STOCHASTIC THEORY FOR IRREGULAR STREAM MODELING. PART I: FLOW RESISTANCE

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ABSTRACT: A stochastic theory is developed for predicting flow resistance in natural rivers. Irregularly varying river width and bed elevation are represented as one-dimensional spatial random fields. Large-scale random flow acceleration and deceleration in response to boundary variations are described by the stochastic differential flow equation. Analytical stochastic flow solutions are developed for the case when boundary variations are small and statistically homogeneous. In particular, closed-form expressions for the effective flow resistance coefficient and flow variance are obtained. The results indicate that flow resistance in natural rivers is strongly influenced by cross-sectional nonuniformity and mean flow condition, in addition to relative boundary roughness and mean cross-sectional shape. The results also show that effective resistance is always greater than uniform resistance in a corresponding mean straight channel. This difference increases as the mean Froude number increases for a given mean bed slope or as mean bed slope decreases for a given mean Froude number. Part II of this paper will be published in the future.

INTRODUCTION

Flow resistance in straight uniform channels results predominantly from skin friction along channel boundaries. It is determined by boundary roughness and mean cross-sectional shape. The state of knowledge of turbulent flows has progressed to the point that it is now possible to predict uniform flow resistance with sufficient accuracy for most engineering purposes (Alam and Kennedy 1969).

It hardly needs to be emphasized, however, that resistance processes in natural rivers are much more complicated. Flow accelerates and decelerates in response to macro-scale boundary irregularities. Indeed, few river channels are straight and uniform over a distance exceeding a few times the channel width (Leopold and Wolman 1957). The required assumptions of uniform resistance theory are probably never satisfied, except for short stretches.

It has been long recognized that large-scale, cross-sectional nonuniformities cause significant form drag, which contributes importantly to the total flow resistance in natural rivers (Einstein and Barbarossa 1952; Rouse 1965). During the past few decades, there have been many efforts to quantify complex nonuniform flow resistance processes (Einstein and Barbarossa 1952; Leopold and Wolman 1957; Leopold et al. 1960; Rouse 1965; Engelund 1966; Alam and Kennedy 1969; Parker and Peterson 1980; Pres-
tegaard 1983; Bathurst 1985; Miller and Wenzel 1985; Hey 1988; and Shen et al. 1990). These investigations have met with varying degrees of success in accounting for the complicated resistance processes encountered in natural rivers. They have produced some useful formulas for predicting non-uniform flow resistance. However, most studies of nonuniform natural channels have been largely empirical, based on dimensional analysis and regression, or semi-empirical, based on ad hoc assumptions that may not be well justified in many situations. Extrapolation of these studies to general situations, other than those for which data were collected, can be quite unreliable.

Calibration is another technique frequently used by practical engineers for estimating flow resistance in natural streams. The resistance coefficient is treated as a tuning parameter. Flow calculation can be always brought into close agreement with field measurements using an appropriate value of the resistance coefficient. But such calibrated values generally lack a physical basis and provide little physical insight into the resistance processes found in natural streams (Hey 1988). More importantly, calibrated resistance coefficients do not generalize to flow conditions other than those for which calibration was performed, since flow resistance in natural rivers is influenced importantly by mean flow conditions. This characteristic of natural streams makes it difficult to apply the calibration approach when predictions are required over a wide range of flow conditions.

A systematic theoretical analysis of flow resistance in natural rivers, based on a rigorous mathematical description of the physical flow processes, seems highly desirable. The new analysis should explicitly recognize the important role of large-scale stream irregularities. In a conventional framework of a deterministic analysis, however, this can be analytically difficult. We adopt in this paper a probabilistic description of river geometry. Variations in river width and bed elevation are conveniently represented as one-dimensional random fields, characterized by their autocorrelation or covariance functions (Chiu 1971). Random flow acceleration or deceleration in response to these boundary variations are described by the stochastic one-dimensional flow equation. When boundary variations are small and statistically homogeneous (more on these assumptions is presented later), the theory of stationary random functions may be used to obtain closed-form stochastic flow solutions. In particular, simple expressions may be obtained for the effective resistance coefficient and flow variances.

