A Normative Theory of Decision-making From Multiple Stimuli

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Drift Diffusion Model and the Two Alternative Forced Choice Tasks

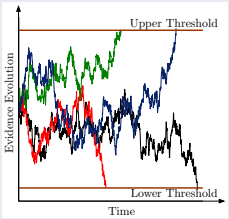
- Models human decision making in two alternative choice tasks
- Evidence evolution in a two alternative choice task is modeled by

$$dx(t) = \beta dt + dW(t), \quad x(t) = x_0$$

- Decision process at time τ is

$$\begin{cases} x(\tau) > \eta, & \text{choose alternative 1,} \\ x(\tau) < -\eta, & \text{choose alternative 2,} \\ \text{else,} & \text{collect more evidence.} \end{cases}$$

- **Threshold dictates the speed-accuracy trade-off**



- The Ornstein-Uhlenbeck process has an additional $-\lambda x\, dt$ term in the above SDE

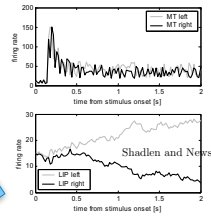
Bogacz et. al. The physics of optimal decision making: A formal analysis of performance in two-alternative forced choice tasks. Psych Rev, 2006

DDM connects several approaches to model decision-making

Wong and Wang. Jrnl Neuroscience, 2006.
Usher and McClelland. Psych Rev, 2001.

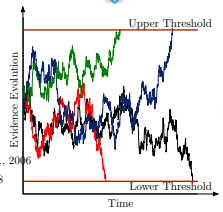
Reduced two−variable model

Linearization
&
Spectral Reduction



Shadlen and Newsome. Jrnl Neurophysiology, 2001

Neural Recordings

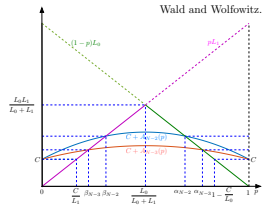


Bogacz et. al. Psych. Rev., 2006
Ratcliff. Psych. Rev., 1978

Drift-Diffusion Model

Continuum
Approximation

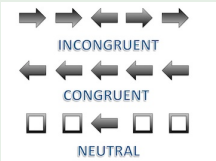
Wald and Wolfowitz. Annals of Math. Stats, 1948



Optimal Statistical Decision-Making

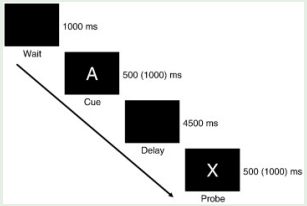
Slightly More Complex Two Alternative Choices Tasks

Eriksen-Flanker Task



- Congruent stimulus– call for the same response as the target
- Incongruent stimulus– call for the opposite response of the target
- Response rule:
  Respond per the target stimuli

AX-CPT Task



- Cue stimulus – A or B
- Probe stimulus – X or Y
- Symmetric Response Rule:
  Click left if AX or BY and right o/w
- AX condition happens 50% of the times
- Others: AY– 20%, BX– 20%, BY– 10%

A Bayesian Perspective on Multiple Stimuli Integration in AX-CPT

- For now, assume context and target appear together
- Suppose context (A or B) can be sampled independently of target (X or Y)
- Let e^c_τ and e^g_τ be the log-likelihood ratios at time τ of A v/s B and X v/s Y, resp.
- Let $\pi_{AX}, \pi_{AY}, \pi_{BX}$ and π_{BY} be the prior probabilities of four conditions

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- Then, the log-likelihood ratio of responding left and responding right is

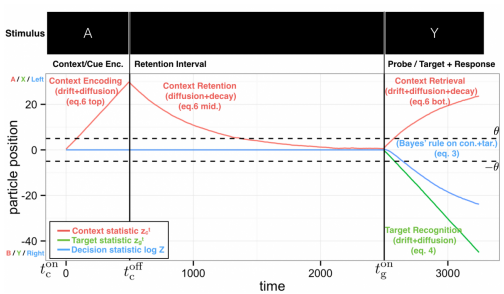
$$Z_t = \log \frac{\mathbb{P}(\text{AX or BY}|e^c_{1:t}, e^g_{1:t})}{\mathbb{P}(\text{AY or BX}|e^c_{1:t}, e^g_{1:t})} = \log \frac{\pi_{AX}\exp(z^c_t)\exp(z^g_t) + \pi_{BY}}{\pi_{AY}\exp(z^c_t) + \pi_{BY}\exp(z^g_t)},$$

where $z^g_t = \sum_{\tau=1}^t e^g_\tau$ and $z^c_t = \sum_{\tau=1}^t e^c_\tau$.

