

Optimization of Diffusion Encoding Gradients in Axisymmetric Diffusion Tensor Imaging Using *A Priori* Structure Information

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Introduction:

The Diffusion Tensor Imaging ([1]) formulation can be modified by assuming isotropic water diffusion in the direction that is transverse to the fiber orientation. Such a modification results in an axisymmetric DTI (ADTI) model with parameters $\{D_{\parallel}, D_{\perp}, \theta_F, \phi_F\}$: longitudinal and transverse diffusion coefficients and orientation angles. Axisymmetric diffusion models have been discussed previously [2, 3]. In this work, a D-optimality based robust technique [4] has been implemented to select the diffusion encoding gradient directions for ADTI model to minimize the uncertainty in parameter estimation. The robust optimization makes use of *a priori* knowledge of the structures to be imaged. In spinal cord or skeletal muscle, fiber orientations may be described as belonging to a cone of possible directions, with a mean orientation and a half-aperture α . In such tissues, fibers are oriented in relatively known directions and the cone of distribution of fiber directions is applicable to the ADTI model. *A priori* information of the structure can be obtained from, for example, a preliminary DTI scan on the subject. In this work, the technique was evaluated to image a spinal cord.

Method:

From statistical estimation theory [5], the Cramer-Rao Lower Bound (CRLB) on the variance of estimation parameters for normally distributed observations is given by $\Sigma_{CR} = \sigma^2 (X^T X)^{-1}$, where σ^2 is the noise variance assuming zero-mean Gaussian noise and X is the model sensitivity matrix which depends on the experimental parameters such as diffusion encoding gradients and model parameters. To reduce the uncertainty in parameter estimation, a measure of uncertainty, such as the hypervolume of uncertainty which can be calculated as $\det \Sigma_{CR} = \sigma^{2M} / \det (X^T X)$, is minimized, where M is the number of parameters to be estimated (for ADTI, $M = 4$). To do so, a set of optimal gradient directions is chosen which minimizes $\det \Sigma_{CR}$ (σ is assumed to be fixed, so $\det (X^T X)$ is the quantity that is actually maximized). The optimization is made robust by considering a range of possible angular deviations from the mean fiber direction in the fiber distribution and applying a "minimax" algorithm to optimize for the worst case. This ensures that the optimized sampling scheme provides a better performance for a set range of fiber directions and avoids any bias. *A priori* knowledge of the potential ranges of fiber orientations and diffusion coefficient values is included in the optimization procedure.

Experiment:

The application of this technique was evaluated for spinal cord imaging. Based on the structure of the spinal cord (Fig. 1(a)), we estimated that mean $\theta_F = 19.5^\circ$, $\phi_F = -10^\circ$ and the maximum angular deviation $\alpha = 35^\circ$ could be chosen. The mean values of $D_{\parallel} = 1.367 \times 10^{-3} \text{ mm}^2 \text{ s}^{-1}$, $D_{\perp} = 0.6227 \times 10^{-3} \text{ mm}^2 \text{ s}^{-1}$ and $\sigma = 20\%$ were used based on a preliminary DTI scan. Utilizing *a priori* information of the structure, a 30-direction diffusion encoding gradient optimized scheme was developed and the expected performance was computed. Fig. 1(b) shows the variation of expected hypervolume of uncertainty ($\det \Sigma_{CR}$) with angular deviation (α) from a mean fiber direction. For comparing performance, the standard diffusion encoding gradient scheme (MF30 [7]) was used. The comparison shows that our optimized scheme is expected to perform better in terms of $\det \Sigma_{CR}$. This was validated by performing ADTI experiments on a scanner. The T_2 and diffusion-weighted images were acquired on a healthy 27 year old volunteer with a spin echo EPI sequence on a 3T GE Signa EXCITE scanner (GE Healthcare, Waukesha, WI) with an 8-channel head coil with the following parameters: 30 contiguous 3-mm axial slices, TR = 8000 ms, TE = 76 ms, matrix size = 128x128, FOV = 22 cm x 22 cm, number of excitation = 2, parallel imaging acceleration factor = 2, $b = 1000 \text{ s/mm}^2$, and scan time = 8 min 32 s. Five complete ADTI scans were acquired for each scheme (standard and optimized). From each scheme, a bootstrap method was used to produce 1000 additional ADTI scan data such that the covariance matrix of the parameter estimation could be estimated. Finally, means and standard deviations were computed for the model parameters (D_{\parallel} , D_{\perp} , θ_F , ϕ_F) at selected voxels in the spinal cord section and the results for the two diffusion encoding gradient schemes were compared.

