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MODULAR MODELING OF ENGINEERING SYSTEMS USING FIXED INPUT-OUTPUT STRUCTURE

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ABSTRACT

Computer modeling is common in the design and development of complex engineering systems. A system model is built up by connecting the inputs and outputs of several subsystem models. This process requires flexible modeling tools. Models with arbitrary input-output structure have this flexibility but must have their internal equations reformulated to agree with the inputs and outputs used. The flexibility achieved with arbitrary input-output structure occurs at the cost of globally reformulating the equations of each subsystem and component model with every change. Each model equation formulation requires performance verification because every formulation does not have the same guaranteed performance. This can be particularly cumbersome in large models. A fixed input-output structure allows elements to be used without modification of their internal equations. Modular simulation models have the property that their elements have fixed input-output structure while maintaining the flexible assembly required in today s large complex modeling environments. Structural and automotive examples are given.

INTRODUCTION

Increasingly large and complex computer models are becoming standard practice in design and development of engineering systems. Models should have sufficient complexity to predict actual behavior of complex engineering systems (Ferris etal, 1994). Chrysler used a large computer model to completely design and analyze the geometry of their 1998 Chrysler Concorde and 1998 Dodge Intrepid. The Chrysler large car models were limited to mechanical geometry studies and still contained representations of over 5500 interconnected physical subsystems (Computers In Engineering: Chrysler designs paperless cars , 1998). Chrysler engineers resolved design issues by developing this computer model instead of physical prototypes reducing the cycle time from 39 to 31 months saving the company more than $75 million (Jost, 1998). Efficient design, development, and refinement of large computer models of complex engineering systems are critical to engineers and their companies. A systematic approach to design and development is the most efficient (Shigley and Mischke, 1989).

Several existing modeling methodologies use systematic techniques for the design and development of engineering system computer models. Kinematic and dynamic models of mechanical systems are developed using the systematic method of generalized Cartesian coordinates (Haug, 1989 and Nikravesh, 1988). Electrical system models are developed using the systematic method of applying Kirchhoff Laws from a network topology (Chua and Lin, 1975, Vlach and Singhal, 1983, and Calahan, 1972). These methods are useful but are limited to their respective energy domains.

A systematic method that includes different energy domains is Finite Element Analysis (FEA). FEA methods systematically construct grids of similar elements to model engineering components and systems in mechanical and thermal energy domains (Zienkiewiez, 1977). These methods are useful for systematic generation of model equations and models grow to be quite large and complex. However, FEA models are not generated with an input-output structure that allows them to be easily interconnected. Therefore, assembling several of these independent models requires reformulating an entirely new model or using special purpose software like PDESolve (PDESolve, BEAM Technologies) to connect them, which is both cumbersome and expensive.

Another systematic inter-energy domain modeling method is bond graphs. Bond graphs have a systematic approach of using graphical multi-port elements and junctions to develop component and system models in mechanical, electrical, hydraulic, and thermal energy domains (Karnopp etal, 1990). System model equations are systematically generated with some hierarchical design but the assembly reformulation issue still exists due to arbitrary input-output definitions (Karnopp etal, 1990). Recent research has further enhanced the hierarchical design of bond graph models such that some reformulation can be avoided (Hales, 1995). Reformulation of model equations is the practice that prevents efficient development of large, complex, models.

A new approach to systematic modeling across multiple energy domains provides a modular fixed input-output structure modeling method. Modular in the sense that the physics that describes each subsystem model remains the same whether the subsystems are separate or assembled into a system (Hogan, 1987). Fixed input-output structure in the sense that the inputs and outputs are standardized so the internal equations of mathematical subsystem models of engineering systems have the same modularity as the engineering system. Modular modeling with fixed input-output structure is a power based physically intuitive top-down methodology, which allows development of large simulation models without model internal equation reformulation. Modular modeling makes the large model design, development, and refinement process systematic, functional, and physically intuitive.

MODULAR MODELING: THE IMPACT OF CAUSALITY

Modular modeling is defined by a fixed input-output structure. This strict input-output approach standardizes the internal equation formulation of multi degree of freedom (DOF) subsystem models. The objective is mathematical models of engineering systems with equations that have the same modularity as the engineering system. This method is not intended for single DOF model elements or single component modeling. It is intended for large system models with many multi DOF subsystems. Modular modeling is particularly advantageous for large system models because the multi DOF subsystem models fixed formulation simplifies large model design, development, and refinement.

