Fundamentals of Metal Forming

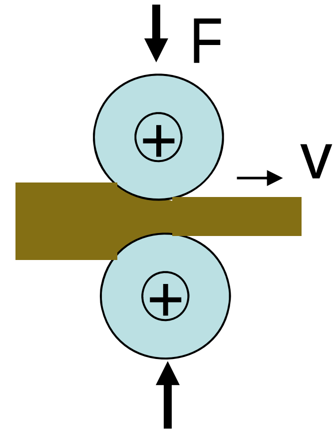
1. Overview
2. Material Behavior
3. Temperature Effects
4. Strain Rate Effect
5. Friction and Lubrication

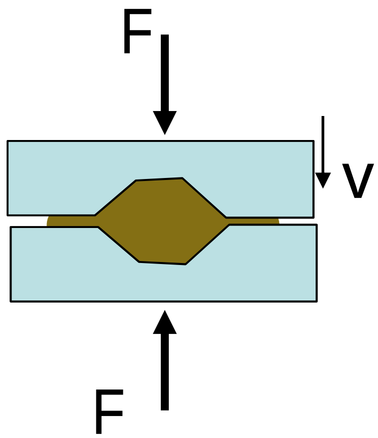
Introduction

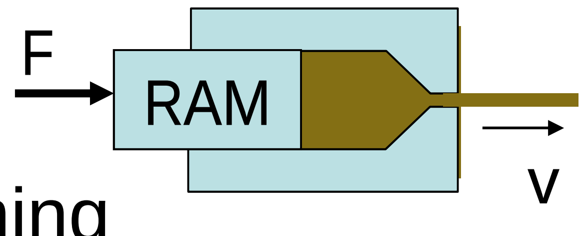
- Metal Forming uses plastic deformation to change the shape of metal workpieces
  - Materials (metals) – plastic deformation
  - External loads – Typically compressive
    - Sometimes **Stretch** the metal (*tensile*), **bend** the metal (*tensile* and *compressive), **shear*** stresses
  - Shape - die and tools
- Classification
  - Bulk Deformation
  - Sheet Metal Forming – High surface area-to-volume ratio
    - Parts are called *stampings*
    - Usual tooling: *punch* and *die*

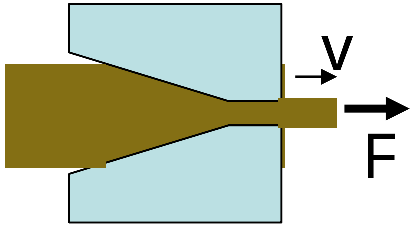
Introduction

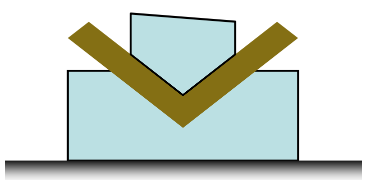
- Bulk Deformation Process
  - Rolling
  - **Forging**
  - Extrusion
  - Drawing
- Sheet metal forming
  - Bending
  - Drawing
  - Shearing
  - (Stamping)

