Example 4.9 Thermal efficiency of a Diesel engine

Develop an expression for the thermal efficiency of the air-standard diesel cycle as a function of the compression ratio \( rc = V_1/V_2 \) and the expansion ratio \( re = V_4/V_3 \). Assume the working fluid is an ideal gas, and the volume effect of moles of gas generated is small relative to the effect of heating from combustion.

**Solution:** This process is a little more complicated than the Otto cycle because of the heat addition at constant pressure. Since the volume is changing, the energy balance for the combustion is:

\[
Q_H + W_{S,comb} = dU \Rightarrow \Delta U - W_{S,comb} = U_3 - U_2 = (U + PV) = \Delta H
\]

\[
\Rightarrow Q_H = \Delta H = C_p(T_3 - T_2)
\]

\[
Q_C = C_V(T_1 - T_4)
\]

\[
-W_{S,net}/Q_H = (Q_H + Q_C)/Q_H = 1 + \frac{C_V(T_1 - T_4)}{C_p(T_3 - T_2)} = 1 + \frac{1}{\gamma} \left[ \frac{T_1}{T_3 - T_2} - \frac{T_4}{T_3 - T_2} \right]
\]

\[
\frac{T_3 - T_2}{T_1} = \frac{T_3 - T_2}{T_1} = \frac{P_3 V_3}{P_2 V_2} = rc^{\gamma - 1} = rc^{\gamma - 1} / re - rc^{\gamma - 1}
\]

\[
\frac{T_4 - T_3}{T_4} = \frac{T_4 - T_3}{T_4} = \frac{P_4 V_4}{P_3 V_3} = re^{\gamma - 1} - \frac{P_4 V_4}{P_3 V_3} = re^{\gamma - 1} - re^{\gamma - 1} / rc
\]

\[
\eta = 1 + \frac{1}{\gamma} \left[ \frac{1}{rc^{\gamma - 1} / re - rc^{\gamma - 1}} - \frac{1}{re^{\gamma - 1} - re^{\gamma - 1} / rc} \right]
\]

\[
= 1 + \frac{1}{\gamma} \left[ \frac{rc}{re^{\gamma - 1} - re \cdot rc^{\gamma - 1}} - \frac{rc}{rc \cdot re^{\gamma - 1} - re^{\gamma - 1}} \right]
\]