ME222: BEAM

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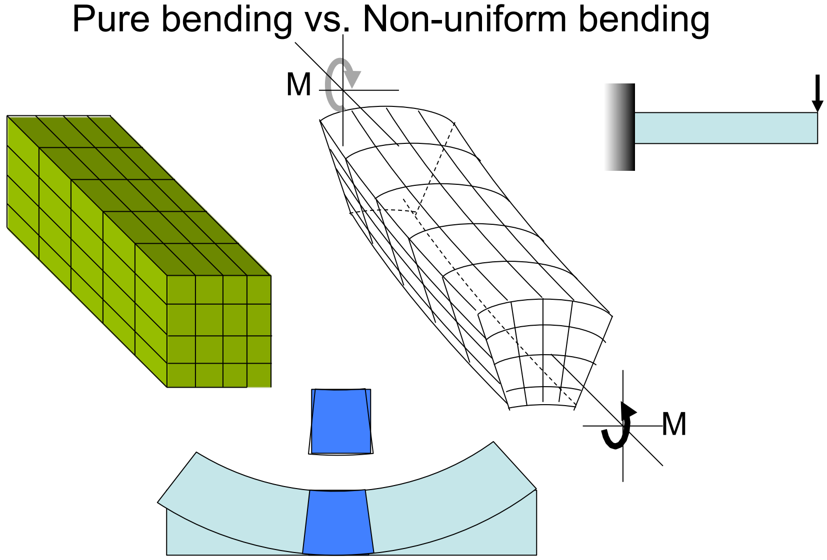
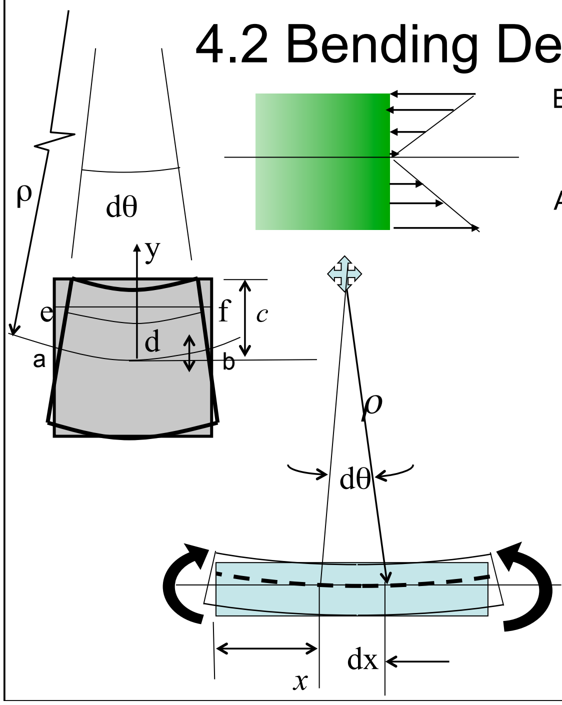
4.1 Introduction

Pure bending vs. Non-uniform bending



4.2 Bending Deformation



Before Bending,

$$L_{ab} = L_{ef} = \rho d\theta$$

After Bending,

$$L_{ab} = dx = \rho d\theta$$

$$L_{ef} = (\rho - y)d\theta$$

$$\Delta L_{ef} = dx - (\rho - y)d\theta = yd\theta$$

$$\varepsilon_x = \frac{\Delta L_{ef}}{L_{ef}} = -\frac{yd\theta}{\rho d\theta} = -\frac{y}{\rho}$$

$$\sigma_x = E\varepsilon_x = -E\frac{y}{\rho} = -\left(\frac{y}{c}\right)\sigma_{\max}$$

4.3 Flexure Formula

- No resultant force on the surface: Neutral Plane (N.P.)

$$\sum F_x = 0; \qquad \int_A \sigma_x dA = -\frac{\sigma_{\max}}{c}\int_A ydA = 0 \qquad \text{First Moment of An Area}$$

- The integral of the elemental moment over the surface must equal to the bending moment

$$\sum M = 0; \qquad M = -\int_A \sigma_x ydA = -\frac{E}{\rho}\int_A y^2 dA$$

- Moment of Inertia: $I = \int_A y^2 dA$
(Second Moment of Area)

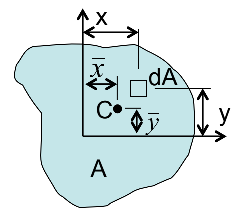
- Flexure Formula: $\sigma_x = -\frac{My}{I}\left(= -\frac{Ey}{\rho} = -\frac{\sigma_{\max} y}{c}\right)$

Moments of Areas

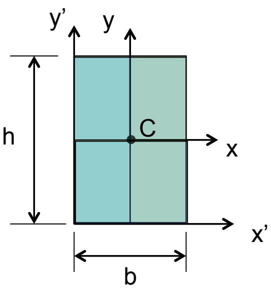
- First Moment of An Area: Centroid of An Area

First Moment of Area A w.r.t. x-axis: $Q_x = \int_A y\,dA = A\bar{y}$

First Moment of Area A w.r.t. y-axis: $Q_y = \int_A x\,dA = A\bar{x}$



Centroid of Area A: C $(\bar{x}, \bar{y})$ at which Q_x and Q_y will be zeros.