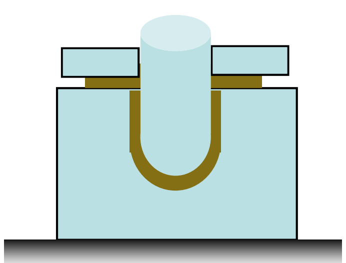


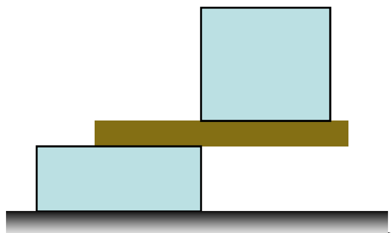












Forging Sequence of Part and Die

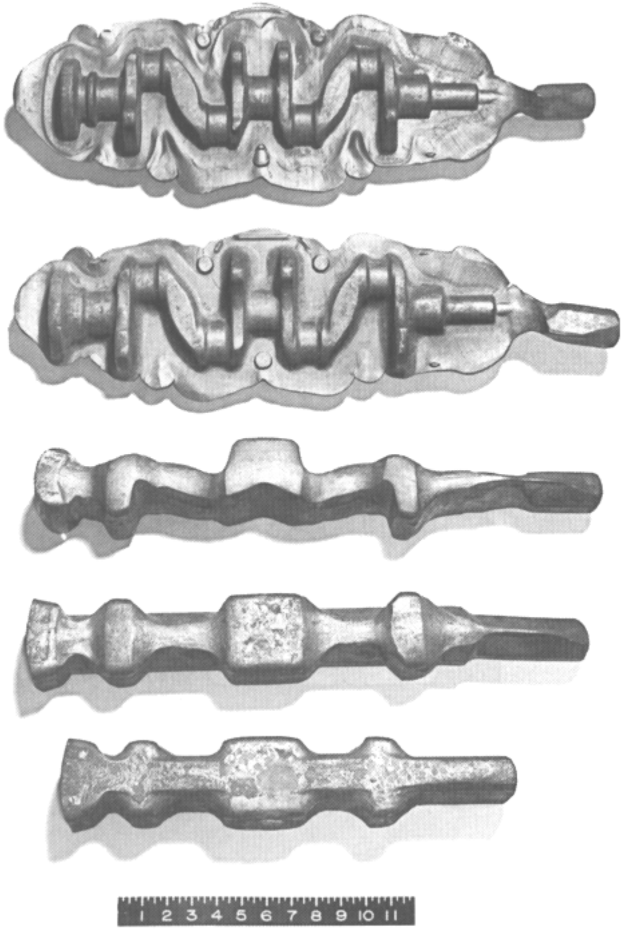


FIGURE 9.35
Stages in the formation of a crankshaft by hot forging from bottom to top (Courtesy of National Machinery LLC).

FIGURE 9.33
Hand-fed crankshaft progressive die used on a 3000-ton press (Courtesy of National Machinery LLC).

Material Properties in Forming

- Desirable material properties:
  - *Yield strength ?*
  - *Ductility ?*
  - *Second phase or Inclusion* ?
- These properties are affected by *temperature*:
  - When work temperature is raised, ductility increases and yield strength decreases while loosing on surface finish and dimension accuracy.
- Other factors:
  - Strain rate and friction

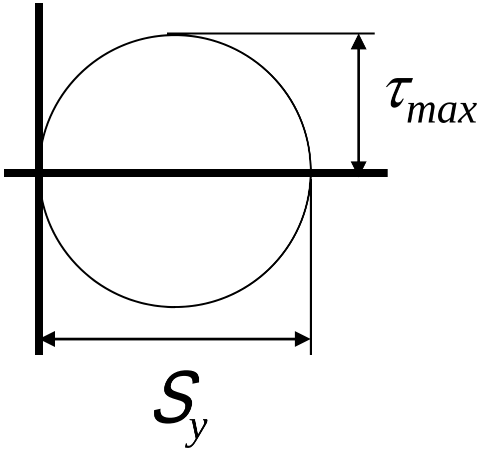
Theories of Failure

- The limit of the stress state on a material
  - Ductile Materials - Yielding
  - Brittle Materials - Fracture
- In a tensile test, Yield or Failure Strength of a material.
- In a multiaxial state of stress, how do we use Yield or Failure Strength?

Yielding: Ductile Materials

A. Maximum Shear Stress Theory
(Tresca Criterion)

In a tension specimen: $\tau_{\max} = \dfrac{S_y}{2}$



The diameter of the Mohr circle = S_y

For **Plane** Stress: (σ_3=0)

$$\left.\begin{aligned} |\sigma_1| &= S_y \\ |\sigma_2| &= S_y \end{aligned}\right\} \sigma_1,\ \sigma_2 \text{ have same signs.}$$

$$\left.|\sigma_1 - \sigma_2| = S_y \right\} \sigma_1,\ \sigma_2 \text{ have opposite signs.}$$

Ductile Materials

B. Maximum Distortion Energy Theory (von Mises Criterion)

For the three principal stresses;

$$u = \frac{1}{2}\left[\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_3\varepsilon_3\right] = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu\left(\sigma_1\sigma_2 + \sigma_3\sigma_2 + \sigma_1\sigma_3\right)\right]$$

After taken out the hydrostatic stress ($\sigma_{\mathrm{ave}} = (\sigma_1 + \sigma_2 + \sigma_3)/3$)

Now substitute σ_1, σ_2 and σ_3 with $(\sigma_1 - \sigma_{ave}), (\sigma_2 - \sigma_{ave}), (\sigma_3 - \sigma_{ave})$

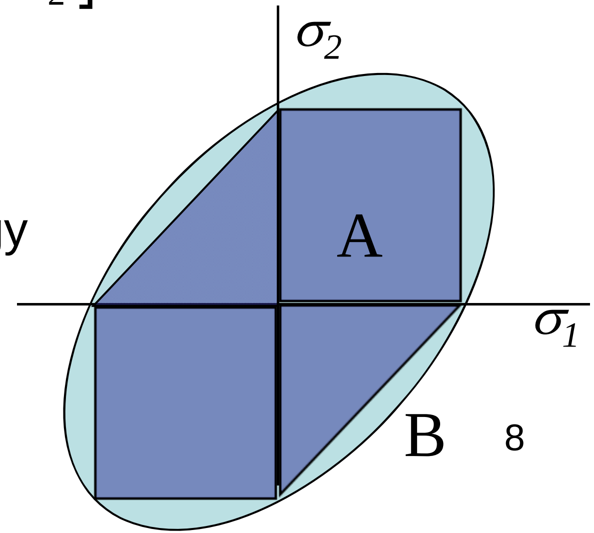
For plane stress; $u_d = \dfrac{1+\nu}{3E}\left[\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\right]$

$u_d = \dfrac{1+\nu}{3E} S_y^2$ in an uniaxial tension test.

