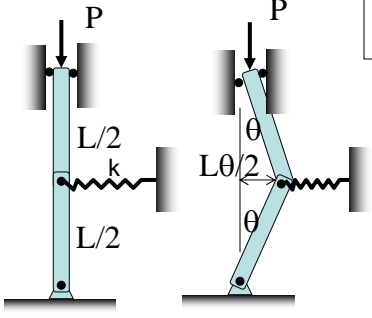
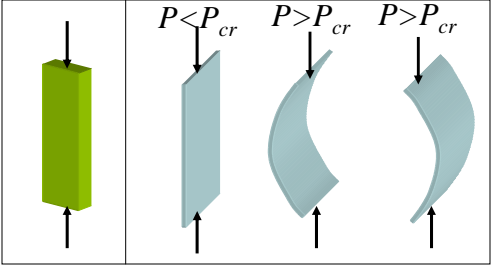


10.1-2 Introduction & Stability

Strength
Stiffness
Stability - Critical Load

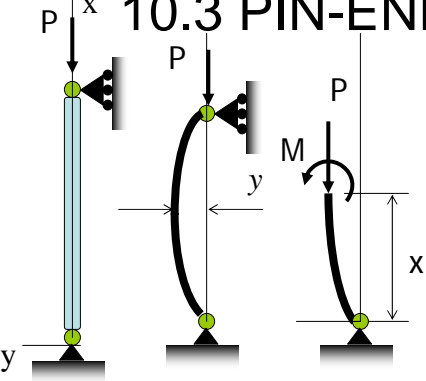



$P \tan \theta \approx P \theta$
 $P \tan \theta \approx P \theta$

$F = k\theta L/2$ $\sum F = 0; \quad 2P_{cr}\theta = \frac{k\theta L}{2}$
 $P_{cr} = \frac{kL}{4}$

1

10.3 PIN-ENDED COLUMNS



Define $k^2 = \frac{P}{EI}$

$$\frac{d^2y}{dx^2} + k^2y = 0$$

$$y = C_1 \sin kx + C_2 \cos kx$$

- Boundary Conditions
 - $y(0) = 0 \rightarrow C_2 = 0$
 - $y(L) = 0 \rightarrow C_1 \sin kL = 0$
 - $kL = n\pi$ or $C_1 = 0$

$\sin kL = 0$ Buckling equation

$$P_{cr}^{(n)} = \frac{n^2 \pi^2 EI}{L^2} \quad n = 1, 2, 3, \dots$$

$$P_{cr}^{(1)} = P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$P_{cr}^{(2)} = P_{cr} = \frac{4\pi^2 EI}{L^2}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EAr^2}{L^2 A} = \frac{\pi^2 E}{(L/r)^2}$$

where $I = Ar^2$

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10.4 Columns Fixed-Free Ends

Solution : $y = y_H + y_P$
 $= C_1 \sin kx + C_2 \cos kx + \delta$

Boundary Conditions:
 $y(0) = 0, \frac{dy}{dx}(0) = 0$ and $y(L) = \delta$
 $y = C_1 \sin kx + C_2 \cos kx + \delta$
 $C_2 = -\delta$ and $C_1 = 0$
 $y = \delta(1 - \cos kx)$

$EI \frac{d^2 y}{dx^2} = M = P(\delta - y)$

$\frac{d^2 y}{dx^2} + k^2 y = k^2 \delta$

where $k^2 = P/EI$

Solution: $\delta = 0$ or $\cos kL = 0$

$kL = \frac{n\pi}{2} \quad n = 1, 3, 5, \dots$

$P_{cr} = \frac{\pi^2 n^2 EI}{4L^2}$

$P_{cr}^{(1)} = \frac{\pi^2 EI}{4L^2} \quad P_{cr}^{(2)} = \frac{9\pi^2 EI}{4L^2}$

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10.4 Columns Having Various Types of Supports

(a) Pinned-Pinned Column	(b) Fixed-Free Column	(c) Fixed-Fixed Column	(d) Fixed-Pinned Column
$P_{cr} = \frac{\pi^2 EI}{L^2}$	$P_{cr} = \frac{\pi^2 EI}{4L^2}$	$P_{cr} = \frac{4\pi^2 EI}{L^2}$	$P_{cr} = \frac{2.046\pi^2 EI}{L^2}$
$L_e = L$	$L_e = 2L$	$L_e = 0.5L$	$L_e = 0.699L$
$K = 1$	$K = 2$	$K = 0.5$	$K = 0.699$

$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$

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