Solution to HW #1

2-1 From Tables A-20, A-21, A-22, and A-24c,
(a) UNS G10200 HR: \( S_{ut} = 380 \) (55) MPa (kpsi), \( S_{yf} = 210 \) (30) Mpa (kpsi) \( \textit{Ans.} \)
(b) SAE 1050 CD: \( S_{ut} = 690 \) (100) MPa (kpsi), \( S_{yf} = 580 \) (84) Mpa (kpsi) \( \textit{Ans.} \)
(c) AISI 1141 Q&T at 540°C (1000°F): \( S_{ut} = 896 \) (130) MPa (kpsi), \( S_{yf} = 765 \) (111) Mpa (kpsi) \( \textit{Ans.} \)
(d) 2024-T4: \( S_{ut} = 446 \) (64.8) MPa (kpsi), \( S_{yf} = 296 \) (43.0) Mpa (kpsi) \( \textit{Ans.} \)
(e) Ti-6Al-4V annealed: \( S_{ut} = 900 \) (130) MPa (kpsi), \( S_{yf} = 830 \) (120) Mpa (kpsi) \( \textit{Ans.} \)

2-4

AISI 1018 CD steel: Table A-5
\[
\frac{E}{\gamma} = \frac{30.0(10^6)}{0.282} = 106(10^6) \text{ in } \textit{Ans.}
\]

2011-T6 aluminum: Table A-5
\[
\frac{E}{\gamma} = \frac{10.4(10^6)}{0.098} = 106(10^6) \text{ in } \textit{Ans.}
\]

Ti-6Al-6V titanium: Table A-5
\[
\frac{E}{\gamma} = \frac{16.5(10^6)}{0.160} = 103(10^6) \text{ in } \textit{Ans.}
\]

No. 40 cast iron: Table A-5
\[
\frac{E}{\gamma} = \frac{14.5(10^6)}{0.260} = 55.8(10^6) \text{ in } \textit{Ans.}
\]

2-6

(a) \( A_0 = \pi \left(0.503\right)^2 / 4, \ \sigma = P_1 / A_0 \)

For data in elastic range, \( \epsilon = \Delta l / l_0 = \Delta l / 2 \)

For data in plastic range, \( \epsilon = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0} - 1 = \frac{A_0}{A} - 1 \)

On the next two pages, the data and plots are presented. Figure (a) shows the linear part of the curve from data points 1-7. Figure (b) shows data points 1-12. Figure (c) shows the complete range. \textbf{Note:} The exact value of \( A_0 \) is used without rounding off.

(b) From Fig. (a) the slope of the line from a linear regression is \( E = 30.5 \) Mpsi \( \textit{Ans.} \)

From Fig. (b) the equation for the dotted offset line is found to be

\[
\sigma = 30.5(10^6)\epsilon - 61 000 \quad (1)
\]

The equation for the line between data points 8 and 9 is

\[
\sigma = 7.60(10^5)\epsilon + 42 900 \quad (2)
\]
Solving Eqs. (1) and (2) simultaneously yields $\sigma = 45.6$ kpsi which is the 0.2 percent offset yield strength. Thus, $S_y = 45.6$ kpsi  \textit{Ans.}

The ultimate strength from Figure (c) is $S_u = 85.6$ kpsi  \textit{Ans.}

The reduction in area is given by Eq. (2-12) is

$$R = \frac{A_0 - A_f}{A_0} \times 100 = \frac{0.1987 - 0.1077}{0.1987} \times 100 = 45.8\% \quad \text{\textit{Ans.}}$$

