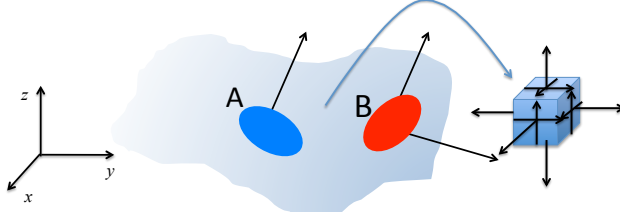


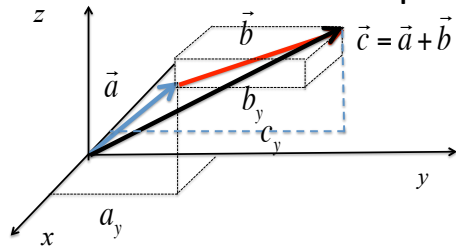
### 0.0 Background

- **Scalar** – Example: Temperature @ (x,y,z)
  - Temperatures at A,  $T_A$ , and at B,  $T_B$ .
- **Vector** – Forces & Moments
  - Statics & Dynamics - Force & Moment Equilibriums
- **Tensor** – Stress and Strain



The diagram shows a 3D coordinate system with x, y, and z axes. A blue irregularly shaped body is shown with two points, A (blue oval) and B (red oval), marked. Arrows point from A and B towards a small blue cube representing a stress element. The cube has arrows on its faces representing forces or moments.

### Vector Example



The diagram shows a 3D coordinate system with x, y, and z axes. Two vectors,  $\vec{a}$  and  $\vec{b}$ , are shown originating from the origin.  $\vec{a}$  has components  $a_x$  and  $a_y$ .  $\vec{b}$  has components  $b_x$  and  $b_y$ . Their sum,  $\vec{c} = \vec{a} + \vec{b}$ , is shown as a red vector with components  $c_x$  and  $c_y$ . Dotted lines illustrate the parallelogram rule for vector addition.

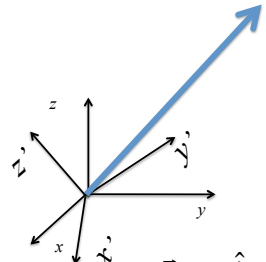
$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$$

$$\vec{c} = \vec{a} + \vec{b} = (a_x + b_x) \hat{i} + (a_y + b_y) \hat{j} + (a_z + b_z) \hat{k}$$

$$\therefore c_x = a_x + b_x; \quad c_y = a_y + b_y; \quad c_z = a_z + b_z$$

### Vector Transformation



The diagram shows two coordinate systems,  $(x, y, z)$  and  $(x', y', z')$ , with a vector  $\vec{a}$  shown in both. The axes are rotated relative to each other.

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k} = a'_x \hat{i}' + a'_y \hat{j}' + a'_z \hat{k}'$$

$$\therefore a_x \neq a'_x; \quad a_y \neq a'_y; \quad a_z \neq a'_z$$

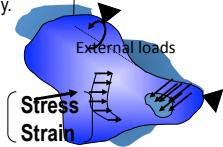
### 1. Concept of Stress

#### 1.1 Introduction

- **Statics (and Dynamics)**
- **Mechanics of Materials (or Deformable Solids)**
  - Understanding the relationships between **external loads** and **internal reactions** on a body.
  - Deals with the integrity, deformation and stability of a body.
- **Pressure, Force & Moment**
  - Stresses
  - Not measurable quantity
- **Displacement, Deformation & Distortions**
  - Strains
  - Measurable quantity
- **Material Behavior**
  - Chalk (**Axial, Bending & Torsion**)
  - Rubber vs. Steel

Deformable Solids

Effect: Displacement Deformation Distortion



The diagram shows a blue irregularly shaped body with arrows representing external loads. Inside the body, arrows represent internal reactions. A box labeled 'Stress Strain' is shown with arrows pointing to the internal reactions.

Cause: Pressure Force Moment

## 1.2 Review on Statics

**Free Body Diagram**

- External Force**
  - Surface Force & Traction
  - Body Force (weight)
  - Reactions

**Equation of Equilibrium**

- Balance of Force:  $\Sigma F = 0$ 

$$\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma F_z = 0 \end{cases}$$
- Balance of Moment:  $\Sigma M_p = 0$ 

$$\begin{cases} \Sigma M_x = 0 \\ \Sigma M_y = 0 \\ \Sigma M_z = 0 \end{cases}$$

2-D

## Boundary Conditions

**Two-force member In equilibrium**

**Pin joints**

## Tension in Cable

## 1.3 STRESSES

**Stresses**  
The reaction of a deformable body due to external loads.

Why do we use Stresses rather than Loads and Moments?

$$\Sigma F_x = 0; \Sigma F_y = 0;$$

$$\Sigma F_z = 0; \int dF = \int \sigma dA; \sigma = \frac{P}{A}$$

$$\Sigma M_x = 0; \Sigma M_y = 0; \Sigma M_z = 0$$

English unit - psi (Pound per inch<sup>2</sup>), ksi(10<sup>3</sup>psi)  
SI Unit - Pa (N/m<sup>2</sup>), kPa (10<sup>3</sup>Pa), MPa(10<sup>6</sup>Pa), GPa (10<sup>9</sup>Pa)

## 1.4 Analysis and Design

- Allowable Stress – Related to Material Properties (e.g. Strength)
- Allowable Load - Related to Material Properties and Geometry
- Analysis - Check for failure
  - Material Limitation - Allowable Stress (e.g. Yield Strength)

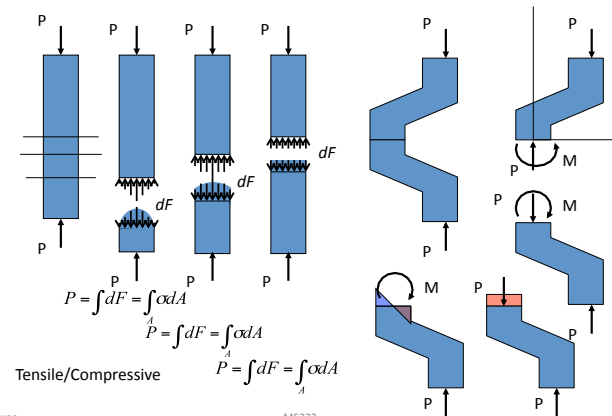
$$\sigma = \frac{P}{A} < \sigma_{allow}$$

- Design
  - Based on Material Limitation, the size of a structure can be designed.

$$\sigma_{allow} = \frac{P}{A}, \quad \text{Solve for Area or Diameter}$$

9

## 1.5 Axial Loading-Normal Stress



Kwon

ME222