Large complex models of engineering systems contain a large number of multi DOF subsystem models (5500 in the Chrysler large car models). Each physical multi DOF subsystem model has one or more connections through which it is attached to other subsystem models. For example, the transmission subsystem model of a pick-up truck has connections to the engine model, the frame model, the driver controls model, and the drive shaft model. Each physical subsystem model connection has an input-output definition, conveys input-output variables to interconnected subsystem models, and hence defines the internal formulations of the subsystem models. Controlling the number of internal formulations of interconnected multi DOF subsystem models is key for design, development, and refinement of large models.

Subsystem model internal formulations result from input-output connections defined here in two general forms: signal-type and natural-type.

Signal-type model connections contain single variables and can only be defined as input or an output. For example, the driver controls-to-transmission model connection is a signal-type model input connection. The only reasonable model is a selected-gear control signal input to the transmission model. Indicator lights are an example of a signal-type model connection only reasonably defined as a model output. Once defined, signal-type connections have only one possible definition and their effects on the model s internal equations is fixed.

Natural-type model connections may have many variables and many valid input-output definitions resulting in many useful subsystem model equation formulations. For example, a transmission-to-drive shaft model connection may have mechanical rotation and mechanical translation variables. Each model must have an input-output definition and hence internal formulation that provides the appropriate input and output variables at the connection. An output of one subsystem must be an input to the other. There are many possible useful input-output definitions, which could be used to assemble these elements. Each definition requires a different, useful, well-posed internal formulation of the connected multi DOF subsystems internal equations.

Power-based models represent natural-type physical model connections with power ports. A power port is a place where physical systems are connected and exchange power. Power is commonly modeled as the product of two variables such as force and velocity, pressure and volume flow rate, or voltage and current. The variable pairs are often referred to as the effort-flow variable pair, e_i-f_i, at each power port (Karnopp etal, 1990). Power-based simulations pair these power variables at each port. Each port has causality, an input-output definition (Karnopp etal, 1990). Causality manages connected power port s physical cause and effect between the variable pairs by defining one variable as a port input and the other variable as a port output. Connected ports must have reciprocal causality (Rosenberg and Karnopp, 1983). In other words, the input of one port is the output of its connected port, and vice versa. Each port affects the model s internal formulation through its input-output causality definition.

The Impact of Causality: Model Internal Formulation

Causality has a great impact on the number of possible different, useful, internal equation formulations of a computer model. Power port causality allows two possible, different, reciprocal, input-output definitions. Let the two variables, e_i and f_I; be the input-output variables at the multi DOF element model port *i*. There are two possible different input-output definitions at a multi DOF element port *i* (Fig. 1). For example, the mechanical shaft of a pump can be modeled with a torque input and an angular velocity output or with the reciprocal causality. Useful pump models can be formulated with either causality.

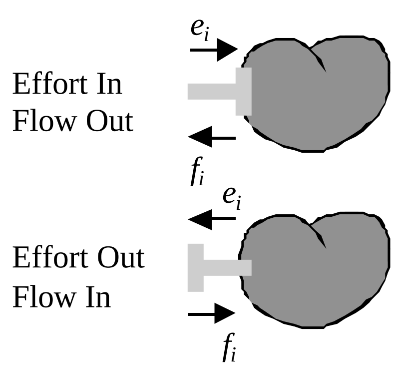

Figure 1: Two Possible Different Input-Output Causal Definitions At A Multi DOF Element Port *i*

A multi DOF element model with *n* power ports with arbitrary causality permitting 2 possible reciprocal input-output definitions at each port will have N_a possible different internal equation formulations (Fig. 2).

$$N_a = 2^n \quad (1)$$

Typical multi DOF subsystem models have 1 or more power ports. For example, a simple hydraulic pump model may have 3 power ports (1 for the mechanical shaft, and 2 for the hydraulic high and low pressure connections). This leads to

$$2^3 = 8 \tag{2}$$

possible specific causal formulations.

