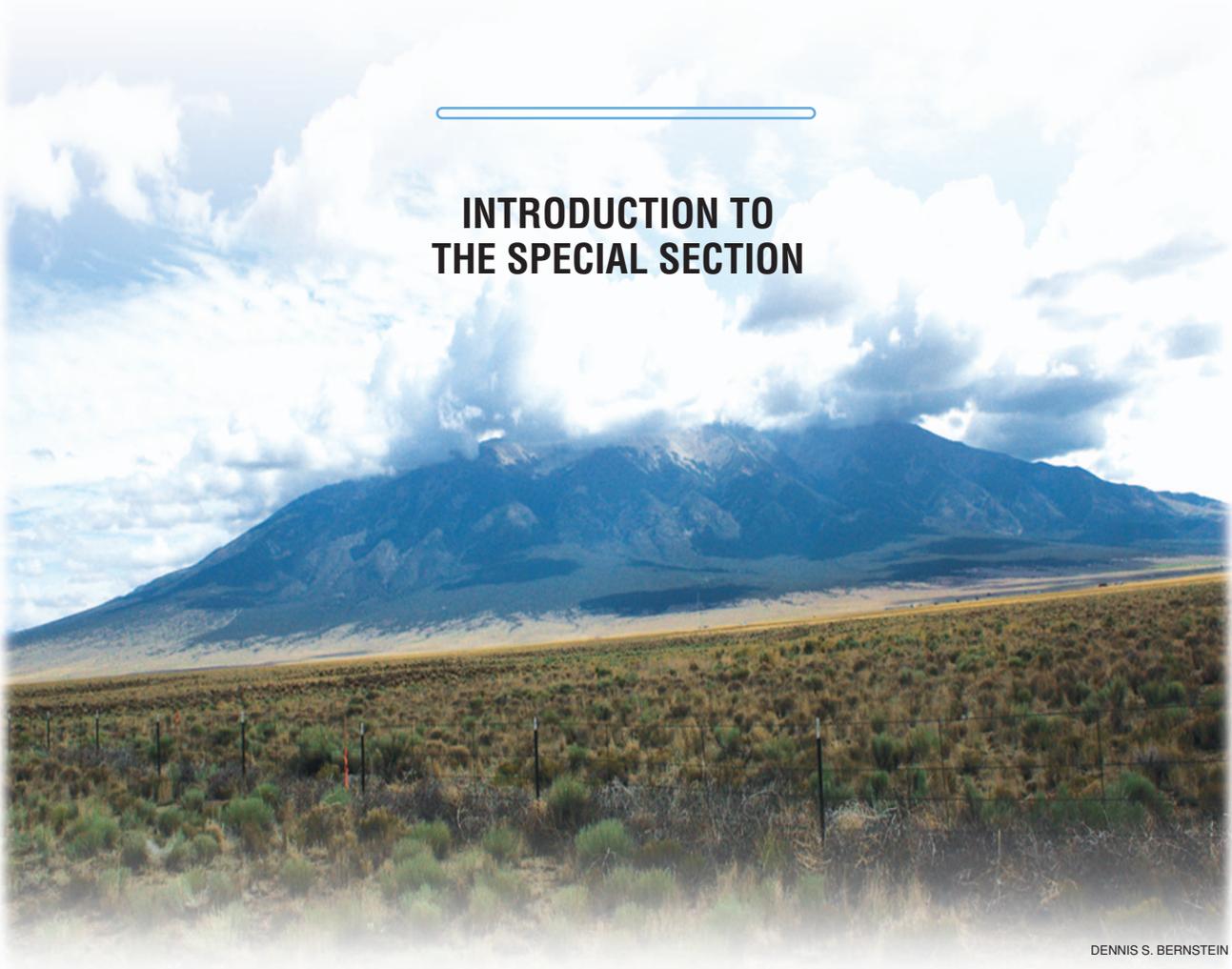


# Modeling and Control of Hysteresis

XIAOBO TAN and RAM V. IYER

---

## INTRODUCTION TO THE SPECIAL SECTION



DENNIS S. BERNSTEIN

**H**ysteresis is a nonlinear effect that arises in diverse disciplines ranging from physics to biology, from material science to mechanics, and from electronics to economics. While ferromagnetism offers a classical example, hysteresis also appears in mechanical systems (in the form of backlash and friction) and geophysical systems. In biology, hysteresis provides a mechanism that enhances the robustness of cell functions against random perturbations. Hysteretic

*Digital Object Identifier 10.1109/MCS.2008.930921*

behavior abounds in economics and finance, where one example is the relationship between the investment decision and the expected profit. In addition, hysteresis is ubiquitous in smart materials, such as piezoelectrics, magnetostrictives, shape memory alloys, and electroactive polymers.

