Chapter 5: Failures Resulting from Static Loading

The concept of failure is central to the design process, and it is by thinking in terms of obviating failure that successful designs are achieved.

Henry Petroski
Design Paradigms

5.1 Static Strength
- Yield Strength
  - Depending on heat treatment, surface finish, size
  - Use in multi-axial loading conditions

5.2 Stress Concentration

\[ \sigma_{\text{max}} = K_f \sigma_{\text{nom}} \]
\[ \tau_{\text{max}} = K_\tau \tau_{\text{nom}} \]

See Table A-15 and 16
Walter D. Pilkey, Peterson’s Stress Concentration Factors, 2nd ed.
FEA analysis can be used.

Stress concentration factors for round bar with fillet. (a) Axial load and (b) Bending.
Stress concentration factors for round bar with groove. (a) Bending and (b) Torsion.

Stress Contours

Figure 6.8 Flat plate with fillet axially loaded showing stress contours for (a) square corners; (b) rounded corners; (c) small grooves; and (d) small holes.

Failure Prediction for Multiaxial Stresses

- The limit of the stress state on a material
  - Ductile Materials - Yielding
  - Brittle Materials - Fracture

- In a tensile test, Yield or Failure Strength of a material.
- In a multiaxial state of stress, how do we use Yield or Failure Strength?
Ductile Material

A. Maximum Shear Stress (MSS) Criterion (Tresca Criterion)

\[ \tau_{\text{max}} = \frac{\sigma_y}{2} \]

In a tension specimen: \( \tau_{\text{max}} = \frac{\sigma_y}{2} \)

The diameter of the Mohr circle = \( \sigma_y \)

For Plane Stress: \( \sigma_A, \sigma_B \) have same signs.

\( \sigma_A, \sigma_B \) have opposite signs.

In a tension specimen: \( \tau_{\text{max}} = \frac{\sigma_y}{2} \)

The diameter of the Mohr circle = \( \sigma_y \)

B. Distortion Energy (DE) Criterion (von Mises)

For the three principal stresses;

\[ u = \frac{1}{2} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] \]

After taken out the hydrostatic stress (\( \sigma_{\text{ave}} = (\sigma_1 + \sigma_2 + \sigma_3)/3 \))

Now substitute \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) with \( (\sigma_1 - \sigma_{\text{ave}}), (\sigma_2 - \sigma_{\text{ave}}), (\sigma_3 - \sigma_{\text{ave}}) \)

For plane stress; \( u_d = \frac{1+\nu}{3E} [\sigma_A^2 - \sigma_A \sigma_B + \sigma_B^2] \)

\( u_d = \frac{1+\nu}{3E} S_y^2 \) in an uniaxial tension test.

Octahedral Stresses

Stresses acting on octahedral planes. (a) General state of stress; (b) normal stress; (c) octahedral stress.

\[ \tau_{\text{oct}} = \frac{1}{3} \left[ (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6 (\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2) \right]^{1/2} \]

\[ \sigma_{\text{oct}} = \frac{\sigma_x + \sigma_y + \sigma_z}{3} \]
Ductile Material

C. Coulomb-Mohr (DCM) theory

Some metals have a higher compressive strength than tensile strength.

- Tensile, shear and compression

\[
\begin{align*}
\frac{\sigma_1 - \sigma_2}{2} & \leq \frac{S_t}{2} \\
\frac{\sigma_2 - \sigma_3}{2} & \leq \frac{S_t}{2} \\
\frac{\sigma_3 - \sigma_1}{2} & \leq \frac{S_t}{2}
\end{align*}
\]

\[
\frac{\sigma_1 + \sigma_2 - \sigma_3}{2} \leq \frac{S_c}{2}
\]

\[
\frac{\sigma_3 + \sigma_1 - \sigma_2}{2} \leq \frac{S_c}{2}
\]

\[
\frac{\sigma_2 + \sigma_3 - \sigma_1}{2} \leq \frac{S_c}{2}
\]

\[
\frac{\sigma_1 + \sigma_2 + \sigma_3}{2} \leq \frac{S_c}{2}
\]

\[
\sigma_1 \leq S_{ul} \quad \text{or} \quad \sigma_1 \leq -S_{uc}
\]

For Plane Stress

\[
\sigma_1 \geq S_{ul} \quad \text{or} \quad \sigma_2 \leq -S_{uc}
\]

B. Modification to Mohr Theory

Brittle-Coulomb-Mohr

\[
\sigma_i = \frac{S_{ul}}{n}
\]

\[
\frac{\sigma_i - \sigma_j}{S_{ul}} = \frac{1}{n}
\]

\[
\sigma_i = 0 \geq \sigma_j = 0
\]

\[
\sigma_i \geq 0 \geq \sigma_j
\]

\[
\sigma_i = 0 \geq \sigma_j \geq 0
\]

\[
\sigma_i \geq 0 \geq \sigma_j \geq 0
\]