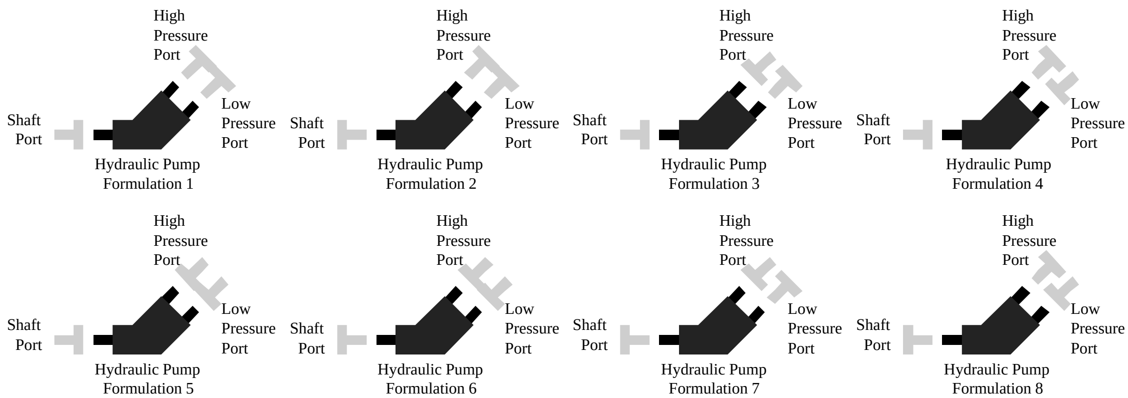

Figure 2: 2 Possible Different Input-Output Causal Definitions At *n* Power Ports Yields 2^n Possible Different Useful Multi DOF Element Formulations Three Port Hydraulic Pump Example

2^n possible useful different element formulations requires 2^n different model verifications. Verifying 2^n possible correct element formulations is a staggering task. Consider interconnecting 5500 multi DOF model elements (e.g. 1998 Chrysler large car models) with reciprocal causality. In the most simplistic interconnected form, 2 power ports per element, there are 5500×2 = 11,000 power ports. There is a 50% probability of a perfect input-output causal match but the number of possible useful system model formulations to verify is an impossible task.

$$50\% \text{ of } 2^{11000} = 1.06885 \times 10^{3310} \tag{3}$$

For this reason users of models with arbitrary causality have made the choice to verify at the component level and reformulate the system equations after every change.

Fixing power port causality at every multi DOF element port selects a single element internal equation formulation (Fig. 3). This standardized functionality simplifies the design, development, and refinement of large models. There is only one system configuration, which enables verification at the subsystem level. The key concept of modular modeling is a physically intuitive fixed causality at every port of every modular modeling element.

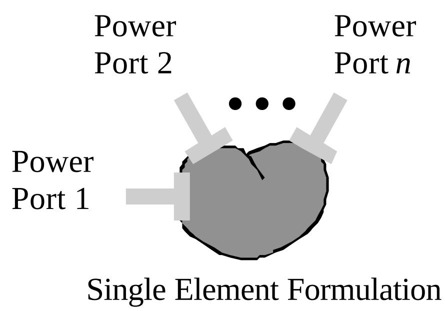

Figure 3: 1 Possible Input-Output Causal Definition At *n* Power Ports Yields A Single Multi DOF Element Formulation

Modular Modeling Elements

Modular modeling elements are multi-port multi DOF subsystem models with a fixed causality at every power port and a single fixed formulation. The element internal equation formulation fits the fixed causality and the number of ports. Since the causality is fixed, the formulation of element s equations does not change. Once formulated modelers can gain validation experience with each specific element. The fixed causality at every port leads to a standardized element functional form.

$$\underset{\sim\, n\times 1}{y} = element(\underset{\sim\, n\times 1}{u}) \tag{4}$$

n is the number of element power ports, $\underset{\sim n\times 1}{u}$ is a vector of inputs and $\underset{\sim n\times 1}{y}$ is a vector of outputs. The multi port multi DOF element calculates a vector of outputs, $\underset{\sim n\times 1}{y}$, given the vector of inputs, $\underset{\sim n\times 1}{u}$, and some internal element parameters. This functional form can be generated for most engineering elements with any number of power ports.

