4200:225 Equilibrium Thermodynamics

Unit I. Earth, Air, Fire, and Water

Chapter 4. Thermodynamics of Processes

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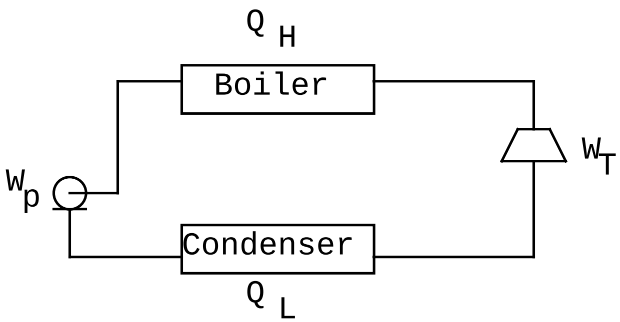
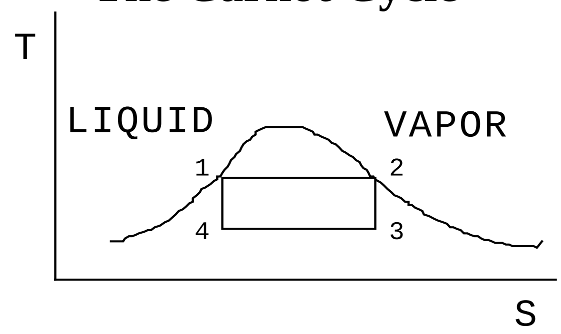
I. Energy and Entropy



The Carnot Cycle



$\eta \equiv -W/Q_H$ **General Formula for the Thermal Efficiency**

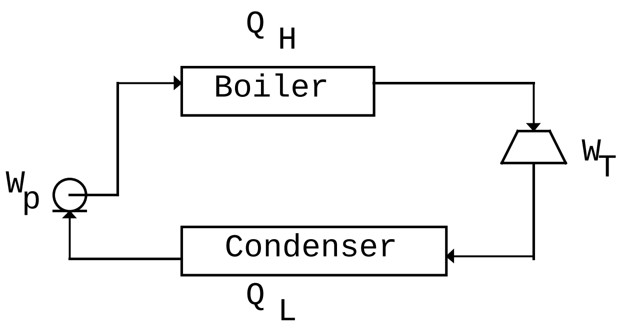
Entire process is reversible: $\Delta S_{1\rightarrow 1}=0 \Rightarrow \frac{Q_H}{T_H}+\frac{Q_L}{T_L}=O \Rightarrow Q_L = -\frac{T_L}{T_H}Q_H$

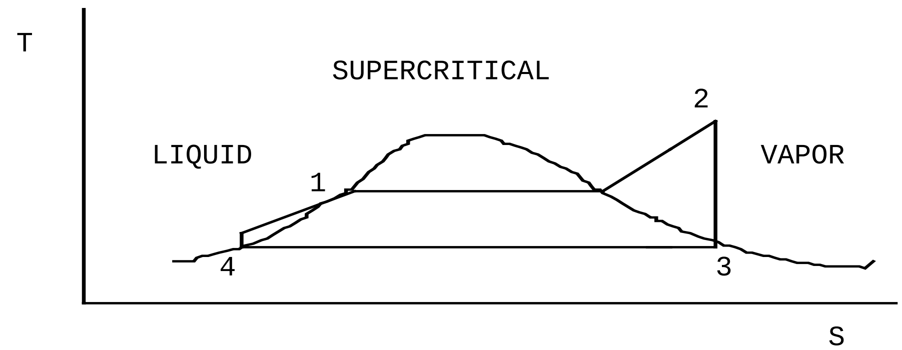
But $-W=Q_H + Q_L = Q_H(1-T_L/T_H)$

$\eta = \frac{-W}{Q_H} = \frac{T_H - T_L}{T_H}$ $\Rightarrow$ Control efficiency by controlling temperature difference.

I. Energy and Entropy

The Basic Rankine Cycle





<u>Advantages:</u>
1. Less wear on turbine and compressor.
2. Compression of liquid is cheap and easy (negligible in most cases).

<u>Disadvantages:</u>
1. Lower thermodynamic efficiency than Carnot or Stirling. This can be somewhat compensated by multiple stages and regeneration.

Example 4.1 Power plant based on the ordinary Rankine cycle

The steam power installation in a chemical plant must satisfy the following requirements:

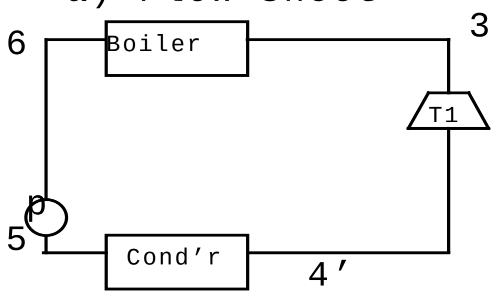
1) 500°C maximum temperature from the boiler.
2) 0.025MPa saturated vapor from the turbine.
3) 1 MW net power output.