In a biaxial case, the same amount of distortion energy

$$\frac{1+\nu}{3E}\left[\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\right] = \frac{1+\nu}{3E} S_y^2$$

$$\boxed{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = S_y{}^2}$$

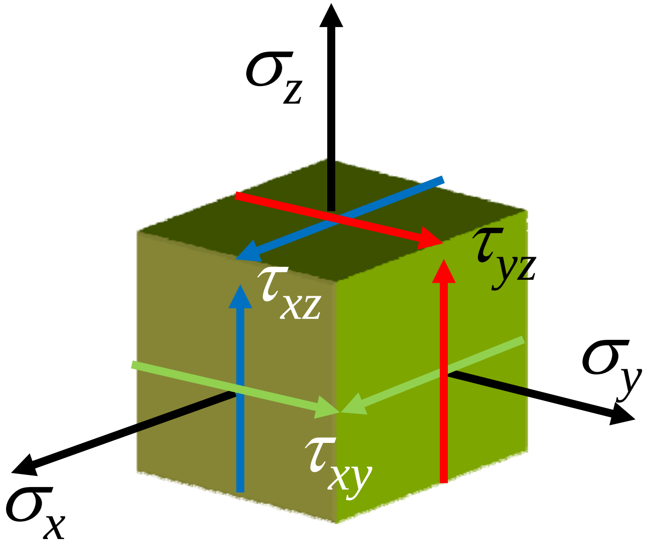


Plasticity

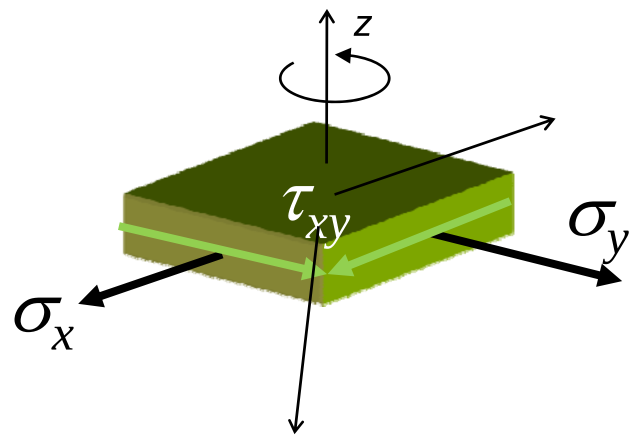
- Flow theory (Classical theory)
  - The current strain rates depend on the stress.
- Deformation theory (Hencky theory)
  - The total strain is related to the stress.
  - Ideal for nonlinear elasticity
  - Still work for monotonically increasing stresses everywhere in a body
- Pressure-independent – Hydrostatic pressure does not affect dislocation motion.
- Bauschinger effect
  - The different behaviors in tension and compression

Stress Representation

General 3-D Stress



Plane Stress



Tensor Notation

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \sigma_{ij}$$

$$\begin{vmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & 0 \end{vmatrix} = \sigma_{ij}$$

Stress Transformation (Plane Stress)

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$
$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\sigma_{x'} + \sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta + \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta = \sigma_x + \sigma_y$$

$$\begin{aligned}
-\sigma_{x'}\sigma_{y'} + \tau_{x'y'}^2 &= -\left(\frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta\right)\left(\frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta\right) + \left(-\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta\right)^2 \\
&= -\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \left(\frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta\right)^2 + \left(-\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta\right)^2 \\
&= \left(\frac{\sigma_x + \sigma_y}{2}\right)^2 - \left(\frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta\right)^2 + \left(-\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta\right)^2 \\
&= -\sigma_x\sigma_y + \tau_{xy}^2
\end{aligned}$$

Yield function in 3D

Stress Tensor : $\left[\sigma_{ij}\right]=\left[\sigma'_{ij}+\sigma_m\delta_{ij}\right]$

where $\sigma'_{ij} = \text{Deviatoric Stress Tensor,}$

$$\sigma_m = \frac{1}{3}\left(\sigma_1+\sigma_2+\sigma_3\right) = \text{Hydrostatic Component of Stress}$$

$$\delta_{ij} = 1 \quad \text{for}\, i = j$$

$$0 \quad \text{for}\, i \neq j$$

Three Stress Invariants

$$\sigma^3 - I_1\sigma^2 - I_2\sigma - I_3 = 0$$

where $I_1 = \sigma_x+\sigma_y+\sigma_z = \sigma_1+\sigma_2+\sigma_3$

$$I_2 = -\left(\sigma_x\sigma_y+\sigma_y\sigma_z+\sigma_x\sigma_z\right)+\tau^2_{xy}+\tau^2_{yz}+\tau^2_{zx}$$

$$= -\left(\sigma_1\sigma_2+\sigma_3\sigma_2+\sigma_3\sigma_1\right)$$

$$I_3 = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{xz} - \sigma_x\tau^2_{yz} - \sigma_y\tau^2_{xz} - \sigma_z\tau^2_{xy}$$

