

ECE 404 - HW #8 - SOLUTION - 30 PTS

A) 1) 
$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(100 + j100) - 50}{(100 + j100) + 50} = \frac{50 + j100}{150 + j100}$$

$$= \frac{111.8 \angle 63.4^\circ}{180.3 \angle 33.7^\circ} = \boxed{620m \angle 29.7^\circ}$$

2) 
$$\beta d = \frac{2\pi}{\lambda} \frac{\lambda}{8} = \frac{1}{4}\pi = 45^\circ$$

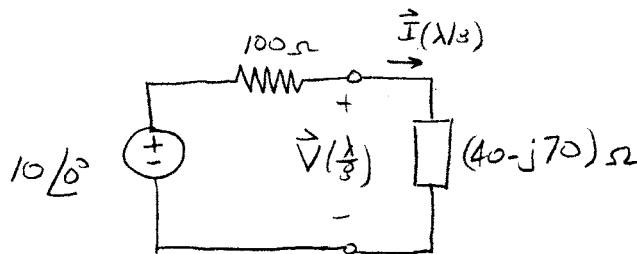
$$Z_{IN} \left(\frac{\lambda}{8}\right) = Z_0 \frac{Z_L + jZ_0 \tan(45^\circ)}{Z_0 + jZ_L \tan(45^\circ)} = 50 \frac{(100 + j100) + j50(1)}{50 + j(100 + j100)1}$$

$$= 50 \frac{100 + j150}{-50 + j100} = 50 \frac{180.3 \angle 56.3^\circ}{111.8 \angle 116.56^\circ}$$

$$= 80.64 \angle -60.26^\circ = \boxed{(40 - j70) \Omega}$$

3) 
$$V_{SWR} = \frac{1 + |\Gamma_0|}{1 - |\Gamma_0|} = \frac{1 + 0.62}{1 - 0.62} = \boxed{4.263}$$

B)



1) 
$$\vec{V} \left(\frac{\lambda}{8}\right) = 10 \angle 0^\circ \frac{40 - j70}{100 + 40 - j70} = 10 \angle 0^\circ \frac{80.64 \angle -60.26^\circ}{156.5 \angle -26.6^\circ}$$

$$= \boxed{5.153 \angle -33.66^\circ \text{ V}}$$

2) 
$$I \left(\frac{\lambda}{8}\right) = \frac{10 \angle 0^\circ}{156.5 \angle -26.6^\circ} = \boxed{63.89m \angle 26.6^\circ \text{ A}}$$

$$\begin{aligned}
3) P(\lambda/8) &= \frac{1}{2} \operatorname{Re} [\vec{V}(\lambda/8) \cdot \vec{I}^*(\lambda/8)] \\
&= \frac{1}{2} \operatorname{Re} [5.153 \angle -33.66^\circ \cdot 63.89_m \angle -26.6^\circ] \\
&= \frac{1}{2} \operatorname{Re} [0.3292 \angle -60.2^\circ] \\
&= \frac{1}{2} \operatorname{Re} [0.1636 - j0.286] = \boxed{81.8 \text{ mW}}
\end{aligned}$$

$$4) \vec{V}(0) = A_1 e^{j\beta d} (1 + T_0 e^{-j2\beta d}) \Big|_{d=0} = A_1 (1 + T_0)$$

$$\vec{V}(\lambda/8) = 5.153 \angle -33.66^\circ = A_1 e^{j\frac{\pi}{4}} [1 + (0.62 \angle 29.7^\circ) e^{-j\pi/2}]$$

$$5.153 \angle -33.66^\circ = A_1 \angle 45^\circ [1 + (0.62 \angle 29.7^\circ)(1 \angle -90^\circ)]$$

$$= A_1 \angle 45^\circ [1 + 0.62 \angle -60.3^\circ]$$

$$= A_1 \angle 45^\circ [1 + 0.307 - j0.539]$$

$$= A_1 \angle 45^\circ [1.414 \angle -22.4^\circ]$$

$$A_1 = \frac{5.153 \angle -33.66^\circ}{(1 \angle 45^\circ)(1.414 \angle -22.4^\circ)} = 3.644 \angle -56.26^\circ$$

$$\vec{V}(0) = (3.644 \angle -56.26^\circ)(1 + 0.62 \angle 29.7^\circ)$$

$$= (3.644 \angle -56.26^\circ)(1 + 0.539 + j0.307)$$

$$= (3.644 \angle -56.26^\circ)(1.569 \angle 11.28^\circ)$$

$$= \boxed{5.717 \angle -45^\circ \text{ V}}$$

$$5) \vec{I}_0 = \frac{\vec{V}(0)}{\vec{Z}_L} = \frac{5.717 \angle -45^\circ}{100 + j100} = \frac{5.717 \angle -45^\circ}{141.4 \angle 45^\circ}$$

$$= \boxed{40.4 \text{ m} \angle -90^\circ \text{ A}}$$

$$6) P(0) = \frac{1}{2} \text{Re} [\vec{V}(0) \cdot \vec{I}^*(0)]$$

$$= \frac{1}{2} \text{Re} [5.717 \angle -45^\circ \cdot 40.4 \text{ m} \angle 90^\circ]$$

$$= \frac{1}{2} \text{Re} [0.231 \angle 45^\circ]$$

$$= \frac{1}{2} \text{Re} [0.1634 + j0.1634]$$

$$= \boxed{81.7 \text{ mW}}$$