Modular modeling standardizes the choice of causality for every port per energy domain to realize the objective of an internal equation formulation with the same modularity as physical systems. The choice of the modular modeling element fixed port causality is motivated by physical measurements. Ideal physical measurements occur at a natural power port with a specific physical location and zero power flow such that there is no effect on the response of the system. In other words, physical measurements have a specific physical location, an externally sensed output at that location, and zero input at that location to zero the power flow. This measurement perspective defines the fixed port causality of modular modeling elements. The port output is the variable related to the externally sensed physical quantity. The port input is the variable related to the internal physical quantity typically assumed zero to attain zero power flow.

The measurement perspective modular modeling element fixed port causality for engineering systems across multiple energy domains are shown in Table 1. Externally sensed physical quantities are electrical potential, curvilinear mechanical motion, angular mechanical motion, hydraulic pressure, acoustic sound pressure, and temperature. Internal physical quantities typically assumed to be zero are electrical current, mechanical force, mechanical torque, hydraulic volume flow rate, acoustic volume velocity, and thermal heat flux. The resulting measurement perspective modular modeling element fixed port causality of electrical, mechanical translation, mechanical rotation, hydraulic, acoustic, and heat transfer systems are current input-potential output, force input-velocity output, torque input-angular velocity output, flow input-pressure output, flow input-pressure output, and heat flux input-temperature output respectively.

ENGINEERING SYSTEM	MEASUREMENT PERSPECTIVE MODULAR MODELING ELEMENT FIXED PORT CAUSALITY	
Electrical	Current Input Potential Output	u = Current, y = Potential
Mechanical Translation	Force Input Velocity Output	u = Force, y = Velocity
Mechanical Rotation	Torque Input Angular Velocity Output	u = Torque , y = Angular Velocity
Hydraulic	Volume Flow Rate Input Pressure Output	u = Volume Flow Rate, y = Pressure
Acoustic	Volume Velocity Input Sound Pressure Output	u = Volume Velocity, y = Sound Pressure
Heat Transfer	Heat Flux Input Temperature Output	u = Heat Flux, y = Temperature

Table 1: Measurement Perspective Modular Modeling Element Fixed Port Causality Across Multiple Energy Domains

Measurement perspective causality is equivalent to implementing nonessential or natural boundary conditions at every port (Meirovitch, 1967). This ensures an internal formulation with a mathematical eigen-structure that does not change whether the model is separate or assembled into a system model. Essential boundary conditions change the model s differential operators and impose specific geometric constraints. Modular modeling implements essential boundary conditions with modeling elements. For example, a mechanical fixed-point element would output a zero velocity regardless of the force input. Measurement perspective fixed port causality enables modular modeling elements with physical system modularity

A benefit of measurement perspective fixed port causality is the flexibility to maintain any number of physically open power ports at discretionary physical locations without affecting the element internal formulation. A physically open power port *i* is physically disconnected with zero input.

$$u_i = 0 \tag{5}$$

The input, u_i, is zero to achieve no power flow but the output, y_i, is open to be defined by the element. For example, a modular (multi DOF) mechanical beam model element whose ports are defined can maintain any number of open zero force-input ports without changing the model s mathematical formulation. A modular modeling beam element will have the same internal formulation and the same performance regardless of the modeling task. The response of the system can be measured at any open port with the power port s output variable. An element formulated from velocity input-force output fixed causality does not have this flexibility. This flexibility enables the single formulation of modular modeling elements to maintain a large number of power ports with no reformulation or revalidation cost.

The modular modeling elements graphical notation represents a multi port multi DOF physical subsystem model with a rectangle (Fig. 4). The bold lines represent the *n* power ports with fixed measurement perspective causality (Table 1). Arrows on the port lines define the standardized direction of positive power for modular modeling elements, when u_1 and y_1 are both positive, power flows into port 1. Port power orientation has a similar 2^n effect on model formulation but does not affect the eigen-structure of the internal formulation. The fixed causality input variable, u_i, is always shown on the top or to the left of the port. The fixed causality output variable, y_i, is always shown on the bottom or to the right of the port.