$$= \sigma_1\sigma_2\sigma_3$$

Three Deviatoric Stress Invariants

$$\sigma'^3 - J_1\sigma'^2 - J_2\sigma' - J_3 = 0$$

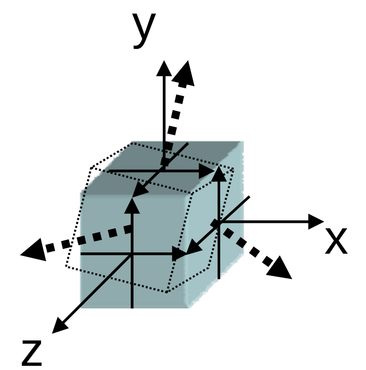
where $J_1 = 0$

$$J_2 = -\left(\sigma'_1\sigma'_2+\sigma'_3\sigma'_2+\sigma'_3\sigma'_1\right)$$

$$J_3 = \sigma'_x\sigma'_y\sigma'_z$$

Yield function: $f\left(\sigma_{ij}\right) = f\left(\sigma_I\right) = f\left(I_1, I_2, I_3\right) = f\left(J_2, J_3\right) = C$

Example: Stresses in 3D



$$[\sigma_{ij}] = \begin{bmatrix} \sigma_{11} & \sigma_{21} & \sigma_{31} \\ \sigma_{12} & \sigma_{22} & \sigma_{32} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

To find Principal Stresses:

$$[\sigma_{ij}] = \begin{bmatrix} 5 & -1 & -1 \\ -1 & 4 & 0 \\ -1 & 0 & 4 \end{bmatrix} \text{MPa}$$

$$\begin{vmatrix} 5-\sigma & -1 & -1 \\ -1 & 4-\sigma & 0 \\ -1 & 0 & 4-\sigma \end{vmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = 0$$

$$(5-\sigma)(4-\sigma)^2 - (4-\sigma) - (4-\sigma) =$$

$$= (4-\sigma)(3-\sigma)(6-\sigma) = 0$$

$$\sigma^3 - 13\sigma^2 + 54\sigma - 72 = 0$$

$$\sigma_1 = 3\text{MPa},\ \sigma_2 = 4\,\text{MPa and } \sigma_3 = 6\,\text{MPa}$$

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3 = 13,$$

$$I_2 = -(\sigma_1\sigma_2 + \sigma_3\sigma_2 + \sigma_3\sigma_1) = -(12 + 24 + 18) = -54 \text{ and}$$

$$I_3 = \sigma_1\sigma_2\sigma_3 = 72$$

Example: Continue

$$\left[\sigma_{ij}\right]=\begin{bmatrix}5 & -1 & -1\\ -1 & 4 & 0\\ -1 & 0 & 4\end{bmatrix}\text{MPa}$$

$$\sigma_{1-3}=3,\ 4\ \&\ 6\text{MPa}$$

$$\sigma_m=\frac{1}{3}\left(5+4+4\right)=\frac{1}{3}\left(3+4+6\right)=4.33\text{MPa}$$

$$\left[\sigma_{ij}\right]=\left[\sigma'_{ij}+\sigma_m\delta_{ij}\right]$$

$$\sigma'_{ij}=\begin{bmatrix}5 & -1 & -1\\ -1 & 4 & 0\\ -1 & 0 & 4\end{bmatrix}-\begin{bmatrix}4.33 & 0 & 0\\ 0 & 4.33 & 0\\ 0 & 0 & 4.33\end{bmatrix}$$

$$=\begin{bmatrix}0.67 & -1 & -1\\ -1 & -0.33 & 0\\ -1 & 0 & -0.33\end{bmatrix}$$

Deviatoric Stress

$$\begin{bmatrix}0.67-\sigma' & -1 & -1\\ -1 & -0.33-\sigma' & 0\\ -1 & 0 & -0.33-\sigma'\end{bmatrix}$$

$$(0.33+\sigma')^2(0.67-\sigma')+2(0.33+\sigma')=0$$

$$(0.33+\sigma')\left(\frac{20}{9}+\frac{1}{3}\sigma'-\sigma'^2\right)=0$$

$$0.74+2.33\sigma'-\sigma'^3=0$$

$$J_1=0;\ J_2=-2.33;\ J_3=0.74$$

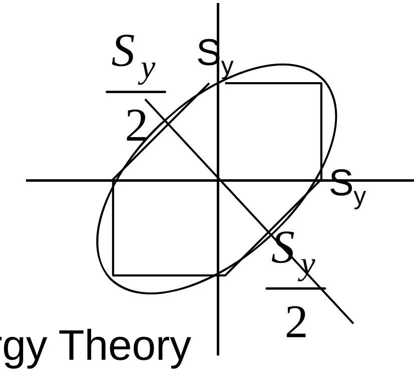
Presentation of Yield Surface

Max. Shear Stress Theory: $\boxed{f\left(\sigma_{1,2,3}\right)=\sigma_1-\sigma_3=C}$

$$f\left(J_2,J_3\right)=4J_2^3-27J_3^2-36k^2J_2^2+96k^4J_2-64k^6=C$$

Maximum Distortion Energy Theory: $\boxed{f\left(J_2,J_3\right)=J_2=k^2=C}$

$$f\left(\sigma_{1,2,3}\right)=J_2=\frac{1}{6}\left[\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2\right]=C$$