**General Resistance Formulation**

Gradually varying steady-state flow in a natural river can be described by the following differential equation:

$$\frac{d}{dx} \left[ \frac{Q^2}{A} \right] + gA \left[ \frac{dh}{dx} + \frac{dz}{dx} + S_f \right] = 0, \quad \text{......................................... (1)}$$

where $Q$ = volumetric flow discharge; $A$ = flow cross-sectional area; $h$ = flow depth (laterally averaged); $x$ = spatial coordinate in the streamwise direction; $g$ = the gravitational constant; $z$ = bed elevation; and $S_f$ = the local resistance coefficient or friction slope. Eq. (1) is a statement of local force balance among gravitational, inertial, pressure, and friction forces. The friction forces characterized by friction slope $S_f$ can be approximately determined from any existing uniform resistance formula, such as the Darcy-Weisbach equation.
\[ S_r = \frac{Q^2 f}{8gA^2R} \]  

(2)  

where \( R \) = hydraulic radius; \( f \) = the dimensionless Darcy-Weisbach resistance coefficient, reflecting local small-scale bed and bank roughness and assumed to be spatially constant.  

For algebraic convenience, we introduce the following dimensionless variables

\[ x^* = \frac{x}{R}; \quad A^* = \frac{A}{\bar{A}}; \quad h^* = \frac{h}{R}; \quad z^* = \frac{z}{R}; \quad R^* = \frac{R}{\bar{R}}; \quad B^* = \frac{B}{\bar{B}} \]  

(3)  

where \( \bar{A}, \bar{R}, \) and \( \bar{B} \) = mean cross-sectional area, mean hydraulic radius, and mean river width, respectively, and are all assumed spatially invariant for analytical convenience. Substituting (3) into (1) and (2) and combining gives the dimensionless flow equation

\[ \frac{F_r^2 f}{8 A^* R^*} - \frac{F_r^2}{A^*^2} \frac{dA^*}{dx^*} + A^* \left( \frac{dh^*}{dx^*} + \frac{dz^*}{dx^*} \right) = 0 \]  

(4)  

where \( F_r = Q/\bar{A} \sqrt{gR} \) is the mean flow Froude number. In what follows, the asterisk superscript is dropped for notational convenience. Each of the spatially varying quantities in (4) is decomposed into its mean (denoted by an overbar) and zero-mean perturbation (denoted by a prime). Taking the mean value on both sides of the equation yields

\[ -\ddot{s} + \frac{fQ^2}{8gA^2R} \left( 1 + \sigma_A^2 + \sigma_{AR}^2 + \sigma_R^2 \right) + A' \ddot{h} + A' \ddot{z} = 0 \]  

(5)  

where \( \ddot{s} = -d\ddot{z}/dx \) is mean bed slope; \( \sigma_A^2 \) and \( \sigma_R^2 \) = variance of the cross-sectional area and hydraulic radius, respectively; and \( \sigma_{AR}^2 \) = the cross-variance between \( A \) and \( R \). Note that the gradients of spatially invariant mean quantities are dropped, and third-order and higher perturbed terms are all assumed to be small, and thus are neglected.

It is instructive to rearrange (5) into the following familiar form

\[ Q = \frac{\sqrt{8gA}}{f_r \sqrt{\bar{A}} \sqrt{\bar{R}}} \]  

(6)  

where

\[ f_r = \left( 1 + \sigma_A^2 + \sigma_{AR}^2 + \sigma_R^2 + \frac{\bar{A}'^2 \ddot{h}}{\ddot{s}} dx + \frac{\bar{A}'^2 \ddot{z}}{\ddot{s}} dx \right) f \]  

(7)  

Eq. (6) shows that the mean behavior of flows in irregular but statistically uniform natural rivers can be described by the same algebraic Chezy equation as for uniform flows, except that the Darcy-Weisbach resistance coefficient \( f \) is replaced by an effective resistance coefficient \( f_r \) to account for the effect of macro-scale roughness or large-scale boundary irregularities. Eq. (7) is a general resistance expression for natural rivers. This expression indicates that the effective resistance coefficient is always greater than the local uniform resistance coefficient as a result of the additional spatial variance of the perturbed quantities. The sum of these perturbed product terms is always positive as will be shown further on.
Evaluation of these variances and cross-variances with the help of the theory of random functions is the focus of the next section.