<table>
<thead>
<tr>
<th>Data Point</th>
<th>$P_i$</th>
<th>$\Delta l, A_i$</th>
<th>$\varepsilon$</th>
<th>$\sigma$</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0.0004</td>
<td>0.00020</td>
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<td>2000</td>
<td>0.0006</td>
<td>0.00030</td>
<td>10065</td>
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<td>4</td>
<td>3000</td>
<td>0.001</td>
<td>0.00050</td>
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<tr>
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<td>0.0013</td>
<td>0.00065</td>
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<td>7</td>
<td>8400</td>
<td>0.0028</td>
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<td>8</td>
<td>8800</td>
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<td>9</td>
<td>9200</td>
<td>0.0089</td>
<td>0.00445</td>
<td>46298</td>
</tr>
<tr>
<td>10</td>
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<td>0.1984</td>
<td>0.00158</td>
<td>44285</td>
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<tr>
<td>11</td>
<td>9200</td>
<td>0.1978</td>
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<td>0.01229</td>
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<tr>
<td>13</td>
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<td>66428</td>
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<tr>
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<td>0.05980</td>
<td>76492</td>
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<tr>
<td>15</td>
<td>17000</td>
<td>0.1563</td>
<td>0.27136</td>
<td>85551</td>
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<tr>
<td>16</td>
<td>16400</td>
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<tr>
<td>17</td>
<td>14800</td>
<td>0.1077</td>
<td>0.84506</td>
<td>74479</td>
</tr>
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</table>

(a) Linear range
(c) The material is ductile since there is a large amount of deformation beyond yield.

(d) The closest material to the values of $S_y$, $S_{ut}$, and $R$ is SAE 1045 HR with $S_y = 45$ kpsi, $S_{ut} = 82$ kpsi, and $R = 40\%$. Ans.
2-9 \( W = 0.20 \),

(a) Before cold working: Annealed AISI 1018 steel. Table A-22, \( S_y = 32 \) kpsi, \( S_u = 49.5 \) kpsi, \( \sigma_0 = 90.0 \) kpsi, \( m = 0.25 \), \( \varepsilon_f = 1.05 \)

After cold working: Eq. (2-16), \( \varepsilon_u = m = 0.25 \)

Eq. (2-14), \( \frac{A_0}{A_f} = \frac{1}{1 - W} = \frac{1}{1 - 0.20} = 1.25 \)

Eq. (2-17), \( \varepsilon_f = \ln \left( \frac{A_0}{A_f} \right) = \ln 1.25 = 0.223 < \varepsilon_u \)

Eq. (2-18), \( S_y' = \sigma_0 \sigma_s^m = 90(0.223)^{0.25} = 61.8 \) kpsi \( \text{Ans.} \) 93% increase \( \text{Ans.} \)

Eq. (2-19), \( S' = \frac{S_y}{1 - W} = \frac{49.5}{1 - 0.20} = 61.9 \) kpsi \( \text{Ans.} \) 25% increase \( \text{Ans.} \)

(b) Before: \( \frac{S_u}{S_y} = 49.5 \) \( \frac{32}{32} = 1.55 \)

After: \( \frac{S_u'}{S_y'} = 61.9 \) \( \frac{61.8}{61.8} = 1.00 \) \( \text{Ans.} \)

Lost most of its ductility

2-14 Eq. (2-21), \( 0.5H_2 = 100 \Rightarrow H_2 = 200 \) \( \text{Ans.} \)

2-20 Appropriate tables: Young's modulus and Density (Table A-5), 1020 HR and CD (Table A-20), 1040 and 4140 (Table A-21), Aluminum (Table A-24), Titanium (Table A-24c)

Appropriate equations:

For diameter, \( \sigma = \frac{F}{A} = \frac{F}{(\pi / 4)d^2} = S_y \Rightarrow d = \sqrt[4]{\frac{4F}{\pi S_y}} \)

Weight/length = \( \mu \), Cost/length = \$/in = (\$/lbf) Weight/length,
Deflection/length = \( \delta / L = F / (AE) \)

With \( F = 100 \) kips = \( 100 \times 10^3 \) lbf,

<table>
<thead>
<tr>
<th>Material</th>
<th>Young's Modulus Mpsi</th>
<th>Density lbf/in³</th>
<th>Yield Strength kpsi</th>
<th>Cost/lbf</th>
<th>Diameter in</th>
<th>Weight/length lbf/in</th>
<th>Cost/length $/in</th>
<th>Deflection/length in/in</th>
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<tbody>
<tr>
<td>1020 HR</td>
<td>30</td>
<td>0.282</td>
<td>30</td>
<td>$0.27</td>
<td>2.060</td>
<td>0.940</td>
<td>$0.25</td>
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<tr>
<td>1020 CD</td>
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<td>57</td>
<td>$0.30</td>
<td>1.495</td>
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<td>$0.15</td>
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<tr>
<td>1040</td>
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<td>1.262</td>
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<td>$0.12</td>
<td>2.66E-03</td>
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<tr>
<td>4140</td>
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<td>0.282</td>
<td>165</td>
<td>$0.80</td>
<td>0.878</td>
<td>0.1709</td>
<td>$0.14</td>
<td>5.50E-03</td>
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<tr>
<td>Al</td>
<td>10.4</td>
<td>0.098</td>
<td>50</td>
<td>$1.10</td>
<td>1.596</td>
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<tr>
<td>Ti</td>
<td>16.5</td>
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</tr>
</tbody>
</table>