Max. Shear Stress Theory

Maximum Distortion Energy Theory

Based on uniaxial yielding: $\sigma_1=\sigma,\sigma_2=\sigma_3=0$

$$f\left(\sigma_{1,2,3}\right)=\sigma_1-\sigma_3=C$$
$$S_y=C$$

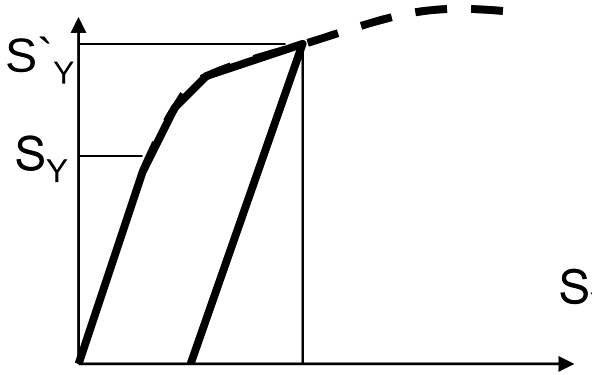
$$f\left(J_2,J_3\right)=J_2=\frac{1}{6}\left(\sigma^2+\sigma^2\right)=C;$$
$$C=\frac{\sigma^2}{3}=\frac{S_y^2}{3}$$

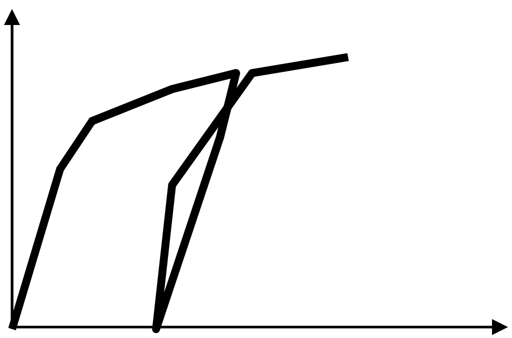
For shear: $\sigma_1=-\sigma,\ \sigma_2=\sigma,\sigma_3=0$

$$f\left(\sigma_{1,2,3}\right)=2\sigma=C=S_y;$$
$$\sigma=\frac{S_y}{2}$$

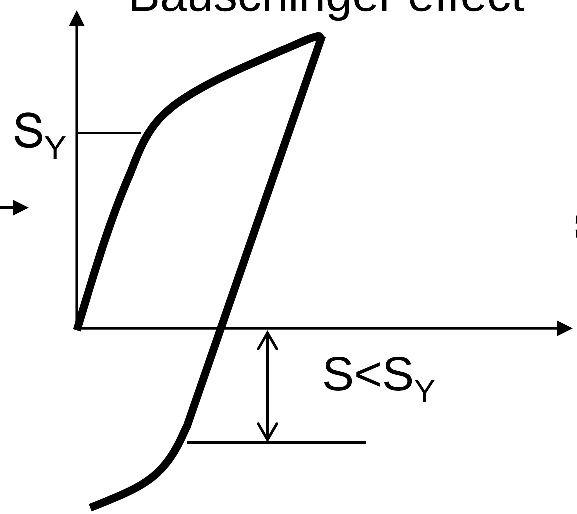
$$f\left(J_2,J_3\right)=J_2=\frac{1}{6}\left(4\sigma^2+\sigma^2+\sigma^2\right)=C;$$
$$\sigma=\frac{S_y}{\sqrt{3}}$$