**Evaluation of Effective Resistance**

**Random Representation of River Geometry**

We have emphasized that natural rivers are characteristically irregular in cross section and bed elevation. Analytical deterministic evaluation of the spatial variance terms in (7) can therefore be extremely difficult, if not impossible. Here, we adopt a probabilistic view point and assume that longitudinal variations of river width and bed elevation can be represented by one-dimensional random functions. These random functions are characterized here by their first two moments, or the mean and autocovariance.

A random function or stochastic process, say \( w(x) \) representing river width or bed elevation, can be considered statistically homogeneous, or wide-sense stationary if it has a constant mean and its autocorrelation function depends only on the separation distance \( \xi \). Physically, this homogeneity indicates that the degree of variability and the correlation structure do not depend on the spatial location. In this case, it is possible to represent the stationary mean-removed process \( w'(x) = w(x) - \bar{w} \) as a Fourier-Stieltjes integral (Priestley 1981)

\[
w'(x) = \int_{-\infty}^{+\infty} e^{ikx} dZ_w(k)
\]

where \( k \) = the wave number; and \( dZ_w(k) \) = the complex amplitude that satisfies the following orthogonality property

\[
dZ_w(k) dZ_{w'}(k') = S_{w w'}(k') \delta(k' - k) \, dk \, dk' \]

where \( \delta(.) \) = the Dirac delta function; \( w_1 \) and \( w_2 \) = two arbitrary stationary random functions; the superscript \( c \) denotes the complex conjugate; and \( S_{w w'}(k) \) = the cross-spectral density function or power spectrum. This spectral representation is unique and provides a convenient formalism for treating stationary random variables. The covariance function of the real-valued stationary process \( w(x) \) can be written in terms of the spectral representation as follows

\[
R_{w}(\xi) = \int_{-\infty}^{+\infty} e^{ik \xi} S_{w w}(k) \, dk
\]

which becomes the variance \( \sigma_w^2 \) when \( \xi = 0 \). This result is the classical Fourier transform relationship between the covariance and the spectrum. For this analysis we assume the following commonly used simple exponential decay function

\[
R_{w}(\xi) = \sigma_w^2 \exp\left(-\frac{\xi}{\lambda_w}\right)
\]

which has the power spectrum

\[
S_{w w}(k) = \frac{\sigma_w^2}{\pi} \frac{\lambda_w}{1 + k^2 \lambda_w^2}
\]

where \( \sigma_w^2 \) and \( \lambda_w \) = the variance and correlation scale of the \( w(x) \) process, respectively.
Stochastic Perturbation Equation and Spectral Solutions

With our random representation for river geometry, the governing equation for flows in natural rivers becomes a stochastic differential equation. Subtracting (6) from (4), and neglecting second- and higher-order terms, we obtain the following equation relating perturbation in flow to that in river geometry

\[ 2\delta A' + F_r \frac{dA'}{dx} - \frac{dh'}{dx} = \frac{dz'}{dx} - \delta R' \]  \hspace{1cm} (13)

The analysis to this point applies to rivers with general arbitrary mean cross-sectional shape. Further analysis requires specification of the functional form of the hydraulic radius, which depends on the shape of the mean river cross section. As an illustration, we assume in what follows that the river in question is wide and shallow, which is usually the case for natural rivers in flat areas downstream of large catchments. In this case, the hydraulic radius is approximately equal to flow depth, or, \( R = h \), and the first-order perturbation in dimensionless cross-sectional area is \( A' = h' + B' \).

Eq. (13) then reduces, after some algebraic manipulations, to

\[ (1 - F_r) \frac{dh'}{dx} - 3\delta h' = 2\delta B' + F_r \frac{dB'}{dx} - \frac{dz'}{dx} \]  \hspace{1cm} (14)

Eq. (14) is a linear stochastic differential equation with constant coefficients. The spectral representation theorem summarized in the previous section can be conveniently applied to obtain its solution. Applying (8) for \( B', h' \), and \( z' \) in (14), gives

\[ \int_{-\infty}^{\infty} [ik(1 - F_r) - 3\delta] e^{ikx} dZ_h = \int_{-\infty}^{\infty} (2\delta + ikF_r) e^{ikx} dZ_B - ik e^{ikx} dZ_z \]  \hspace{1cm} (15)