Example: Rectangle

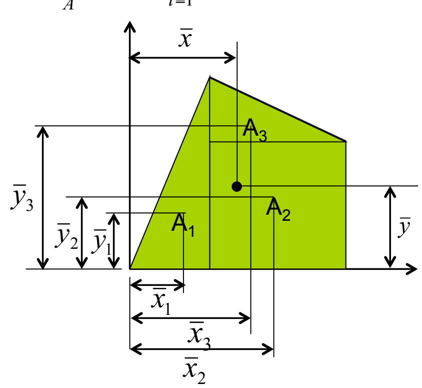
$$Q_x = A\bar{y} = (bh)(0) = 0 \qquad Q_{x'} = A\bar{y} = (bh)\frac{h}{2} = \frac{bh^2}{2}$$

$$Q_y = A\bar{x} = (bh)(0) = 0 \qquad Q_{y'} = A\bar{x} = (bh)\frac{b}{2} = \frac{b^2h}{2}$$

First Moment of An Area

$$Q_x = \int_A y\,dA = \sum_{i=1}^{n} A_i\bar{y}_1$$

$$Q_y = \int_A x\,dA = \sum_{i=1}^{n} A_i\bar{x}_1$$



Centroid $(\bar{x}, \bar{y})$

$$\bar{x} = \frac{\sum_{i=1}^{n} A_i\bar{y}_i}{\sum_{i=1}^{n} A_i}$$

$$\bar{y} = \frac{\sum_{i=1}^{n} A_i\bar{x}_i}{\sum_{i=1}^{n} A_i}$$

Second Moment (Moment of Inertia)

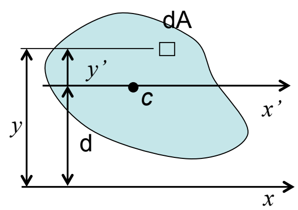
Second Moment w.r.t. x-axis: $I_x = \int_A y^2 dA$

Second Moment w.r.t. y-axis: $I_y = \int_A x^2 dA$

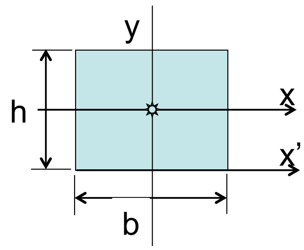
Polar Moment of Inertia: $J = \int_A \rho^2 dA = I_x + I_y$

Parallel-Axis Theorem

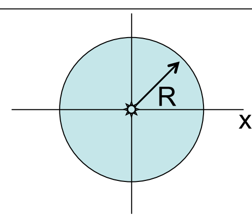
$$I_x = \int_A y^2 dA = \int_A (y'+d)^2 dA$$
$$= \int_A y'^2 dA + 2d\int_A y' dA + d^2 \int_A dA$$
$$= \bar{I}_{x'} + 2dQ_{x'} + Ad^2 = \bar{I}_{x'} + Ad^2$$
$$\because Q_{x'} = A\bar{y}' = A(0) = 0$$



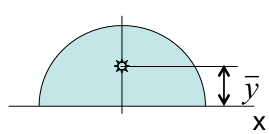
Moment of Inertia



$$I_x = \int_A y^2 dA = \int_{-b/2}^{b/2} \int_{-h/2}^{h/2} y^2 dy dx$$
$$= \int_{-b/2}^{b/2} \left(\frac{y^3}{3} \Big|_{-h/2}^{h/2} \right) dx = \int_{-b/2}^{b/2} \frac{h^3}{12} dx = \frac{bh^3}{12}$$
$$I_{x'} = \frac{bh^3}{12} + bh\left(\frac{h}{2}\right)^2 = \frac{bh^3}{3}$$



$$I_x = I_y = \int_A y^2 dA = \int_0^R \int_0^{2\pi} r^2 \cos^2 \theta r dr d\theta$$
$$= \int_0^R \left(\frac{\theta}{2} + \frac{\sin 2\theta}{4} \Big|_0^{2\pi} \right) r^3 dx = \int_0^R \pi r^3 dx = \frac{\pi R^4}{4}$$
$$J_0 = I_x + I_y = \frac{\pi R^4}{4} + \frac{\pi R^4}{4} = \frac{\pi R^4}{2}$$



$$I_x = I_y = \frac{\pi r^4}{8} \qquad \bar{y} = \frac{4r}{3\pi}$$