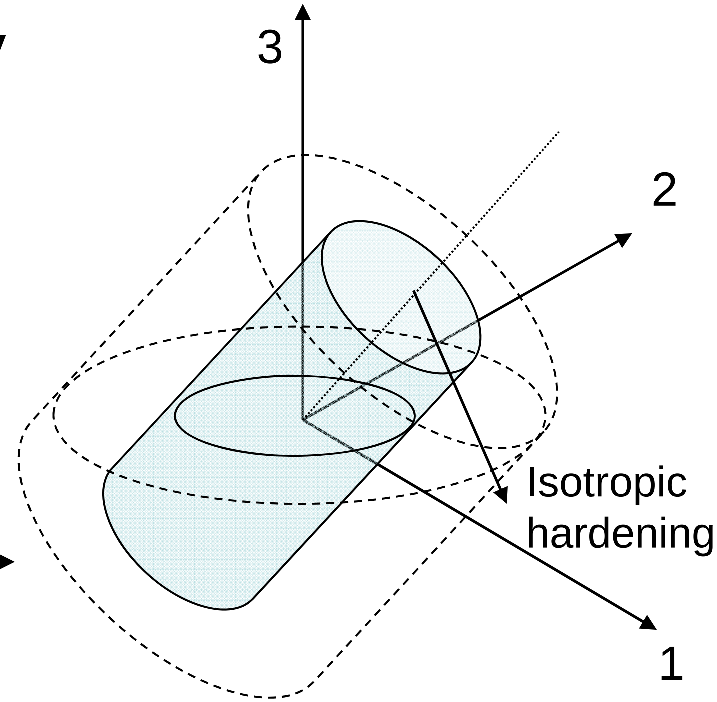
Plasticity





Bauschinger effect





Plastic State: $f\left(\sigma_{ij}\right)=0$

Elastic State: $f\left(\sigma_{ij}\right)<0$

Impossible: $f\left(\sigma_{ij}\right)>0$

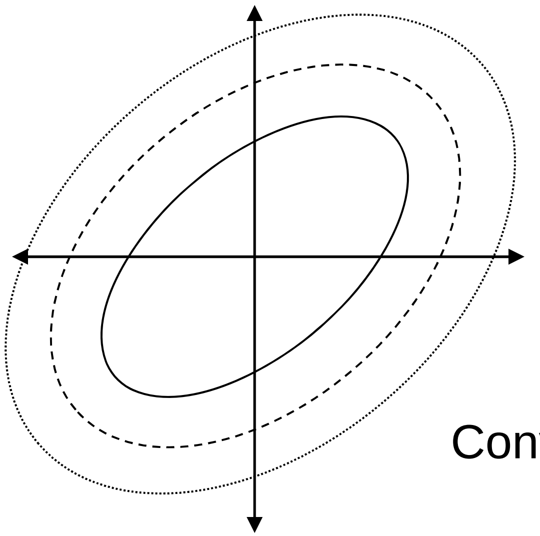
Loading: $f\left(\sigma_{ij}\right)=C, \qquad df=\dfrac{\partial f}{\partial \sigma_{ij}}\sigma_{ij}>0$

Unloading: $f\left(\sigma_{ij}\right)=C, \qquad df=\dfrac{\partial f}{\partial \sigma_{ij}}\sigma_{ij}<0$

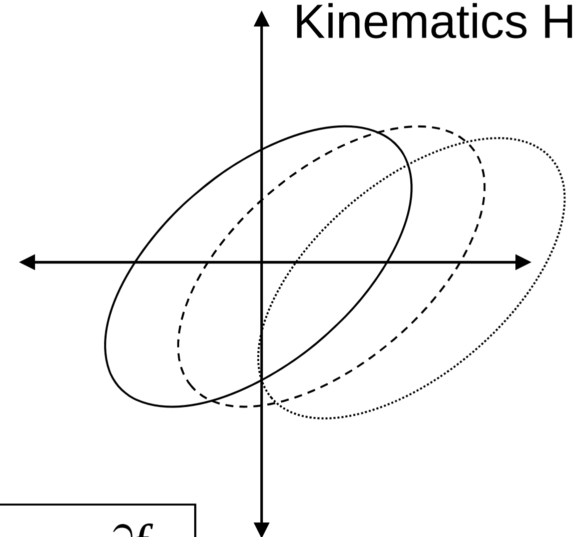
Neutral: $f\left(\sigma_{ij}\right)=C, \qquad df=\dfrac{\partial f}{\partial \sigma_{ij}}\sigma_{ij}=0$