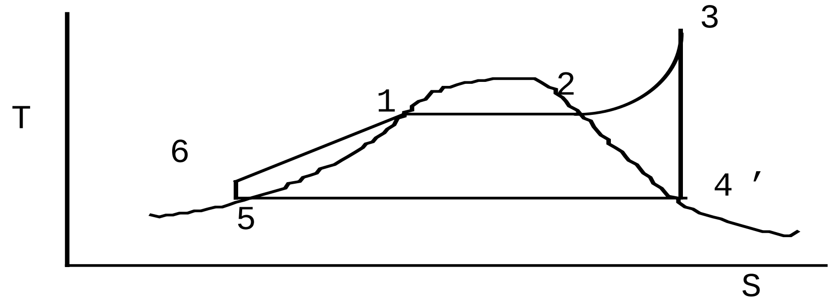
Determine:

a) The operating pressure of the boiler
b) The thermal efficiency
c) The circulation rate

b) T-S Diagram

a) Flow Sheet





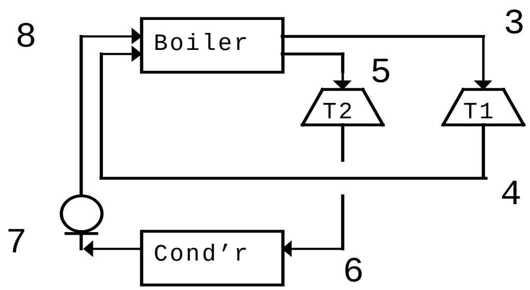
a) Interpolating on the pressure at 500°C and S=7.8314, P = 0.869 Mpa

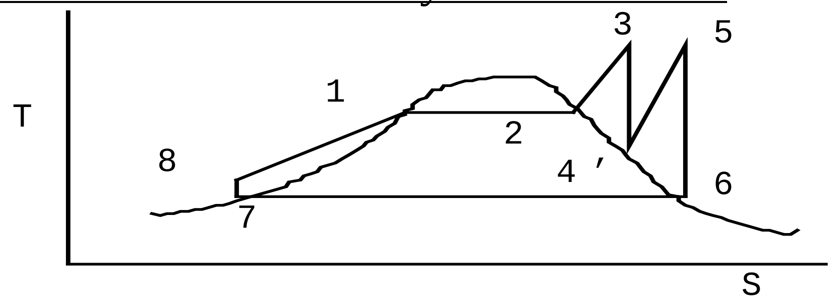
b) The enthalpy at state 3 is interpolated at 500°C and 0.869MPa, H = 3480

W_T = 2618-3480 = -862 J/g; $W_p = \int V dP = V\Delta P$ = 0.79 J/g

$W_{net} = W_T + W_p$ = -861 J/g; Q_L = 2618 - 273 = 2345; η = 861/(2345+861) = 27%

Example 4.3. Power plant based on a Rankine cycle with reheat





Consider the same outlet conditions as above, but interject an extra stage for the pressure drop. That is, consider the case depicted above with stream 3 being at 434°C and 6MPa and stream 5 at 500°C and 0.8 MPa. Compute the thermal efficiency in this case.

434°C, 6MPa⇒H_3 = 3262 J/g; S_3 = 6.6622 J/g-K; State 4' is SatV, 0.8MPa ⇒ $H_{4'}$ = 2769

State 5 is same as previous problem ⇒ H_5 = 3480 J/g, H_6 = 2618 J/g, H_7 = 272.

The pump work has increased because the pressure has increased⇒W_p = 6 J/g, H_8 = 278

$W_{net} = (H_4 - H_3) + (H_6 - H_5) + W_p$ = (2769-3262)+(2618-3480) + 6 = -1349

$Q_{tot} = (H_3 - H_8) + (H_5 - H_{4'})$ = (3262-278)+(3480-2769) = 3695 J/g

η = 36.7 %

This compares to 27% for the cycle without reheat.