The selected materials with minimum values are shaded in the table above. \( \text{Ans.} \)
2-20  Appropriate tables: Young’s modulus and Density (Table A-5) 1020 HR and CD (Table A-20), 1040 and 4140 (Table A-21), Aluminum (Table A-24), Titanium (Table A-24c)

Appropriate equations:

For diameter, \( \sigma = \frac{F}{A} = \frac{F}{(\pi / 4) d^2} = \frac{F}{\pi S_d} \Rightarrow d = \sqrt[4]{\frac{4F}{\pi S_d}} \)

\[
\text{Weight/length} = \rho L, \quad \text{Cost/length} = \frac{\$}{\text{in}} = \left(\frac{\$}{\text{lbf}}\right) \text{Weight/length},
\]

\[
\text{Deflection/length} = \frac{\delta}{L} = F/(AE)
\]

With \( F = 100 \text{ kips} = 100(10^5) \text{ lb} \),

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s Modulus</th>
<th>Density</th>
<th>Yield Strength</th>
<th>Cost/lbf</th>
<th>Diameter</th>
<th>Weight/length</th>
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<tbody>
<tr>
<td></td>
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<td>lb/in^3</td>
<td>kpsi</td>
<td>$/lbf</td>
<td>in</td>
<td>lb/in</td>
<td>$/in</td>
<td>in/in</td>
</tr>
<tr>
<td>1020 HR</td>
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<td>2.060</td>
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<tr>
<td>1040</td>
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<td>0.282</td>
<td>80</td>
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<td>$0.80</td>
<td>0.878</td>
<td>0.1709</td>
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<tr>
<td>Al</td>
<td>16.5</td>
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<tr>
<td>Ti</td>
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<td>0.16</td>
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<td>1.030</td>
<td>0.1333</td>
<td>$0.93</td>
<td>7.273E-03</td>
</tr>
</tbody>
</table>

The selected materials with minimum values are shaded in the table above. \( \text{Ans.} \)

2-28  For strength,

\[
\sigma = \frac{F l}{Z} = S \quad \text{(1)}
\]

where \( F l \) is the bending moment and \( Z \) is the section modulus [see Eq. (3-26b), p. 90].

The section modulus is strictly a function of the dimensions of the cross section and has the units \( \text{m}^3 \) (ips) or \( \text{m}^3 \) (SI). Thus, for a given cross section, \( Z = C (A)^{3/2} \), where \( C \) is a number. For example, for a circular cross section, \( C = \left(\frac{4}{\pi}\right)^{3/2} \). Then, for strength, Eq. (1) is

\[
\frac{F l}{C A^{3/2}} = S \Rightarrow A = \left(\frac{F l}{C S}\right)^{2/3} \quad \text{(2)}
\]
For mass, \( m = Al \rho = \left( \frac{F}{CS} \right)^{\frac{2}{3}} l \rho = \left( \frac{F}{C} \right)^{\frac{2}{3}} \left( \frac{\rho}{S^{\frac{1}{2}}} \right) \)

Thus, \( f_3(M) = \frac{\rho}{S^{\frac{2}{3}}} \), and maximize \( S^{\frac{2}{3}}/\rho \) \( (\beta = 2/3) \)

In Fig. (2-19), draw lines parallel to \( S^{2/3}/\rho \)

From the list of materials given, a higher strength aluminum alloy has the greatest potential, followed closely by high carbon heat-treated steel. Tungsten carbide is clearly not a good choice compared to the other candidate materials. \( \text{Ans.} \)