The origin of hysteresis is the existence of multiple metastable equilibria associated with the system dynamics under consideration. This explanation is due to Landau [1], who developed a qualitative theory of phase transitions, which explains various kinds of hysteretic phenomena. Hysteresis is typically viewed as an undesirable, detrimental effect in that it complicates our perception or control of interentity relations in physical, biological, and engineering systems. On the other hand,

Prandtl (1924) considered a scalar model of elastoplasticity, which was later rediscovered by Ishlinskii (1944). Haines (1930) [8] discovered a hysteretic relationship between moisture content in the soil and capillary pressure. Hysteresis due to sunk-costs was studied as a fundamental economic phenomenon in the 1980s [9].

The mathematical study of hysteresis was initiated not by a mathematician, but by an engineer Bouc (1966) [10], who studied scalar hysteresis as a map between function spaces. Starting in 1970, systematic study was undertaken by Krasnoselskii, Pokrovskii, and others, who constructed rate-independent hysteresis operators from elementary units called hysterons [11]. Here, rate independence refers to the idea that only the past maxima and minima of an input signal affect future outputs,

**The mathematical study of hysteresis was initiated not by a mathematician, but by an engineer, Bouc, who studied scalar hysteresis as a map between function spaces.**

hysteresis can be exploited in positive ways both in nature and in engineering applications. For example, hysteresis is the fundamental mechanism in magnetic data storage and emerging computer memory technologies, such as ferroelectric nonvolatile thin-film memories, which are major enablers of the information-technology industry. As another example, hysteretic switching prevents chattering and the associated consequences in switched systems, such as thermostats, digital circuits, and power electronics.

The study of hysteresis has a long history (see [2, p. 9]). Although James A. Ewing [3] coined the word hysteresis during his study of ferromagnetism in 1881, the special properties of magnetite, that is, loadstone, seem to have been known and utilized by ancient Greek and Chinese civilizations at least in first and second millennia BC respectively (see [4, p. 1]). The ancients constructed compasses that take advantage of the remanance property of magnetite, which is a consequence of hysteresis in the magnetic-field input versus the magnetization output. Contributions to the study of hysteresis were made by Madelung (1905) [5], who discovered the rules for scalar hysteresis; Weiss (1907) [6], who introduced the concept of a magnetic domain based on spontaneous magnetization; Heisenberg (1928), who explained spontaneous magnetization in terms of quantum mechanics; Preisach (1935) [7] who introduced a model for scalar hysteresis; and Landau (1937) [1]. By the 1920s, researchers in plasticity and soil mechanics had discovered the hysteresis phenomenon as well.

and the rate at which these maxima or minima are achieved does not matter. Further work by Mayergoyz [12], Visintin [2], Brokate and Sprekels [13], and Krejci [14] in the 1980s led to an understanding of general scalar hysteresis operators, including, in particular, the Preisach operator. These authors also studied the existence and uniqueness of solutions of ordinary differential equations (ODEs) and partial differential equations (PDEs) coupled with hysteresis operators.

A surge in interest in smart materials and their control took place in the 1990s spurred by the development of new materials that showed significant magnetostrictive and electrostrictive properties. The constitutive coupling between mechanical and electrical/magnetic/thermal properties of these materials at the microscale enables their inherent sensing and actuation capabilities, while, at the same time, at the microstructure level, multiple stable equilibria exist for each fixed external input. This property leads to the hysteretic behavior of smart material sensors and actuators. When hysteresis is present along with the electro-magneto-mechano-chemical dynamics of the materials, the overall behavior of smart sensors and actuators can be very complex. Nevertheless, many applications, such as atomic force microscopy, noise reduction, and ultrahigh density data storage, demand robust, high-precision control of smart material actuators. The need for better control motivates the need for identification and control methods for systems described by hysteresis operators [15]–[22]. Though many basic problems have been solved, technically difficult problems remain to be addressed.

## The origin of hysteresis is the existence of multiple metastable equilibria associated with the system dynamics under consideration.

The intellectual appeal and practical importance of hysteretic phenomena motivated us to prepare this special section for the broader controls community. The modeling, identification, and control techniques, as well as the applications examined in this special section, reflect the wide diversity of the field. Recent progress in understanding and exploiting hysteresis from the systems perspective is highlighted, and major challenges and opportunities faced by the controls community are discussed. In particular, applications to economics, terrestrial hydrology, and smart materials are presented to illustrate the ubiquitous nature of hysteresis. Although hysteresis is understood to be a quasi-static, rate-independent phenomenon in applied mathematics, the notion of rate-dependent hysteresis is also found in engineering literature. In particular, rate-dependent hysteresis in linear systems with nonlinear feedback is shown to be ubiquitous.