Note: The Carnot cycle gives the upper limit of η_{carnot} = (500-65)/773 = 56%

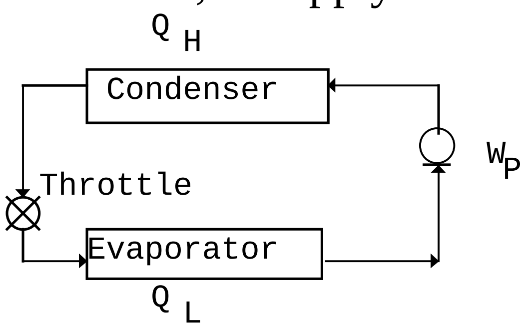
I. Energy and Entropy

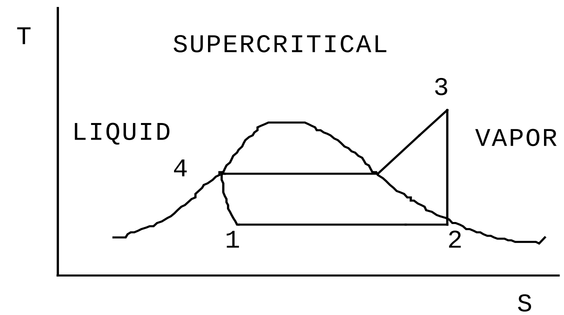
Refrigeration

COP = Q_L/W_{net} $\Rightarrow$ For ***Carnot***: COP = $(Q_H/W_{net})*(Q_L/Q_H) = T_L/(T_H - T_L)$

But the Carnot cycle is not always practical.

Therefore, we apply the **ORDINARY VAPOR COMPRESSION (OVC) CYCLE**





<u>COP for ordinary vapor compression cycle</u>

COP = Q_L/W; $Q_L = (H_2 - H_1)$

Energy balance: $W = \Delta H_{2\rightarrow 3} = (H_3 - H_2)$

$\Rightarrow$ COP = $(H_2 - H_1)/(H_3 - H_2) = (H_2 - H_4)/(H_3 - H_2)$

I. Energy and Entropy

Example 4.5 Refrigeration by vapor-compression cycle

A cold storage room is to be maintained at -15°C and the available cooling water exits the evaporator at 298K. The refrigerant temperature exiting the condenser is to be 30°C. The refrigeration capacity is to be 120,000 Btu/hr (126,500 kJ/hr). (This is the cooling rate required to freeze ten tons of 32°F water to 32°F ice in a 24 hr period. It is known in the trade as ten "tons" of refrigeration.) Freon-134a will be used for the vapor compression cycles. Calculate the COP and recirculation rate (except part a for the following cases:

a) Carnot cycle

b) Ordinary vapor compression cycle.

c) Vapor compression cycle with expansion engine

d) Ordinary vapor compression cycle for which compressor is 80% efficient.

State	H	S	Comment
1	241.5	---	Throttle from 4
1'	235.0	1.1428	Tur from 4, q=.29
2	386.5	1.7414	Sat V, 253K
3'	424	1.7414	$S_3=S_2$, read chart
4	241.5	1.1428	Sat L, 303K

(a) Carnot

$$\text{COP} = \frac{T_L}{(T_H - T_L)} = \frac{253}{(303-253)} = 5.06$$

(b) OVC cycle $\Rightarrow COP = \dfrac{(H_2 - H_1)}{(H_3 - H_2)} = \dfrac{(H_2 - H_4)}{(H_3 - H_2)} = \dfrac{386.5 - 241.5}{424 - 386.5} = 3.87$

$$\dot{m} = \frac{126{,}500}{386.5 - 241.5} = 872 \frac{\text{kg}}{\text{hr}}$$

(c) VC cycle with turbine expansion

$qS_1^V + (1\text{-}q)\, S_1^L = S_4^L = 1.1428 = q(1.7414) + (1\text{-}q)\, 0.8994 \;\Rightarrow\; q = 0.289 \;\Rightarrow H_1' = 235$

$$\Rightarrow COP = \frac{(H_2 - H_1^{rev})}{(H_3 - H_2) + (H_1' - H_4)} = \frac{386.5 - 235.0}{(424 - 386.5) + (235.0 - 241.5)} = 4.89$$

$$\dot{m} = \frac{126{,}500}{386.5 - 235.0} = 835 \frac{\text{kg}}{\text{hr}}$$

(d) States (1), (2) & (4) are the same as in (b)

The only difference is that $W = (424\text{-}386.5)/0.8 = 46.9$

$\Rightarrow COP = \dfrac{H_2 - H_4}{H_3 - H_2} = \dfrac{386.5 - 241.5}{46.9} = 3.09$ and $m = 872$ kg/hr

Note: Choice of refrigerant dictated by

(1) Toxicity (Freon-12, 22 are bad for ozone and is being phased out)