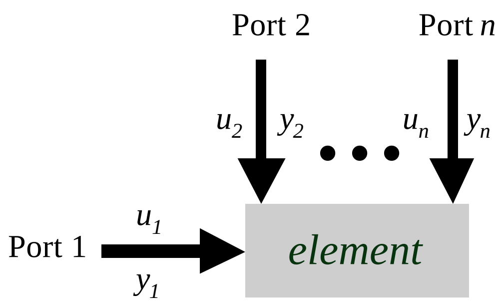

Figure 4: Modular Modeling Element Graphical Notation

Connector for Element Interconnections

Modular modeling requires a connector to join incompatible modular modeling element power ports. The connector provides the compatible port causality. Modular modeling connectors provide the proper physical connection constraints for connecting the ports of engineering subsystems. The connector is not a model of a physical connection subsystem. It is a power transmission mechanism that enforces constraints between subsystems.

The connector graphical notation represents the connector with a circle (Fig. 5). The bold lines represent the connected modular modeling element ports, *i* and *j*. Arrows on the port lines define the standardized direction of positive power for connectors, when u_i and y_i are both positive, power flows out of port i. The input variable, u, is always shown on the top or to the left of the port. The output variable, y, is always shown on the bottom or to the right of the port.

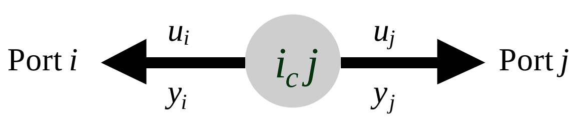

Figure 5: Modular Modeling Connector Graphical Notation

Modular modeling connectors provide a power constraint. Power is conserved across modular connectors because modular connectors are power transfer mechanisms. The power at the connected modular modeling element ports sum to zero.

$$\sum_{k=1,2} P_k = 0 \tag{6}$$

Recall that the product of port variables, u_i and y_i, is power.

$$P_i = u_i y_i \tag{7}$$

An additional constraint is required for the connected modular modeling element ports. Connectors are defined to constrain connected modular modeling element ports outputs to be the same implementing connections of elements.

$$y_i = y_j \tag{8}$$

From (6) - (8) the inputs at the connected ports must be equal and opposite.

$$u_i = -u_j \tag{9}$$

The functional definition of the connector is a 2-port power constraint with port causality compatible with modular modeling element port causality (4). In modular modeling, (8) and (9) are the defining equations for all modular connectors.

There are two important characteristics of modular modeling elements and connectors to aid in the assembly of a modular modeling system models. First, properly connecting any number of power ports of modular modeling elements with the 2-port connector requires a junction structure inside modular modeling elements for each port. The internal junction structure has the affect of summing port inputs and constraining the port outputs to have the same output. This is consistent with the measurement perspective that defines the fixed modular modeling causality. For example, measuring a voltage at a point on an electrical circuit will have one voltage but there may be several currents input to that point. Second, the explicit difference between modular modeling elements and connectors is that modeler defined equations are in modular modeling elements. Modular connector equations are fixed (8) and (9). All modeling is done in modular modeling elements, none in modular connectors. Modeling a physical connection requires modular modeling elements.

Structural models of 3 bars connected with a shear pin will provide a demonstration (Fig. 6). The simplest model (Fig. 6 b) neglects deflection of the pin. In this simple case, no pin deflection model is required and the bars are connected with displacement and force constraints. When the pin deflection is large (Fig. 6 c), a model for the physical deflection of the pin is required and a pin modeling element is used.