The opening article by Rod Cross, Michael Grinfeld, and Harbir Lamba presents the notion that rate-independent hysteresis is a fundamental aspect of economic systems. Sunk costs associated with switching between strategies lead naturally to the individual economic unit acting as a hysteron, in a role similar to a thermostat relay. The thresholds of the hysteron depend on the individual, which, upon aggregation, give rise to a Preisach hysteresis operator between macroeconomic variables. This hysteresis operator is a mechanism by which memory is reflected in future events and actions: such nonlocal memory is not accounted for in classical and neoclassical economic theories. Applications to macroeconomics and financial markets are presented.

The article by Brian Appelbe, Denis Flynn, Hugh McNamara, Philip O'Kane, Alexander Pimenov, Alexei Pokrovskii, Dmitrii Rachinskii, and Andrey Zhezherun presents an application of hysteresis modeling to terrestrial hydrology. The discovery of hysteresis in the relation between soil moisture and capillary pressure was made in 1930 by Haines [8]. The application of modern mathematical models of hysteresis to this phenomenon reveals insights into global phenomena that are relevant to the study of greenhouse warming effects. In particular, the authors argue that hysteresis between soil moisture and capillary pressure causes dissipation of heat energy, which may be significant on a global scale. Verification of this hypothesis is an open problem. The authors develop

the FEST model describing a fully vegetated slab of soil and transpiring plants. Validation of the model with real data is presented.

The third article by Kam Leang, Qingze Zou, and Santosh Devasia focuses on feedforward control of piezoactuators in an atomic force microscope (AFM). In this application, the AFM is used to create nanosized features, such as quantum dots, and to probe biological specimens such as cells. The AFM relies on a piezoactuator to move and position the probe tip relative to the sample surface. The piezoactuator exhibits hysteresis and creep phenomena, which makes controlling its movements a challenge. The authors describe an inversion-based feedforward approach to compensate for dynamic and hysteresis effects in piezoactuators and present experimental evidence of its efficacy.

The article by Ram Iyer and Xiaobo Tan studies precision control of smart actuators. Modeling, identification, and adaptive control are considered. Practical problems, such as the lack of sufficient data in offline identification, as well as online adaptive identification, are tackled. Approximate inversion of Preisach operators has emerged as a key component of feedforward controller design for smart material systems. The authors present two algorithms for approximate inversion with experimental validation. FPGA implementation of the inversion algorithm along with experimental validation is presented as well.

The final article by JinHyoungh Oh, Bojana Drincic, and Dennis Bernstein studies the appearance of hysteresis in SISO systems that have a feedforward linear system and a nonlinear, memoryless, feedback component. The notions of rate-independent and rate-dependent hysteresis are clarified with examples. The authors show that many simple nonlinearities, such as a cubic function or a deadzone, can give rise to rate-dependent hysteresis. The notion of step convergence is developed, and is shown to be related to the input-output map for a hysteretic system. A graphical method for analyzing feedback systems with nonlinearities is presented, and numerical examples are given to illustrate the ideas.

### REFERENCES

- [1] L.D. Landau, *Phys. Z. Soviet Un.*, vol. 11, p. 26, 1937.
- [2] A. Visintin, *Differential Models of Hysteresis* (Applied Mathematical Sciences). New York: Springer-Verlag, 1994.

**Hysteresis is a nonlinear effect that arises in diverse disciplines  
ranging from physics to biology, from material science to mechanics,  
and from electronics to economics.**

[3] J.A. Ewing, "On the production of transient electric currents in iron and steel conductors by twisting them when magnetized or by magnetising them when twisted," *Proc. R. Soc. Lond.*, vol. 33, pp. 21–23, 1881.

[4] H. Kronmüller and M. Fähnle, *Micromagnetism and the Microstructure of Ferromagnetic Solids*. Cambridge, U.K.: Cambridge Univ. Press, 2003.

[5] E. Madelung, "Über magnetisierung durch schnellverlaufende ströme und die wirkungsweise des rutherford-marconischen magnetdetektors," *Ann. Phys.*, vol. 17, pp. 861–890, 1905.

[6] P. Weiss, "Hypothesis of the molecular field and ferromagnetic properties," *J. Phys.*, vol. 4, no. 6, pp. 661–690, 1907.

[7] F. Preisach, "Über die magnetische nachwirkung," *Zeitschrift für Physik*, vol. 94, pp. 277–302, 1935.