Results and Discussion:

Table shows the results for two sample voxels from the middle of the axial section of the cervical spinal cord (around level C1/C2). The uncertainty in parameter estimation is quantified by $\det \Sigma$ (Σ is the computed covariance matrix) and has been improved by our optimized sequence. The standard deviation (SD) of fiber direction (θ_F, ϕ_F) also show improvement. The last two columns (r) show the ratios of standard deviations estimation from the data and prediction based on the optimized design. The above results indicate that the robust optimization of the diffusion encoding gradient schemes based on *a priori* structure information significantly reduces the uncertainty in the parameter estimates for ADTI.

References: [1] P. J. Basser, J. Mattiello, D. Le Bihan, *Biophys J* 66, 259–267, 1994; [2] Y. Assaf, R. Z. Freidlin, G. K. Rohde, and P. J. Basser. *Magn. Reson. Med.*, 52:965-978, 2004.; [3] L. G. Raguin et al. . In IFMBE Proc. 3rd European Medical and Biological Engineering Conference, vol. 11, 2005. [4] L. G. Raguin, S. Majumdar, and S. S. Udpa. *Int. J. Appl. Electromagn. Mech.*, 2008. in press. [5] S. M. Kay. *Fundamentals of Statistical Signal Processing: Estimation Theory*. Prentice Hall, Ney Jersey, USA, 1993. pp. 47-49. [6] S. Majumdar, S. Udpa, L. G. Raguin. In Proc 6th ICIPE, Dourdan, France, June 15–19, 2008; [7] K. M. Hasan, D. L. Parker, and A. L. Alexander. *J. Magn. Reson. Imag.*, 13:769-780, 2001.

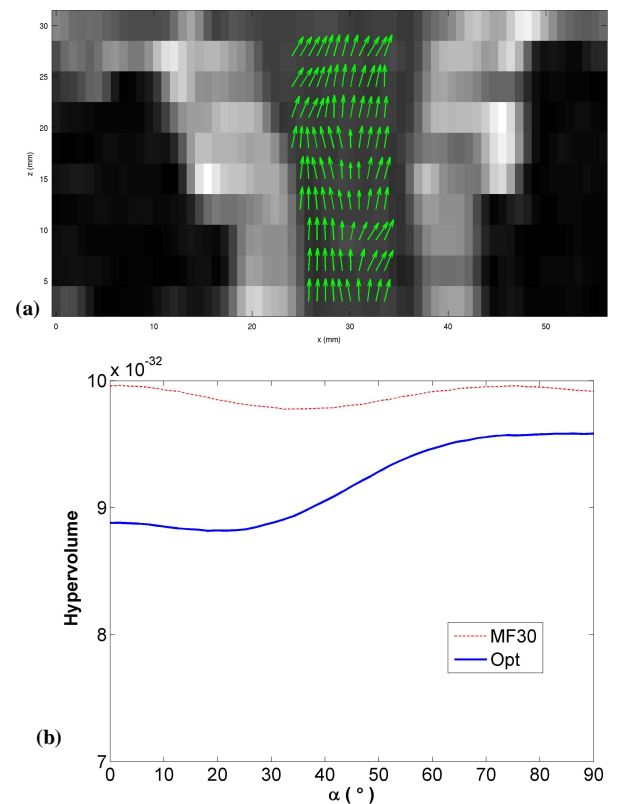


Fig.1 (a) Spinal cord section showing fiber orientations, (b) Variation of expected uncertainty ($\det \Sigma_{CR}$) with respect to angular deviation (α) from mean fiber direction. Opt = optimized scheme.

		MF30	Opt	r(Est.)	r(Pred.)
Voxel 1	$\det \Sigma$ ($\times 10^{-32}$)	5.152	2.613	0.507	0.535
	SD θ_F ($^\circ$)	1.891	1.532	0.810	0.923
	SD ϕ_F ($^\circ$)	1.580	1.374	0.870	0.819
Voxel 2	$\det \Sigma$ ($\times 10^{-32}$)	9.360	5.627	0.601	0.542
	SD θ_F ($^\circ$)	2.199	1.479	0.673	0.921
	SD ϕ_F ($^\circ$)	1.628	1.355	0.832	0.807