(2) vapor pressure > atmospheric @ T_{fridge}

(3) vapor pressure not too high @ T_H

(4) high heat of vaporization per unit mass

(5) small Cp/Cv

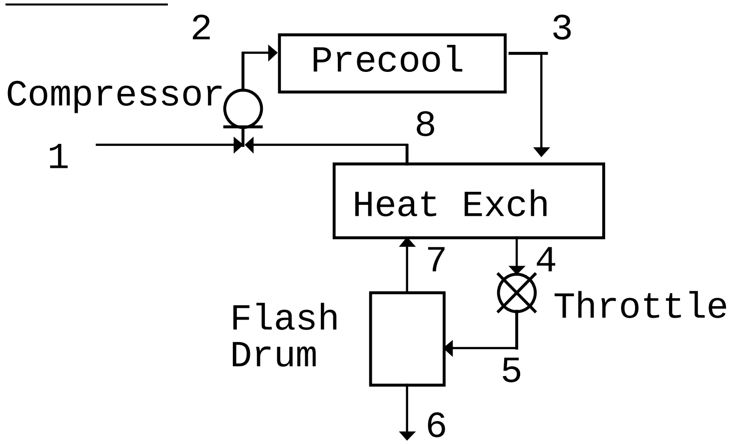
(6) high heat transfer coefficients in vapor and liquid

I. Energy and Entropy

Example 4.6. Liquefaction of methane by the Linde process.

Natural gas, assumed here to be pure methane, is liquefied in a simple Linde process. Compression is to 60 bar and precooling is to 300 K. The separator is maintained at a pressure of 1 bar and unliquefied gas at this pressure leaves the cooler at 295 K.

Solution



a) Bottom half E-Bal: $H_3 - [qH_8 + (1-q)H_6] = 0$

$$\Rightarrow q = \frac{H_3 - H_6}{H_8 - H_6} = \frac{H(60{,}300) - H(1, SATL)}{H(1{,}295) - H(1, SATL)}$$

$$= \frac{1130 - 284}{1195 - 284} = 0.9286 \Rightarrow 7.14\% \textit{liquefied}$$

b) E-Bal around HXER: $H_4 - H_3 = q(H_8 - H_7)$

= .9286(1195-796.1)= -370.5

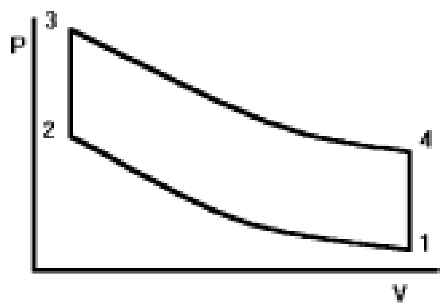
$\Rightarrow H_4$=780 @ 60 BAR $\Rightarrow$ chart gives 203K

I. Energy and Entropy

Example 4.8 Thermal Efficiency of the Otto Engine

Determine the thermal efficiency of the air-standard Otto cycle as a function of the specific heat ratio γ (= Cp/Cv) and the compression ratio $r=V1/V2$.

Solution:



$QH = Cv(T3-T2)$

$QL = Cv(T1-T4)$

$W=QH+QL = Cv(T3-T2+T1-T4)$

$\eta = Cv(T3-T2+T1-T4)/[Cv(T3-T2)] + 1-(T3-T2)/(T3-T2) = 1 + (T1-T4)/(T3-T2)$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{R/Cv} ; \frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{R/Cv} = \left(\frac{V_2}{V_1}\right)^{R/Cv}$$

$\Rightarrow$ $T4 = T3\, r^{-R/Cv}$; $T1 = T2\, r^{-R/Cv}$

$\eta = 1 - r^{-R/Cv} = 1 - r^{(1-\gamma)}$

I. Energy and Entropy

In a large refrigeration plant it is necessary to compress a fluid which we will assume to be an ideal gas with constant heat capacity, from a low pressure P1 to a much higher pressure P2. If the compression is to be done in two stages, first compressing the gas from P1 to P*, then cooling the gas at constant pressure down to the compressor inlet temperature T1, and then compressing the gas to P2, what should the value of the intermediate pressure be to accomplish the compression with minimum work?

Solution:

E-Bal: $\Delta H = Q + W = W = Cp\Delta T$

S-Bal: $\Delta S = 0 \Rightarrow T^*/T_1 = (P^*/P_1)^{R/Cp}$; $T_2/T_1 = (P_2/P^*)^{R/Cp}$

$W_{tot} = Cp(T_2 - T_1) + Cp(T^* - T_1) = CpT_1\{[(P_2/P^*)^{R/Cp} - 1] + [(P^*/P_1)^{R/Cp} - 1]$

To maximize function, set derivative to zero.

$$\frac{dW}{dP^*} = CpT_1\left\{\frac{-R/Cp}{P^*}\left(\frac{P_2}{P^*}\right)^{R/Cp} + \frac{R/Cp}{P^*}\left(\frac{P^*}{P_1}\right)^{R/Cp}\right\} = 0$$

$$\left(\frac{P_2}{P^*}\right)^{R/Cp} = \left(\frac{P^*}{P_1}\right)^{R/Cp} \Rightarrow P^* = \sqrt{P_2 P_1}$$

Note: Equal compression ratios per stage is optimal for multistage compressors also.