[8] W.B. Haines, "Studies in the physical properties of soil: V. the hysteresis effect in capillary properties, and the modes of moisture distribution associated therewith," *J. Agricultural Sci.*, vol. 20, pp. 97–116, 1930.

[9] A. Dixit, "Hysteresis, import penetration, and exchange rate pass-through," *The Quart. J. Econ.*, vol. CIV, pp. 205–228, 1989.

[10] R. Bouc, "Solution périodique de l'équation de la ferorésonance avec hystérésis," *C. R. Acad. Sci. Paris, Serie A*, vol. 263, pp. 497–499, 1966.

[11] M. Krasnoselskii and A. Pokrovskii, *Systems with Hysteresis*. New York: Springer-Verlag, 1989.

[12] I. Mayergoyz, *Mathematical Models of Hysteresis*. New York: Springer-Verlag, 1991.

[13] M. Brokate and J. Sprekels, *Hysteresis and Phase Transitions*. New York: Springer-Verlag, 1996.

[14] P. Krejci and V. Lovicar, "Continuity of hysteresis operators in Sobolev spaces," *Aplikace Matematiky*, vol. 35, no. 1, pp. 60–66, 1990.

[15] G. Tao and P. Kokotović, *Adaptive Control of Systems with Actuator and Sensor Nonlinearities* (Wiley Series on Adaptive and Learning Systems for Signal Processing, Communications, and Control). New York: Wiley, 1996.

[16] W. Galinaitis and R. Rogers, "Control of a hysteretic actuator using inverse hysteresis compensation," in *Mathematics and Control in Smart Structures*, vol. 3323, V. Varadhan, Ed. Bellingham, WA: SPIE, 1998, pp. 267–277.

[17] R. Iyer and M. Shirley, "Hysteresis parameter identification with limited experimental data," *IEEE Trans. Magn.*, vol. 40, no. 5, pp. 3227–3239, 2004.

[18] X. Tan and J.S. Baras, "Modeling and control of hysteresis in magnetostrictive actuators," *Automatica*, vol. 40, no. 9, pp. 1469–1480, 2004.

[19] K. Kuhnen, "Modeling, identification and compensation of complex hysteretic nonlinearities: A modified Prandtl-Ishlinskii approach," *Euro. J. Contr.*, vol. 9, pp. 407–418, 2003.

[20] K.K. Leang and S. Devasia, "Iterative feedforward compensation of hysteresis in piezo positioners," in *Proc. 42nd IEEE Conf. Decision and Control*, Dec. 2003, pp. 2626–2631.

[21] R. Smith, *Smart Material Systems: Model Development*. Philadelphia, PA: SIAM, 2005.

[22] G. Song, J. Zhao, X. Zhou, and J.A.D. Abreu-García, "Tracking control of a piezoceramic actuator with hysteresis compensation using inverse Preisach model," *IEEE/ASME Trans. Mechatron.*, vol. 10, no. 2, pp. 198–209, 2005.

### AUTHOR INFORMATION

**Xiaobo Tan** received the bachelor's and master's degrees in automatic control from Tsinghua University, Beijing, China, in 1995 and 1998, and his Ph.D. in electrical engineering from the University of Maryland at College Park, in 2002. From September 2002 to July 2004 he was a research associate with the Institute for Systems Research at the University of Maryland. In August 2004 he joined the Department of Electrical and Computer Engineering at Michigan State University as an assistant professor. His research interests include electroactive polymer sensors and actuators, modeling and control of smart materials, biomimetic robotics, bio/micro-manipulation, and collaborative control of unmanned vehicles. He was a recipient of the NSF CAREER award in 2006.

**Ram Iyer** (ram.iyer@ttu.edu) received the Ph.D. degree in electrical engineering from the University of Maryland, College Park in 1999. He is an associate professor in the Department of Mathematics and Statistics, Texas Tech University, Lubbock. He was a postdoctoral fellow during 1999–2000 at the University of Maryland, College Park, and a visiting scientist at the Control Center of Excellence, AFRL, Wright-Patterson, AFB, Ohio during 2000–2001. His interests include modeling, model identification, and control of smart materials and smart structures; numerical methods for control and observation of mechanical systems defined on Lie groups; and bio-inspired vision systems and their application to unmanned air vehicles. He was a NRC/AFOSR Summer Faculty Fellow for 2002–2004 and an ASEE/AFOSR Summer Faculty Fellow during 2005–2006. He is a member of IEEE, AIAA, and SPIE. He can be contacted at the Department of Mathematics and Statistics, Boston and Broadway, Texas Tech University, Lubbock, TX 79409-1042 USA.