Strain Hardening

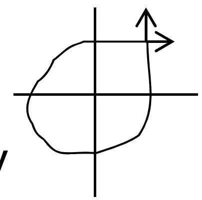
Isotropic Hardening



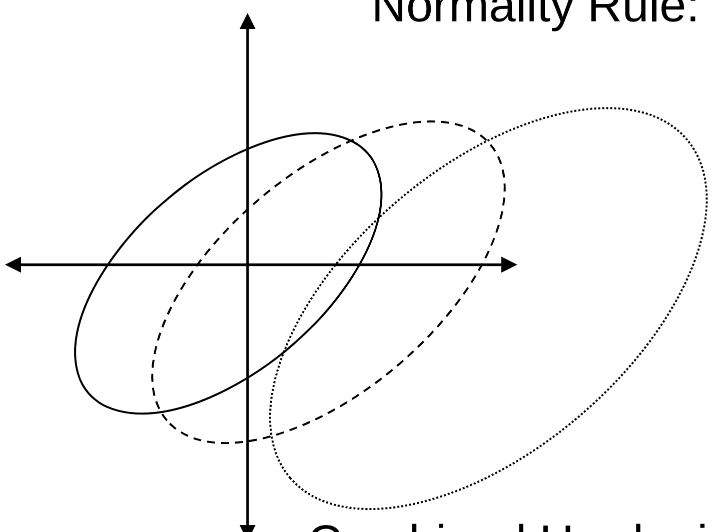
Kinematics Hardening



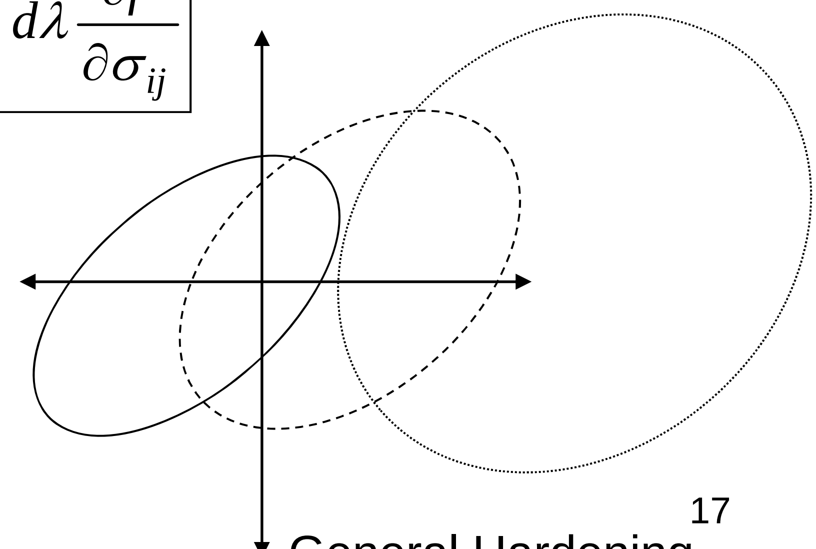
Convexity



Normality Rule: $$\boxed{d\varepsilon_{ij} = d\lambda \frac{\partial f}{\partial \sigma_{ij}}}$$



Combined Hardening



General Hardening

2. Behavior in Metal Forming

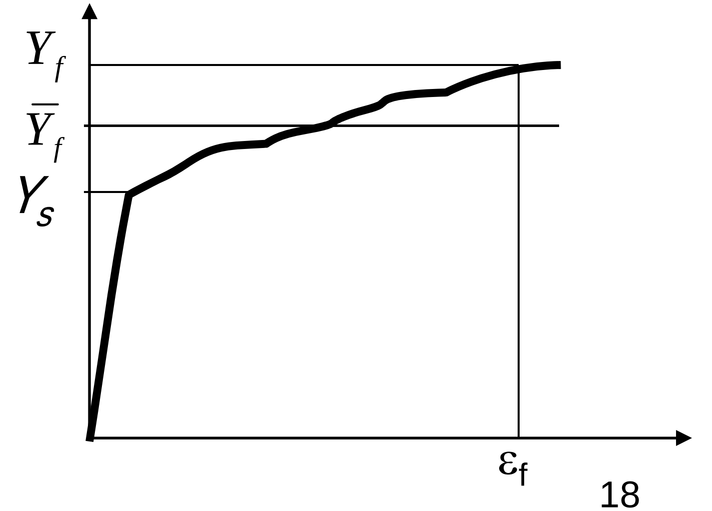
- The stress-strain relationship beyond elastic range assuming no unloading at anytime and anywhere.

$$\boxed{\sigma = K\varepsilon^n}$$

- Flow stress – The instantaneous value of stress required to continue deforming the material. $\boxed{Y_f = K\varepsilon^n}$
- Average Flow Stress

$$\boxed{\bar{Y}_f = \frac{\int_0^{\varepsilon_f} \sigma d\varepsilon}{\varepsilon_f} = \frac{\int_0^{\varepsilon_f} K\varepsilon^n d\varepsilon}{\varepsilon_f} = \frac{K\varepsilon_f^{\ n}}{1+n}}$$

- For any metal, *K* and *n* in the flow curve depend on temperature



3. Temperature in Metal Forming

- Cold Working – Performed at room temperature or slightly above
  - *Near net shape* or *net shape*
  - Better accuracy, closer tolerances & surface finish
  - Strain hardening increases strength and hardness
  - Grain flow during deformation can cause desirable directional properties in product
  - No heating of work required
  - Higher forces and power required
  - Surfaces must be free of scale and dirt
  - Ductility and strain hardening limit the amount of forming

Warm Working

- Performed at above room temperature but below recrystallization temperature
- $0.3T_m$, where T_m = melting point (absolute temperature)
  - Lower forces and power than in cold working
  - More intricate work geometries possible
  - Need for annealing may be reduced or eliminated
- Isothermal Forming – eliminate surface cooling especially highly alloyed steels and Ti and Ni alloys