By the uniqueness of the representation theorem (Priestley 1981), (15) holds only if

\[ dZ_h = \frac{ik dZ_z - (2\delta + ikF_r) dZ_B}{3\delta - ik(1 - F_r)} \]  \hspace{1cm} (16)

which relates the complex amplitude of flow depth to that of width \( dZ_B \) and bed elevation \( dZ_z \). The spectrum of flow depth and cross-spectra of flow depth with width and bed elevation can then be obtained using the property-of-representation theorem, given in (9) as

\[ S_{hh} = \frac{S_{hh}(4\delta^2 + F_r^2 k^2)}{9\delta^2 + k^2(1 - F_r)^2} \]  \hspace{1cm} (17)

\[ S_{hb} = \frac{S_{hb}(-6\delta^2 + 2i\delta k + iF_r^2 k + F_r^2 k^2 - F_r^2 k^2)}{9\delta^2 + k^2(1 - F_r)^2} \]  \hspace{1cm} (18)

\[ S_{hz} = \frac{S_{hz}(3i\delta k - k^2 + F_r^2 k^2)}{9\delta^2 + k^2(1 - F_r)^2} \]  \hspace{1cm} (19)

Here, \( S_{rb} \) has been assumed to be zero or equivalently, the cross-correlation between river width \( B \) and bed elevation \( z \) has been assumed to be small, and thus is neglected. The cross-spectra of \( B \) with \( dh/dx \) and \( h \) with \( dz/dx \) are related to \( S_{rb} \) and \( S_{hz} \) by

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\[ S_{hdx} = -ikS_{hx} \]  \hspace{1cm} \text{(20)}
\[ S_{hd} = -ikS_{hx} \]  \hspace{1cm} \text{(21)}

Noting the relationship between flow velocity and cross-sectional area, and making use of (17) and (18), we can similarly relate the spectrum of flow velocity to the input geometrical spectrum by
\[ S_{uu} = \frac{S_{uu}(\bar{u}^2 + k^2)}{9s^2 + k^2(1 - F_2)^2} \]  \hspace{1cm} \text{(22)}

Having obtained all the necessary flow spectra, we can proceed to evaluate the statistics of the flow variables. The variances of flow depth and velocity, and covariances between flow depth and river width or bed elevation, can be calculated from (10). By integrating the spectra \( S_{uu}, S_{uw}, S_{wh}, \) and \( S_{hh} \) over the wave number domain, the following variance expressions are obtained
\[ \sigma_h^2 = \frac{\sigma_y^2}{1 - F_2^2(3\delta\lambda_y + [1 - F_2])} + \frac{\sigma_h^2(3F_y^2 + 4\delta\lambda_y[1 - F_2])}{3[1 - F_2^2(3\delta\lambda_y + [1 - F_2])]}, \]  \hspace{1cm} \text{(23)}
\[ \sigma_u^2 = \frac{\sigma_y^2}{1 - F_2^2(3\delta\lambda_y + [1 - F_2])} + \frac{\sigma_h^2(3 + 4\delta\lambda_y[1 - F_2])}{3[1 - F_2^2(3\delta\lambda_y + [1 - F_2])]}, \]  \hspace{1cm} \text{(24)}
\[ \sigma_{wh} = \frac{-\sigma_h^2(1 + F_y^2 + 2\delta\lambda_y[1 - F_2])}{1 - F_2^2(3\delta\lambda_y + [1 - F_2])}, \]  \hspace{1cm} \text{(25)}
\[ \sigma_{he} = \frac{-\sigma_h^2}{1 - F_2^2(3\delta\lambda_y + [1 - F_2])}. \]  \hspace{1cm} \text{(26)}

To evaluate the effective resistance from (7), we also need to calculate the correlations of \( B \) with \( dh/dx \) and \( h \) with \( dz/dx \). Integrating (20) and (21) over \( k \), and making use of (18) and (19), gives
\[ h' \frac{dz'}{dx} = \frac{3\delta\sigma_y^2}{1 - F_2^2(3\delta\lambda_y + [1 - F_2])} \]  \hspace{1cm} \text{(27)}
\[ B \frac{dh'}{dx} = \frac{(2 + F_y^2)\delta\sigma_y^2}{1 - F_2^2(3\delta\lambda_y + [1 - F_2])} \]  \hspace{1cm} \text{(28)}