- z^g_t and z^c_t are aggregate log-likelihood ratios and converge to DDM in continuum limit

A Bayesian Perspective on Multiple Stimuli Integration in AX-CPT II

- Now, context and target may appear at different times



$0 \le t \le t_c^{\text{off}}$

$$dz^c = \bar{a}_c dt + dW$$
$$z^g(t) = z^g_0$$

$t_c^{\text{off}} \le t \le t_g^{\text{on}}$

$$dz^c = -\lambda_f z^c dt + dW$$
$$z^g(t) = z^g_0$$

$t > t_g^{\text{on}}$

$$dz^c = (-\lambda_f z^c + a_c)dt + dW$$
$$dz^g = a_g dt + dW$$

Fixed Threshold Policy: Compare Z_t with fixed thresholds $\pm\theta$.

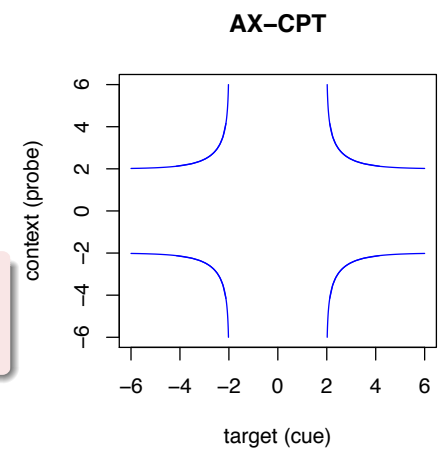
A 2D Integration Perspective on AXCPT

$t > t_g^{\text{on}}$

$$dz^c = (-\lambda_f z^c + a_c)dt + dW$$
$$dz^g = a_g dt + dW$$

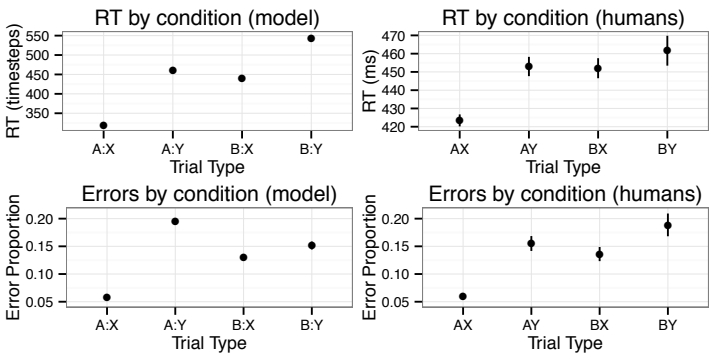
$Z_t = \pm\theta \implies$

$$\exp(\mp\theta)\big(\pi_{AX}\exp(z^c_t)\exp(z^g_t) + \pi_{BY}\big) = \pi_{AY}\exp(z^c_t) + \pi_{BY}\exp(z^g_t)$$

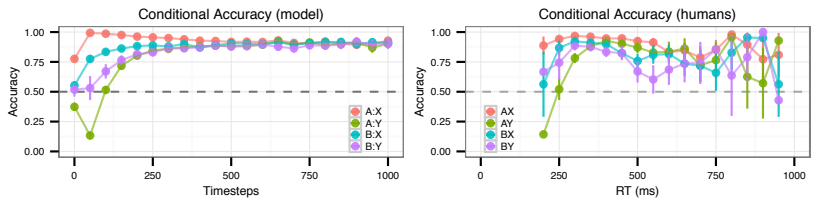


2D LCA in disguise with boundaries determined by the task and response rule

Model Predicts AXCPT Behavioral Data



Model Predicts Signature AXCPT Behavior



- A region of below-chance performance early in AY trials but not other trials
- Below chance performance in AY trials due to a high AX prior in AX-CPT
- Model predicts lower accuracy in early B* trials: possibly due to global left response bias

A Normative Account of Proactive and Reactive Control

Braver (2012): Dual Mechanisms of Control.

"Proactive" control:

- effortful maintenance of context / task rules.
- errors based on misidentifying X/Y (more AY than BX errors)

"Reactive" control:

- usage of target stimulus as cue for memory retrieval
- errors based on misremembering A/B (more BX than AY errors)

Lositsky et al. 2015

Proactive strategy with bad memory looks reactive, reactive strategy with bad perception looks proactive

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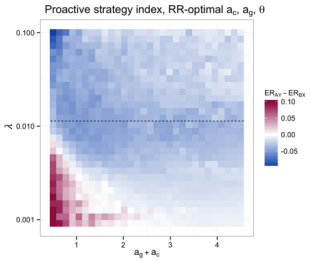


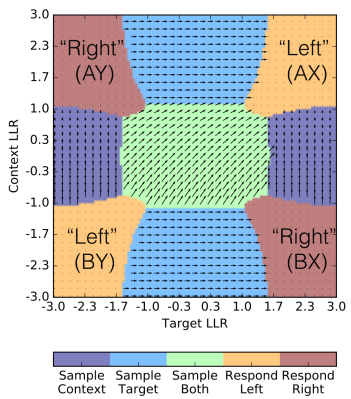
- Explore threshold θ and drift rates a_c and a_g that maximize the reward rate ($= \frac{1-\text{ER}}{\text{RT}}$) for fixed $a_c + a_g$
- Proactive strategies are observed for low λ and $a_c + a_g$

What is the optimal decision rule?