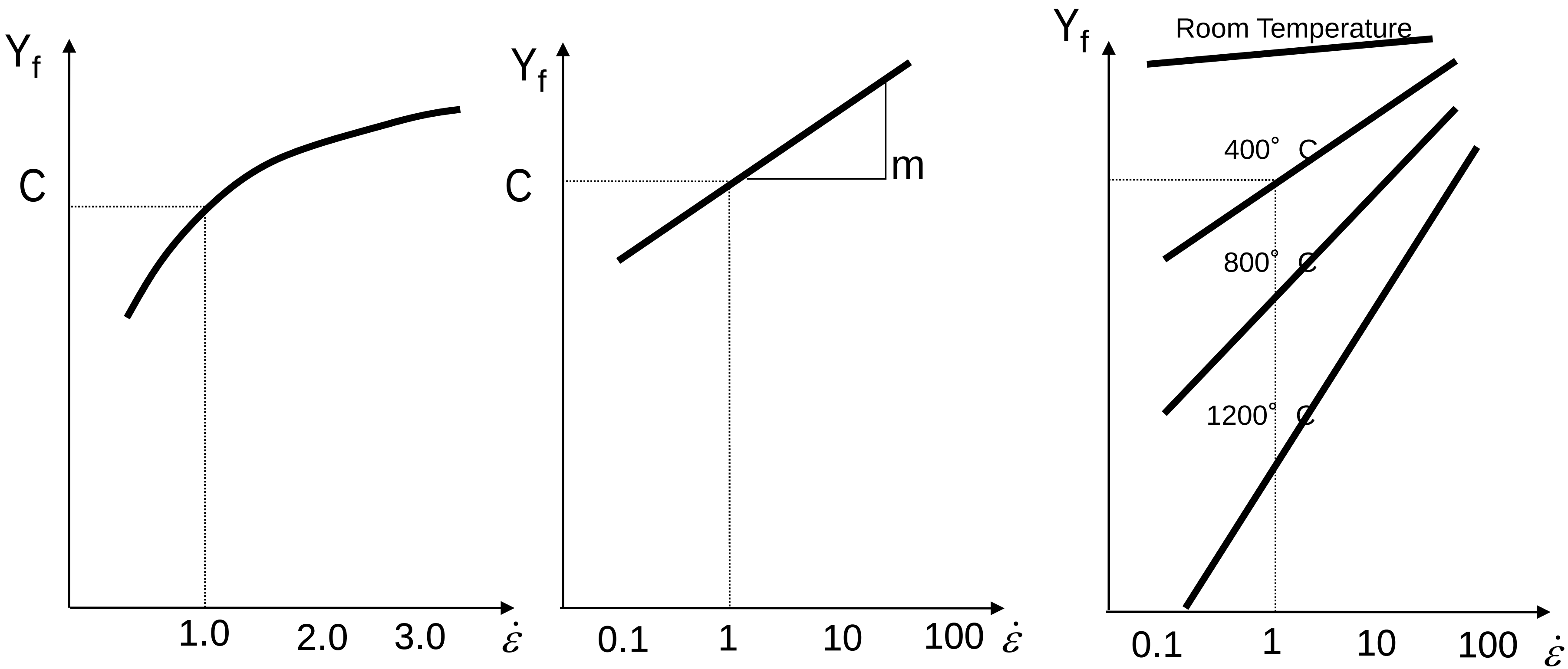
Hot Working

- Deformation process at temperatures above *recrystallization temperature* ($0.5T_m$)
- A perfectly plastic material - Strain hardening exponent is zero (theoretically)
  - Lower forces and power required
  - Metals become ductile
  - Strength properties are generally isotropic
  - No work hardening of part
  - part can be subsequently cold formed
  - Lower dimensional accuracy
  - Higher total energy required
  - Poorer surface finish including oxidation (scale),
  - Shorter tool life

4. Effect of Strain Rate

- Sensitive to strain-rate at elevated temperatures
- Strain rate: $\dot{\varepsilon} = \dfrac{v}{h}$
- Relationship: $Y_f = C\dot{\varepsilon}^m$
- A more complete relationship: $Y_f = A\varepsilon^n\dot{\varepsilon}^m$
- Evaluation of strain rate is complicated by
  - Workpart geometry
  - Variations in strain rate on the part
- Strain rate can reach 1000 s^{-1} or more for some metal forming operations

Effect of temperature on flow stress



Increasing temperature decreases *C* & increases *m*

At room temperature, effect of strain rate is almost negligible

5. Friction and Lubrication

- Friction – retard metal flow and increase power and wear

Categories	Temp. Range	Strain-rate Sensitivity exponent	Coefficient of Friction
Cold Working	$<0.3T_m$	$0<m<0.05$	0.1
Warm Working	$0.3T_m$-$0.5T_m$	$0.05<m<0.1$	0.2
Hot Working	$0.5T_m$-$0.75T_m$	$0.05<m<0.4$	0.4-0.5

- Lubrication – reduce friction & heat, improve surface finish
  - Choosing a Lubricant – Type of operation, reactivity, work materials, cost and ease of applications
    - Cold working – mineral oil, fats, fatty oils, water-based emulsions, soaps and coating
    - Hot working – mineral oil, graphite and glass