Substitution of (23)–(28) in (7) leads to the final expression for the effective resistance coefficient
\[ f_e = \left[ 1 + \frac{3\delta\sigma_y^2}{1 - F_2^2(3\delta\lambda_y + [1 - F_2])} \right. \]
\[ + \left. \frac{\sigma_h^2(1 + F_y^2 + F_y^2 + 2\delta\lambda_y[1 - F_2])}{1 - F_2^2(3\delta\lambda_y + [1 - F_2])} \right] f \]  \hspace{1cm} \text{(29)}

\text{RESULTS AND COMMENTS}

We have shown in the last section that a simple Chezy equation can be used to describe the mean behavior of flows in natural rivers, based on an effective resistance coefficient given by (29). Deviations from the mean flow condition at any particular location along the river can be statistically quan-
tified in terms of the flow parameter standard deviations given in (23) and (24). It is important to recognize from (23)–(29) that both effective resistance and flow variances are strongly influenced by river cross-sectional nonuniformity, characterized by the statistical parameters \( \sigma_p \), \( \sigma_\epsilon \), \( \lambda_p \), and \( \lambda_\epsilon \), as well as by mean flow conditions, characterized by the mean Froude number. This is clearly different from the simple straight channel situation, in which flow variance is identically zero. In straight channels, flow resistance is independent of flow condition and determined only by relative boundary roughness and mean cross-sectional shape. In natural channels, (29) shows that total resistance includes both skin friction induced by fine-scale boundary roughness, and form resistance induced by large-scale nonuniformities in river bed and width. These two resistance processes are not independent, as has been assumed so in many existing analyses. They are, in fact, constantly interacting. The form resistance is, to first order, proportional to the skin friction resistance, with the proportionality coefficient being a function of river flow and geometrical conditions. To examine how different factors affect mean flows and resistance processes in natural rivers, we have plotted the relative effective resistance \( f_e/f \) and flow standard deviations \( \sigma_p \) and \( \sigma_\epsilon \) versus different characteristic parameters, including the mean Froude number, mean bed slope, and the correlation scales and standard deviations of the river boundary variations. The calculations are based on the following set of nominal parameters: \( F_e = 0.4 \), \( \lambda_p \delta = \lambda_\epsilon \delta = 0.1 \), and \( \sigma_p = \sigma_\epsilon = 0.25 \), unless otherwise specified. The results are summarized in Figs. 1–4.

Figs. 1(a) through 1(c) show contours of relative effective resistance, flow depth standard deviation, and velocity standard deviation plotted against the standard deviations of river width, \( \sigma_p \), and bed elevation, \( \sigma_\epsilon \). As we expect, the effective resistance is always greater than the local skin resistance due to river cross-sectional nonuniformity. Both effective resistance and flow standard deviations increase quadratically with geometrical nonuniformity, as measured by \( \sigma_p \) or \( \sigma_\epsilon \), even though their sensitivities to \( \sigma_p \) and \( \sigma_\epsilon \) are generally different. Fig. 1(b) shows that \( \sigma_p \) is much more sensitive to variations in bed elevation than in river width, at least for the nominal case under consideration. This agrees with our intuition, since flow depth is directly related to bed elevation but only indirectly related to river width.

The correlation scale of boundary variation, as measured by \( \lambda_p \) or \( \lambda_\epsilon \), indicates the distance over which variations of river width or bed elevation tend to persist. Larger correlation scales correspond to statistically slower variation in the river boundary. Therefore, both flow resistance and flow standard deviations generally decrease with increasing \( \lambda_p \) or \( \lambda_\epsilon \), as is shown respectively in Figs. 2 and 3. An exception is shown in Fig. 2(b), in which \( \sigma_p \) increases slowly with \( \lambda_p \) at very small Froude numbers.