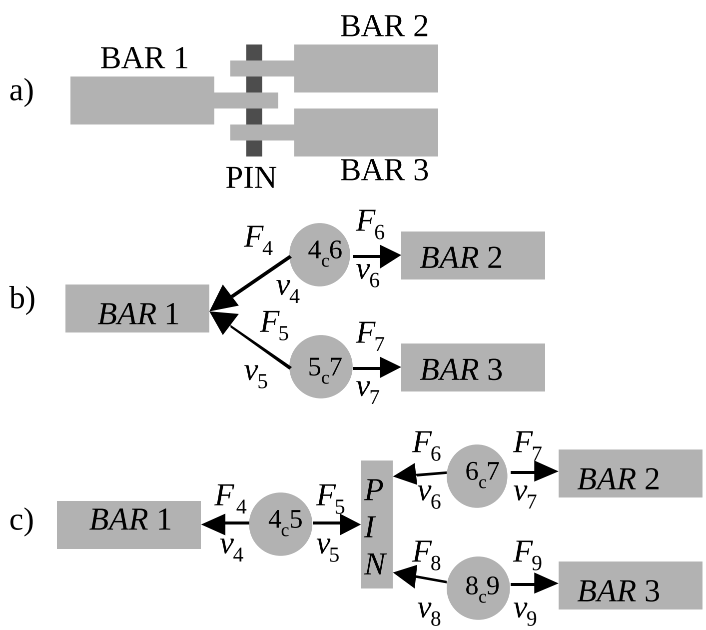

Figure 6: Structural Models Of 3 Bars Connected With A Shear Pin: a) Physical System b) Negligible Pin Deflection c) Large Pin Deflection

The first rigid pin modular model is a demonstration of the choice to not model the pin. The pin is ignored and 2 modular connectors join the 3 bars. The modeler defines the multi-port multi-DOF modular modeling elements *BAR* 1, *BAR* 2, and *BAR* 3. The modular modeling element *BAR* 1 has a port 45 with two input-output pairs 4 and 5 where power is transferred from *BAR* 2 and *BAR*

3 respectively. Each modular connector is associated with a power transfer mechanism between subsystem models. More than one power transfer mechanism can occur at a single port of a model element because of the internal junction structure. Port 45 power pair variables 4 and 5 are input and output from the same apparent geometric point on *BAR* 1. The internal junction structure has the affect of summing the 4 and 5 inputs and constraining the 4 and 5 to have the same output at that port. The right end of *BAR* 1 has the input force F_{45} and the output velocity v_{45}.

$$\begin{aligned} F_{45} &= F_4 + F_5 \\ v_{45} &= v_4 = v_5 \end{aligned} \tag{10}$$

Modular connectors 4_c6 and 5_c7 are the power transfer mechanisms that join port 45 of *BAR* 1 to port 6 of *BAR* 2 and port 7 of *BAR* 3 respectively. The connectors enforce the fixed power constraints (8) and (9) to the power transfer such that the respective outputs are the same and the respective inputs are equal and opposite. Two 2-port modular connectors properly constrain the power transfer to join 3 modular modeling element ports at the apparent same geometric point. This can be extended to any number of connected modular modeling element ports. Multiple ports at one-point increases the number of ports in the model but modular modeling elements can maintain any number of open ports with no internal formulation change.

The second flexible pin modular model is a demonstration of the choice to model the pin. The modeler defines the multi-port multi-DOF modular modeling elements *PIN*, *BAR* 1, *BAR* 2 and *BAR* 3. *PIN* has three power ports 5, 6, and 8 at the connection or power transfer points with *BAR* 1, *BAR* 2 and *BAR* 3 respectively. Modular connectors 4_c5, 6_c7, and 8_c9 are the power transfer mechanisms that join the power ports 4, 7, and 9 of *BAR* 1, *BAR* 2 and *BAR* 3 to the *PIN* power ports 5, 6, and 8 respectively. The connectors facilitate the power transfer between power ports implementing (8) and (9). For example, connector 4_c5 enforces power conservation constraints on the power transfer between power ports 4 and 5 such that v_4 and v_5 are equal and F_4 and F_5 are equal and opposite. Connectors 6_c7 and 8_c9 facilitate power transfer on their respective ports enforcing the same constraints.

An automotive model of a conceptual rear-wheel drive power train (Fig. 7) is another modular modeling example. The physical system (Fig. 7a) consists of an engine, a transmission, three mounts, a flex plate, and a frame. The engine has physical connections to the transmission through a flex plate and to the frame through mounts on the either side of the engine. The transmission has physical connections to the engine through a flex plate and to the frame through a transmission mount. The frame has physical connections to the engine and the transmission through the mounts. The modular model of the drive train (Fig. 7 b) shows the modeling choice not to model the flex plate deflection. The deflection of the flex plate is small relative to the deflection of three mounts. The rigid flex plate is not considered an element, so the engine and transmission transfer power to one another through modular connectors. The flexible mounts require modeler-defined equations to describe their deflections and are considered modular modeling elements. Modular connectors are power transfer mechanisms composed of standard constraints to conserve power. This is quite different from multi DOF modular subsystem elements, which implement all modeling analysis.