- Cost Structure:

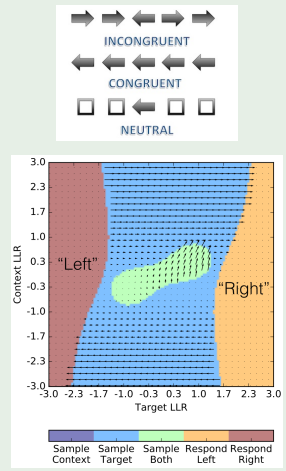
c_1,	sample only the context,
c_2,	sample only the target,
$c_1 + c_2 - \epsilon$,	sample both target and context,
ℓ_1,	click left erroneously,
ℓ_2,	click right erroneously.

- Solve the POMDP using backward induction
- Fixed threshold policies only optimal in some limiting cases

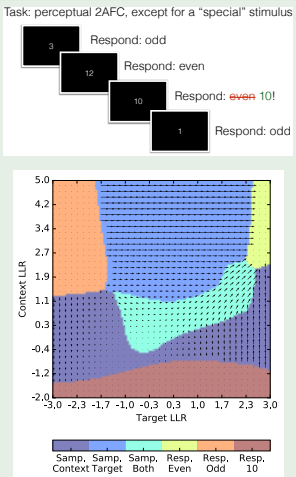


Other Behavioral Tasks

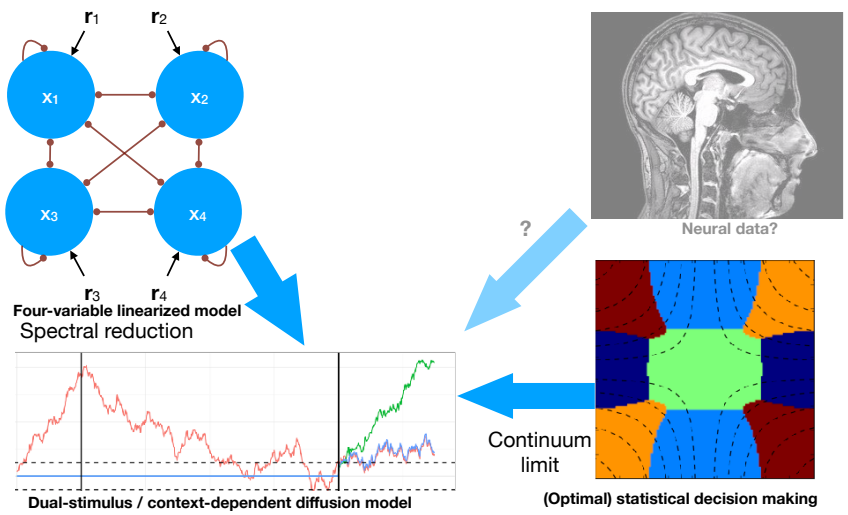
Eriksen-Flanker Task



Prospective Memory Task



CDDM as a Unifying Model for Multiple Stimuli Decision Making



Recap

- A normative theory based on Bayesian inference for two-stimuli decision-making
- Can model several TAFC tasks with contextual information including, Ericksen-Flanker, AXCPT, Prospective memory tasks
- A 2d diffusion model in which the context sets threshold for target and vice-versa
- Model captures behavioral signatures in these tasks
- Fixed-threshold policies need not be Bayes-optimal in the SPRT sense
- In some limits, the CDDM unifies the PDP model and the Bayes-optimal model

Acknowledgment and References

Mike Shvartsman

Narayanan Sundaram

Jonathan Cohen

References:

1. M. Shvartsman, V. Srivastava, and J. D. Cohen. **A Theory of Decision Making Under Dynamic Context**. *Advances in Neural Information Processing Systems 28*, Montréal, Canada, pages 2476-2484, December 2015.
2. M. Shvartsman, V. Srivastava, N. Sundaram, and J. D. Cohen. **Using Behavior to Decode Allocation of Attention in Context Dependent Decision Making**. *Proceedings of the International Conference on Cognitive Modeling*, University Park, PA, pages 65-71, 2016.
3. M. Shvartsman, V. Srivastava, and J. D. Cohen. **Exploring Fixed-Threshold and Optimal Policies in Multi-alternative Decision Making**. *Multidisciplinary Conf. on Reinforcement Learning and Decision Making*, June 2017.