An important characteristic of flows in natural rivers is the mean flow dependence of effective resistance. Fig. 4(a)–4(c) show that both relative effective resistance \( f_e/f \) and flow standard deviations are influenced by the mean flow Froude number and mean bed slope, increasing as the Froude number increases or mean bed slope decreases, especially at higher Froude numbers. This reflects the fact that flows in natural rivers become increasingly sensitive to river boundary variations, and that the boundary-nonuniformity–induced resistance becomes more and more important with increasing Froude number, finally dominating relative to boundary skin friction. Note that flow variances and effective resistance become unbounded as the Froude number approaches unity. This resonant behavior corresponds to the physical flow transition from a supercritical state to a subcritical state,
or a hydraulic jump. In fact, local supercritical flow may have occurred well before the mean Froude number exceeded one, causing local hydraulic jumps and rendering the flow equation inapplicable. We may therefore expect our results to be quantitatively meaningful only when the Froude number is well below one.

It is instructive to examine the asymptotic flow behavior for the case of very large correlation scales, which corresponds to smooth natural rivers. The expressions of flow variances and effective resistance in this case are greatly simplified as follows

\[ \sigma_{nh} = \frac{2}{3} \sigma_h^2, \quad \sigma_{hz} = 0; \quad \sigma_h = \frac{2}{3} \sigma_n; \quad \sigma_n = \frac{1}{3} \sigma_h; \]

\[ f_c = \left( 1 + \frac{1}{3} \sigma_n^2 \right) f \] ................................. (30)
Eq. (30) shows that flow statistics and effective resistance are no longer dependent on mean flow, mean bed slope, or the standard deviation of bed elevation. Correlation of flow depth with bed elevation vanishes, i.e., the river's free surface varies in phase with the large-scale or slowly varying fluctuating bed profile, with flow depth remaining constant along the flow (if $\sigma_n = 0$). The variation in river width becomes the only factor that affects large-scale mean flow behavior. Depth correlation with river width is always nonzero and negative. Any variation in river width would force flow to distort in order to satisfy the underlying physical conservation principles, resulting in variations in flow depth, velocity, and therefore nonzero form resistance.

**Conclusions**

In this paper, we have developed a predictive model for mean flow in irregular natural rivers. Practical site-specific applications of this model require information on the statistical parameters of the boundary variations $\sigma_B$, $\sigma_z$, $\lambda_B$, and $\lambda_z$. These are all specific, physically observable quantities in contrast to the nonphysical, empirical calibration constants used in a
conventional regression analysis. The standard deviation and correlation
scale of a river's boundary variation correspond to (but are not equal to)
its wavelength and amplitude in a deterministic sense. These parameters
can be obtained, for example, from a statistical analysis of space series of
observed river width and bed elevation.

There are many different ways to obtain a space series of stream geometrical parameters. Direct measurement is one way. In this case, the re-
quired data-sampling interval depends on the degree of stream irregularity.
Measurements should be taken where there is significant change in stream
cross sections. Remote sensing is another way to obtain stream topographical
data. Various remote sensors are available for surveying rivers, including
the multispectral scanner subsystem (MSS) on Landsat satellites, the very
high resolution radiometer (VHRR) on NOAA meteorological satellites,
and basic multispectral camera arrays flown on high-altitude aircraft such
as the NASA U-2 and RB-57 (Salomonson and Rango 1979). These remote
sensors can be used to determine the detailed longitudinal variation of the
river width \( B(x) \). The property of penetration of blue and green wavelengths
through clear and turbid waters can be used to estimate variations in stream
bottom topography, or bed elevation \( z(x) \). These detailed spatial measure-
ments are particularly useful for estimating correlation scales of stream
boundary variations.

It is necessary at this point to emphasize several assumptions we have
made in this theoretical analysis. The ergodicity assumption of flow and
boundary variations is implicit here. That is, we are assuming that flow
occurring in an ensemble of natural streams with the assigned statistical
properties approximates the real field situation, which involves flow in a
single irregular natural stream (a realization of the stochastic process). This
assumption will be reasonable only if river geometry is statistically uniform
or mean river geometrical statistics are spatially invariant, and the problem
scale is much larger compared with the scale of longitudinal variation of
geometry [see Li and McLaughlin (1991) for an extension of this analysis
to statistically nonuniform or nonstationary problems]. Therefore, ensemble
statistics, such as the mean flow variances and the effective resistance coef-
cefficient developed here, are meaningful only when we are interested in be-
havior on a scale much larger than the correlation scale of the boundary
variations. Another essential assumption made here is that of small per-
urbation, which permits the higher-order terms in \( (5) \) to be neglected. This
assumption may not hold in many situations, particularly for mountain streams
in which flow is usually low and bed variation is relatively high. But we may
still expect the results developed here to be qualitatively correct. Systematic
Monte-Carlo simulations and field applications are needed before we can
have a definitive evaluation of the applicability of this first-order analysis
to highly nonuniform rivers.