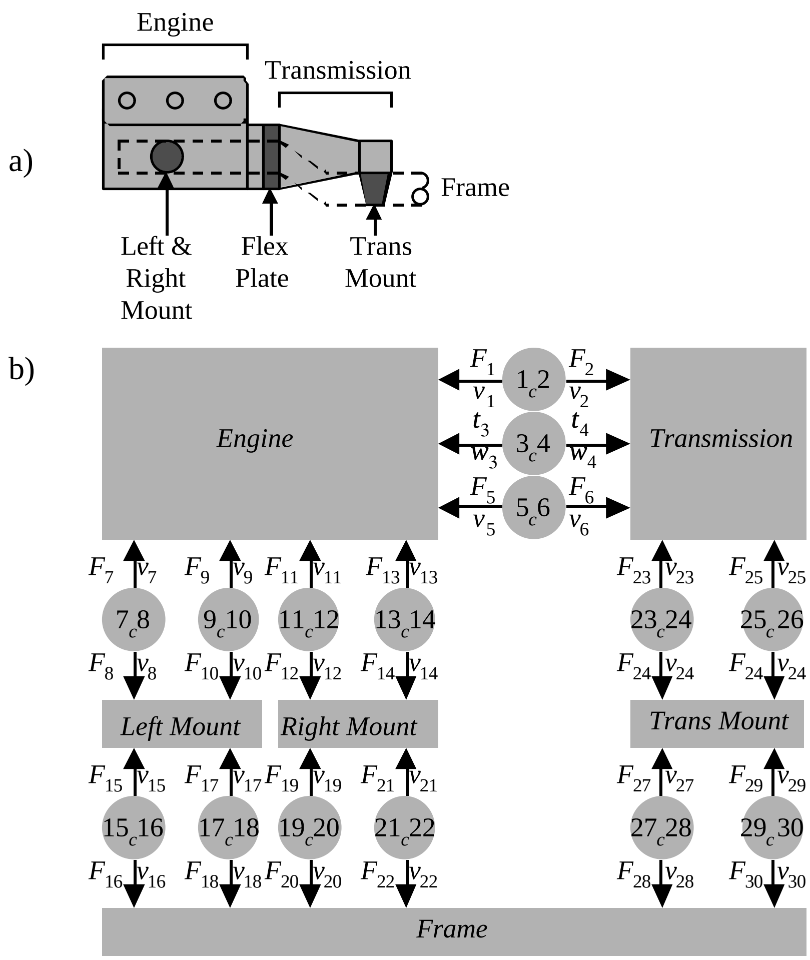

Figure 7: A Conceptual Rear-Wheel Drive Power Train Model: a) Physical System b) Modular Model

The modular connector can not implement the constraint (8) and (9) on ports of different energy domains. For example, from Table 1 and constraint (8), an electrical potential output and a mechanical velocity output can not be equal. The solution is to define an appropriate modular transducer model element with fixed measurement perspective port causality, then use modular connectors to join the modular transducer element ports and the element ports in different energy domains. Modeler-defined multi-port modular transducer elements have the same form and function as any modular modeling elements and are simply another element in the modular model.

The standardized port causality and sign conventions of modular modeling give modular connectors the exact same appearance (Fig. 7). The modular connector notation (Fig. 5) can be replaced with a single line. This simplified modular modeling notation for the conceptual power train example has a traditional block diagram appearance (Fig. 8). This simpler notation aids only in graphically communicating the model. The power-based fixed measurement perspective port causality and sign convention input-output structure implicit in every port line is critically important. For example, implicit in the port line connecting the *Engine* and *Transmission* modeling elements are two power variables $F_{12}^{\ c}$ and $v_{12}^{\ c}$ following the input-output causality in Table 1 constrained by (8) and (9).

$$Input \;\Rightarrow \begin{cases} F_1 = F_{12}^{\ c} \\ F_2 = -F_{12}^{\ c} \end{cases} \qquad Output \;\Rightarrow \{ \; v_1 = v_2 = v_{12}^{\ c} \qquad (11)$$

The dogmatic input-output structure of modeling elements is the key concept of modular modeling.

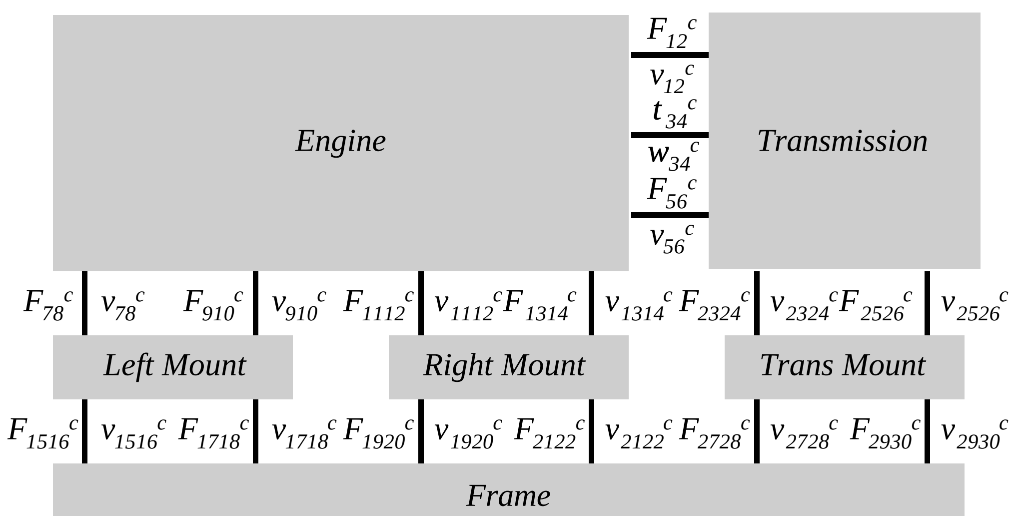

Figure 8: Simplified Modular Modeling Notation of the Conceptual Power Train Model in Figure 7

CONCLUSIONS

We have successfully developed mechanisms for modular modeling of engineering systems with fixed input-output structure. Modular modeling elements with a power-based fixed input-output structure have fixed power port input-output causality, fixed sign convention, and a fixed internal equation formulation. Modular modeling connectors join modular element ports with fixed power constraints. The standardized internal formulation of modular modeling elements enables top-down modeling and enhances model verification via modularity. Because a modular modeling element s formulation does not change, modular modeling yields more easily verified models.

Unverified models are of no use in today s industrial environment. A model with n power ports has 2^n possible formulations. The performance of all 2^n formulations simply cannot be simultaneously verified. Reducing the number of verifications from 2^n different verifications to a single verification is an enhancement in modeling technology. Modular modeling elements with a known physically validated performance are an asset to modelers of engineering systems.

Model complexity increases with the modular modeling method because of the connector requirement. Modular connectors have a fixed definition, which is compatible with the modular modeling element s fixed functional definition. The compatible fixed functional definitions enable a systematic assembly method for modular models with a fixed mathematical formulation and a fixed computational sparseness. A modular modeling assembly method and its computational attributes are the topic of a future paper.

The measurement perspective fixed port causality used in modular modeling has the ability to avoid using energic junctions to maintain modularity upon assembly. Hogan concluded that use of energic junctions guaranteed modularity at the cost of stiff system equations with widely spread eigenvalues. Modular modeling can assemble modular modeling elements with fixed measurement perspective causality and maintain modularity with the simple power constraint of the modular modeling connector. Assembly and solution of linear and nonlinear modular models using the connector are topics of future papers

Modular modeling reduces the verification task of large model design, development, and refinement by standardizing the functional form of all multi degree of freedom modeling elements. The fixed power-based measurement perspective port causality results in a standardized multi degree of freedom modular modeling element with a single mathematical model formulation. This formulation has the flexibility to support any number of physically disconnected ports without reformulation. Assembling incompatible modular modeling elements requires a causally compatible 2-port modular connector that facilitates a standardized constrained power transfer between modular modeling element ports. The separate modular modeling elements and connectors with explicitly different functions enable subsystem level modeling with no reformulation. Fixed formulation enhances model verification. Modular modeling has the flexibility to model any engineering system across multiple energy domains with the benefits of a fixed input-output structure.

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