APPENDIX. REFERENCES


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APPENDIX I. NOTATION

The following symbols are used in this paper:

\[ A = \text{flow cross-sectional area}; \]
\[ A' = \text{perturbation of cross-sectional area}; \]
\[ \bar{A} = \text{mean cross-sectional area}; \]
\[ B = \text{river width}; \]
\[ B' = \text{perturbation of river width}; \]
\[ \bar{B} = \text{mean river width}; \]
\[ dZ_w = \text{complex amplitude of river width}; \]
\[ dZ_h = \text{complex amplitude of flow depth}; \]
\[ dZ_e = \text{complex amplitude of bed elevation}; \]
\[ dZ_u = \text{complex amplitude of flow velocity}; \]
\[ dZ_w = \text{complex amplitude of } w(x) \text{ process}; \]
\[ F_r = \text{mean flow Froude number}; \]
\[ f = \text{Darcy-Weisbach resistance coefficient}; \]
\[ f_e = \text{effective resistance coefficient}; \]
\[ g = \text{gravitational constant}; \]
\[ h = \text{flow depth}; \]
\[ h' = \text{mean flow depth}; \]
\[ h' = \text{perturbation of flow depth}; \]
\[ i = \text{pure imaginary number}; \]
\[ k = \text{wave number}; \]
\[ Q = \text{volumetric flow discharge}; \]
\[ R = \text{hydraulic radius}; \]
\[ R' = \text{perturbation of hydraulic radius}; \]
\[ \bar{R} = \text{mean hydraulic radius}; \]
\[ R_{w,w_2} = \text{cross-covariance function between } w_1(x) \text{ and } w_2(x); \]
\[ S_f = \text{local resistance coefficient, friction slope}; \]
\[ S_{BB} = \text{power spectrum of width}; \]
\[ S_{bh} = \text{cross-spectrum of width and depth}; \]
\[ S_{bh} = \text{power spectrum of depth}; \]
\[ S_{h_2} = \text{cross-spectrum of depth and bed elevation}; \]
\[ S_{u_2} = \text{power spectrum of velocity}; \]
\[ S_{z_2} = \text{power spectrum of bed elevation}; \]
\[ S_{w_1w_2} = \text{power spectrum between } w_1 \text{ and } w_2; \]
\[ s = \text{bed slope}; \]
\[ \delta = \text{mean bed slope}; \]
\[ u = \text{velocity of flow}; \]
\[ u' = \text{perturbation of flow velocity}; \]
\[ w = \text{stationary random function or process}; \]
\[ \bar{w} = \text{mean of stochastic process } w(x); \]
\[ w' = \text{perturbation of } w, \text{ mean-removed process}; \]
\[ w_1 = \text{stationary random function or process}; \]
\[ w_2 = \text{stationary random function or process}; \]
\[ x = \text{spatial coordinate in streamwise direction}; \]
\[ z = \text{bed elevation}; \]
\[ z' = \text{perturbation of bed elevation}; \]
\[ \delta(.) = \text{Dirac delta function}; \]
\[ \lambda_n = \text{correlation scale of } w(x) \text{ process}; \]
\[ \lambda_z = \text{correlation scale of bed elevation}; \]
\[ \lambda_R = \text{correlation scale of river width}; \]
\[ \sigma_A^2 = \text{variance of cross-sectional area}; \]
\[ \sigma_{AR}^2 = \text{cross-variance between cross-sectional area } A \text{ and hydraulic radius } R; \]
\[ \sigma_R^2 = \text{variance of hydraulic radius}; \]
\[ \sigma_w^2 = \text{variance of } w(x) \text{ process}. \]